Official nomenclature for material functions describing the response of a viscoelastic fluid to various shearing and extensional deformations

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I. INTRODUCTION

In the spring of 1981, the Executive Committee of the Society appointed an *ad hoc* nomenclature committee to recommend standard names and symbols for material functions arising in the study of nonlinear viscoelasticity and of extensional flows. Two previous nomenclature committees[1,2] had prepared nomenclature lists for linear viscoelastic functions and for viscometric flows, and the recommendations of those committees are summarized in Tables I and II. However, the increased use of large transient shear flows and extensional flows to study viscoelastic materials made it desirable to augment these lists.

A tentative proposal was prepared by the Committee and sent to members of the Society at the end of 1982 along with a request for corrections and comments. The Committee studied the responses to this request and prepared a final report that was

TABLE I. Society of Rheology nomenclature for steady simple shear.

Quantity	Symbol	S.I. units	CGS units
Direction of flow	x_1 or x	m	cm
Direction of velocity gradient	x_2 or y	m	cm
Neutral direction	x_3 or z	m	cm
Shear stress	σ	Pa	dyn cm^{-2}
Shear strain	γ	–	–
Shear rate	$\dot{\gamma}$	s^{-1}	s^{-1}
Viscosity	η	Pa s	P (poise)
First normal stress function	N_1	Pa	dyn cm^{-2}
Second normal stress function	N_2	Pa	dyn cm^{-2}
First normal stress coefficient	Ψ_1	Pa s^2	dyn s^2 cm^{-2}
Second normal stress coefficient	Ψ_2	Pa s^2	dyn s^2 cm^{-2}
Limiting viscosity at zero shear rate	η_0	Pa s	P
Limiting viscosity at infinite shear rate	η_∞	Pa s	P
Viscosity of solvent or of continuous medium	η_s	Pa s	P
Relative viscosity (η/η_s)	η_r	–	–
Specific viscosity ($\eta_r - 1$)	η_{sp}	–	–
Intrinsic viscosity	$[\eta]$	m^3 kg^{-1}	cm^3 g^{-1}

[a)]Initially published in J. Rheol. **28**, 181–195 (1984) and presented here with corrections and additions identified by D. M. Husband, J. Rheol. **36**, 409–410 (1992).

TABLE II. Society of Rheology nomenclature for linear viscoelasticity.

Quantity	Symbol	S.I. units
Simple shear		
Shear strain	γ	–
Shear modulus (modulus of rigidity)	G	Pa
Shear relaxation modulus	$G(t)$	Pa
Shear compliance	J	Pa^{-1}
Shear creep compliance	$J(t)$	Pa^{-1}
Equilibrium shear compliance	J_e	Pa^{-1}
Steady-state shear compliance	J_s^0	Pa^{-1}
Complex viscosity	$\eta^*(\omega)$	Pa s
Dynamic viscosity	$\eta'(\omega)$	Pa s
Out-of-phase component of η^*	$\eta''(\omega)$	Pa s
Complex shear modulus	$G^*(\omega)$	Pa
Shear storage modulus	$G'(\omega)$	Pa
Shear loss modulus	$G''(\omega)$	Pa
Complex shear compliance	$J^*(\omega)$	Pa^{-1}
Shear storage compliance	$J'(\omega)$	Pa^{-1}
Shear loss compliance	$J''(\omega)$	Pa^{-1}
Tensile extension		
Strain (True strain)	ε	–
Young's modulus	E	Pa
Tensile relaxation modulus	$E(t)$	Pa
Tensile compliance	D	Pa^{-1}
Tensile creep compliance	$D(t)$	Pa^{-1}

presented to the Executive Committee at the Knoxville meeting in October 1983. Presented here is the nomenclature approved at that time.

The Committee wishes to acknowledge the helpful suggestions received from a number of members. Particularly useful were the thoughtful and extensive comments received from C. J. S. Petrie, J. Meissner, and W. Philippoff.

This presentation commences with descriptions of a number of flows that have been introduced in the last 15 years for the study of viscoelastic materials. A number of material functions are defined, and symbols are specified for these. The names of the newly defined material functions and the corresponding symbols are listed in Tables III and IV.

Members of the Nomenclature Committee:

R. B. Bird
J. M. Dealy (Chairman)
W. W. Graessley
K. F. Wissbrun

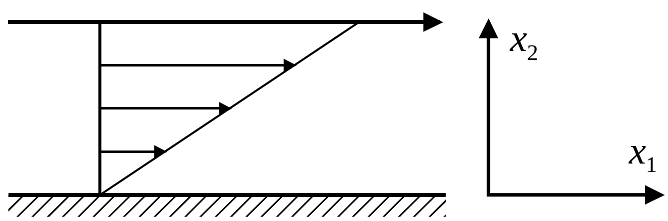
TABLE III. Society of Rheology nomenclature for nonlinear viscoelasticity in shear.

Quantity	Symbol	S.I. units
Start-up flow		
Shear stress growth function	$\sigma^{+}(t,\dot{\gamma})$	Pa
Shear stress growth coefficient	$\eta^{+}(t,\dot{\gamma})$	Pa s
First normal stress growth function	$N_1^{+}(t,\dot{\gamma})$	Pa
First normal stress growth coefficient	$\Psi_1^{+}(t,\dot{\gamma})$	Pa s^2
Second normal stress growth function	$N_2^{+}(t,\dot{\gamma})$	Pa
Second normal stress growth coefficient	$\Psi_2^{+}(t,\dot{\gamma})$	Pa s^2
Cessation of steady shear flow		
Shear stress decay function	$\sigma^{-}(t,\dot{\gamma})$	Pa
Shear stress decay coefficient	$\eta^{-}(t,\dot{\gamma})$	Pa s
First normal stress decay function	$N_1^{-}(t,\dot{\gamma})$	Pa
First normal stress decay coefficient	$\Psi_1^{-}(t,\dot{\gamma})$	Pa s^2
Second normal stress decay function	$N_2^{-}(t,\dot{\gamma})$	Pa
Second normal stress decay coefficient	$\Psi_2^{-}(t,\dot{\gamma})$	Pa s^2
Step strain		
Shear stress relaxation function	$\sigma(t,\gamma)$	Pa
Shear stress relaxation modulus	$G(t,\gamma)$	Pa
First normal stress relaxation function	$N_1(t,\gamma)$	Pa
Second normal stress relaxation function	$N_2(t,\gamma)$	Pa
Creep and recoil		
Shear creep compliance	$J(t,\sigma)$	Pa^{-1}
Steady-state compliance	$J_s(\sigma)$	Pa^{-1}
Recoil strain	$\gamma_r(t,\sigma)$	–
Recoil function	$R(t,\sigma)$	Pa^{-1}
Ultimate recoil	$\gamma_\infty(\sigma)$	–
Ultimate recoil function	$R_\infty(\sigma)$	Pa^{-1}
Superposed steady and oscillatory shear		
Parallel complex viscosity	$\eta_{\parallel}^{*}(\omega,\dot{\gamma}_m)$	Pa s
Orthogonal complex viscosity	$\eta_{\perp}^{*}(\omega,\dot{\gamma}_m)$	Pa s

II. TRANSIENT SHEAR FLOWS

All the tests and material functions defined below are based on a homogeneous simple shear flow, described as follows:

$$v_1 = \dot{\gamma}\, x_2$$
$$v_2 = v_3 = 0$$



A. Start-up flow

A sample, initially in its rest state, is subjected to a constant shear rate $\dot{\gamma}$ at time t = 0. The quantities measured are the shear stress σ and the normal stress differences N_1 and N_2, all as functions of t and $\dot{\gamma}$. The measurable material functions are as follows:

TABLE IV. Society of Rheology nomenclature for nonlinear viscoelasticity in extension.

Quantity	Symbol	S.I. units
Tensile (simple) extension		
Tensile strain	ε	–
Strain rate (≥0)	$\dot{\varepsilon}$	s^{-1}
Net tensile stress	σ_E	Pa
Tensile stress growth function	σ_E^+	Pa
Tensile stress growth coefficient	$\eta_E^+(t,\dot{\varepsilon})$	Pa s
Tensile viscosity	η_E	Pa s
Tensile stress decay coefficient	$\eta_E^-(t,\dot{\varepsilon})$	Pa s
Tensile creep compliance	$D(t,\sigma_E)$	Pa^{-1}
Steady-state tensile compliance	$D_s(\sigma_E)$	Pa^{-1}
Tensile recoil function	$\varepsilon_r(t,\dot{\varepsilon})$	–
Tensile recoil coefficient	$S(t,\dot{\varepsilon})$	Pa^{-1}
Ultimate tensile recoil	$\varepsilon_\infty(\sigma_E)$	–
Ultimate tensile recoil coefficient	$S_\infty(\sigma_E)$	Pa^{-1}
Tensile stress relaxation modulus	$E(t,\varepsilon)$	Pa
Biaxial extension (symmetric)		
Biaxial strain/strain rate (≥0)	$\varepsilon_B / \dot{\varepsilon}_B$	$–/s^{-1}$
Net stretching stress	σ_B	Pa
Biaxial stress growth function	σ_B^+	Pa
Biaxial stress growth coefficient	$\eta_B^+(t,\dot{\varepsilon}_B)$	Pa s
Biaxial stress decay coefficient	$\eta_B^-(t,\dot{\varepsilon}_B)$	Pa s
Biaxial extensional viscosity	η_B	Pa s
Asymmetric extension		
Largest principal strain rate	$\dot{\varepsilon}$	s^{-1}
Strain ratio	m	–
First net stretching stress	$\sigma_1^{(m)}$	Pa
Second net stretching stress	$\sigma_2^{(m)}$	Pa
First extensional viscosity	$\eta_1^{(m)}$	Pa
Second extensional viscosity	$\eta_2^{(m)}$	Pa

Shear stress growth function:

$$\sigma^+(t,\dot{\gamma}) \equiv \sigma$$

Shear stress growth coefficient:

$$\eta^+(t,\dot{\gamma}) \equiv \sigma^+ / \dot{\gamma}$$

First normal stress growth function:

$$N_1^+(t,\dot{\gamma}) \equiv \sigma_{11} - \sigma_{22}$$

First normal stress growth coefficient:

$$\Psi_1^+(t,\dot{\gamma}) \equiv N_1^+ / \dot{\gamma}^2$$

Second normal stress growth function:

$$N_2^+(t,\dot{\gamma}) \equiv \sigma_{22} - \sigma_{33}$$

Second normal stress growth coefficient:

$$\Psi_2^+(t,\dot{\gamma}) \equiv N_2^+ / \dot{\gamma}^2$$

Note 1: If the stresses approach steady-state values, we can make the following correspondences with the viscometric functions:

$$\lim_{t\to\infty}\left[\eta^{+}(t,\dot\gamma)\right]=\eta(\dot\gamma)\,,$$

$$\lim_{t\to\infty}\left[N_1^{+}(t,\dot\gamma)\right]=N_1(\dot\gamma)\,,$$

$$\lim_{t\to\infty}\left[N_2^{+}(t,\dot\gamma)\right]=N_2(\dot\gamma)\,.$$

Note 2: If linear viscoelastic behavior is exhibited by the fluid of interest at very low $\dot\gamma$, the following relationships are valid:

$$\lim_{\dot\gamma\to 0}\left[\eta^{+}(t,\dot\gamma)\right]=\eta^{+}(t)\,,$$

$$\lim_{\dot\gamma\to 0}\left[\Psi_1^{+}(t,\dot\gamma)\right]=\Psi_1^{+}(t)\,,$$

$$\lim_{\dot\gamma\to 0}\left[\Psi_2^{+}(t,\dot\gamma)\right]=\Psi_2^{+}(t)\,.$$

B. Cessation of steady shear flow

A fluid that has been subjected to steady simple shear flow at a rate $\dot\gamma$, until its stresses are steady, is brought suddenly to rest at time t = 0. The stresses are monitored as functions of time, and the measurable material functions are as follows:

Shear stress decay function:

$$\sigma^{-}(t,\dot\gamma)\equiv\sigma$$

Shear stress decay coefficient:

$$\eta^{-}(t,\dot\gamma)\equiv\sigma^{-}/\dot\gamma$$

First normal stress decay function:

$$N_1^{-}(t,\dot\gamma)\equiv\sigma_{11}-\sigma_{22}$$

First normal stress decay coefficient:

$$\Psi_1^{-}(t,\dot\gamma)\equiv N_1^{-}/\dot\gamma^{2}$$

Second normal stress decay function:

$$N_2^{-}(t,\dot\gamma)\equiv\sigma_{22}-\sigma_{33}$$

Second normal stress decay coefficient:

$$\Psi_2^{-}(t,\dot\gamma)\equiv N_2^{-}/\dot\gamma^{2}$$

Note: If the fluid of interest exhibits linear viscoelasticity in the limit of small shear rates, the following relationships are valid:

$$\lim_{\dot\gamma\to 0}\left[\eta^{-}(t,\dot\gamma)\right]=\eta^{-}(t)\,,$$

$$\lim_{\dot\gamma\to 0}\left[\Psi_1^{-}(t,\dot\gamma)\right]=\Psi_1^{-}(t)\,,$$

$$\lim_{\dot{\gamma} \to 0} \left[\Psi_2^-(t, \dot{\gamma}) \right] = \Psi_2^-(t) \,.$$

C. Step strain

A material initially in its rest state is subjected to a sudden shear strain of magnitude γ at time t = 0, and the stresses are observed as functions of time.

Shear stress relaxation function:

$$\sigma(t, \gamma) \equiv \sigma$$

Shear stress relaxation modulus:

$$G(t, \gamma) \equiv \sigma / \gamma$$

Note: In the case of linear viscoelastic behavior,

$$\lim_{\gamma \to 0} \left[G(t, \gamma) \right] = G(t)$$

First normal stress relaxation function:

$$N_1(t, \gamma) \equiv \sigma_{11} - \sigma_{22}$$

Second normal stress relaxation function:

$$N_2(t, \gamma) \equiv \sigma_{22} - \sigma_{33} \,.$$

D. Creep

A material initially at rest is subjected to a constant shear stress σ at time t = 0. The shear strain γ is monitored as a function of time.

Shear creep compliance:

$$J(t, \sigma) \equiv \gamma / \sigma$$

Alternatively, one can monitor the shear rate as a function of time.

Shear creep rate decay function:

$$\dot{\gamma}^-(t, \sigma) \equiv \dot{\gamma}$$

The relationship between the stress and shear rate is defined as the shear creep rate coefficient:

$$\eta_c^+(t, \sigma) \equiv \sigma / \dot{\gamma}^- \,.$$

The shear creep rate coefficient is the quantity determined in creep which is analogous to the shear stress growth coefficient $\eta^+(t, \dot{\gamma})$ determined in a constant shear rate test.

If the material of interest is a fluid, so that the strain ultimately becomes linear with time, the compliance curve can be extrapolated to t = 0 to determine the steady-state compliance J_s. Thus, in the linear portion of the curve, the shear creep rate coefficient becomes constant, and the shear creep compliance is given by

$$J(t, \sigma) = J_s(\sigma) + t/\eta \,,$$

where $\eta \equiv \eta_c^+(\sigma)$ is evaluated at the shear rate corresponding to σ.

Note: When the material of interest exhibits linear viscoelastic behavior at small shear stresses, we have:

$$\lim_{\sigma \to 0} \left[J(t, \sigma) \right] = J(t)$$

and

$$\lim_{\sigma \to 0} \left[\eta_c^+(t,\sigma) \right] = \eta_c^+(t),$$

and, after the strain becomes a linear function of time:

$$J(t) = J_s^0 + t/\eta_0 ,$$

where J_s^0 is the steady-state compliance in the limiting case of small stress and $\eta_0 \equiv \eta_c^+$ is the limiting viscosity at zero shear rate.

E. Recoil

A fluid in which the shear stress σ and the shear rate $\dot{\gamma}$ are constant in time has its shear stress reduced to zero at time $t = 0$ while being constrained in the x_2 direction. The recoil strain is monitored as a function of time, and is considered positive when it is in a direction opposite to that of the original shearing motion:

Recoil strain:

$$\gamma_r \equiv \gamma(0) - \gamma(t)$$

Recoil function:

$$R(t,\sigma) \equiv \gamma_r/\sigma$$

Ultimate recoil:

$$\gamma_\infty(\sigma) = \lim_{t \to \infty} \left[\gamma_r(t,\sigma) \right]$$

Ultimate recoil function:

$$R_\infty(\sigma) \equiv \lim_{t \to \infty} \left[R(t,\sigma) \right]$$

Note 1: Since $\sigma = \eta\dot{\gamma}$ at $t = 0$, the strain rate can be used in place of the stress as an independent variable.

Note 2: If linear viscoelastic behavior is exhibited in the limit of small initial stress, we have:

$$\lim_{\sigma \to 0} \left[R(t,\sigma) \right] = R(t) = J(t) - t/\eta_0 ,$$

$$\lim_{\sigma \to 0} \left[R_\infty(\sigma) \right] \equiv R_\infty^0 = J_s^0 .$$

Note 3: Recoil experiments are sometimes performed by releasing the stress in a creep experiment at some time t_0, *before* the strain has become linear in time. In this case the functions defined above can still be used, but the new independent variable must be noted. For example,

$$\gamma_r(t - t_0, t_0, \sigma) \equiv \gamma(t_0) - \gamma(t) .$$

F. Superposed parallel steady and oscillatory shear flow

The shear rate is the sum of a constant (mean) value ($\dot{\gamma}_m$) and an oscillatory component

$$\dot{\gamma}(t) = \dot{\gamma}_m + \gamma_0 \omega \cos \omega t .$$

If the amplitude of oscillation, γ_0, is sufficiently small that the resulting shear stress is a sum of a mean value σ_m and a sinusoidal component, we have:

$$\sigma_m = \eta(\dot{\gamma}_m)\,\dot{\gamma}_m\,,$$

$$\sigma = \sigma_m + \sigma_0 \sin(\omega t + \delta)\,,$$

where σ_0 is the amplitude of the sinusoidal component and δ is the mechanical loss angle.

The parallel complex viscosity has the following real and imaginary components:

$$\eta'_{||}(\omega, \dot{\gamma}_m) \equiv (\sigma_0/\omega\gamma_0)\sin\delta\,,$$

$$\eta''_{||}(\omega, \dot{\gamma}_m) \equiv (\sigma_0/\omega\gamma_0)\cos\delta\,.$$

G. Superposed orthogonal steady and oscillatory shear flow

In this case the steady shear is in the 1–2 plane, while the oscillatory component is in the 2–3 plane:

$$v_1 = \dot{\gamma}_m\, x_2\,,$$

$$v_2 = 0\,,$$

$$v_3 = (\gamma_0 \omega \cos\omega t)\, x_2\,.$$

In other words, the rate of deformation tensor* has the following form:

$$\begin{bmatrix} 0 & \dot{\gamma}_m & 0 \\ \dot{\gamma}_m & 0 & (\gamma_0\omega\cos\omega t) \\ 0 & (\gamma_0\omega\cos\omega t) & 0 \end{bmatrix}.$$

If the amplitude of oscillation, γ_0, is sufficiently small that the shear stress component σ_{23} is sinusoidal, we have:

$$\sigma_{23} = \sigma_0 \sin(\omega t + \delta)\,,$$

and the orthogonal complex viscosity has the following real and imaginary components:

$$\eta'_{\perp}(\omega, \dot{\gamma}_m) \equiv (\sigma_0/\omega\gamma_0)\sin\delta\,,$$

$$\eta''_{\perp}(\omega, \dot{\gamma}_m) \equiv (\sigma_0/\omega\gamma_0)\cos\delta\,.$$

III. EXTENSIONAL FLOWS

A. Tensile (simple) extension

The material functions defined below are based on the homogeneous simple extension of a fluid with constant density.

$$v_1 = \dot{\varepsilon}\, x_1\,,$$

$$v_2 = -\tfrac{1}{2}\dot{\varepsilon}\, x_2\,,$$

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* The rate of deformation tensor is defined here as $(\nabla\mathbf{v} + \nabla\mathbf{v}^T)$.

$$v_3 = -\tfrac{1}{2}\dot{\varepsilon}\, x_3 ,$$

where $\dot{\varepsilon} \geq 0$.

Note 1: This is an axisymmetric flow and can be described alternatively in terms of cylindrical coordinates:

$$v_z = \dot{\varepsilon}\, z ,$$

$$v_r = -\tfrac{1}{2}\dot{\varepsilon}\, r .$$

Note 2: The parameter $\dot{\varepsilon}$ appearing in the velocity distribution is the true strain rate. Thus, ε is the true strain, defined as

$$\varepsilon \equiv \ln(L/L_0) .$$

1. Tensile start-up flow

A sample initially in its rest state is subjected to a constant extensional (true) strain rate $\dot{\varepsilon}$ at time $t = 0$. The "net tensile stress" σ_E is monitored as a function of time.

$$\sigma_E \equiv \sigma_{11} - \sigma_{22} = \sigma_{11} - \sigma_{33} = \sigma_{zz} - \sigma_{rr} .$$

Tensile stress growth coefficient:

$$\eta_E^+(t,\dot{\varepsilon}) \equiv \sigma_E / \dot{\varepsilon}$$

Tensile viscosity:

$$\eta_E(\dot{\varepsilon}) = \lim_{t \to \infty} \left[\eta_E^+(t,\dot{\varepsilon})\right]$$

Note 1: If the material of interest tends toward linear viscoelastic behavior at small strain rates, the following relationships are valid:

$$\lim_{\dot{\varepsilon} \to 0} \left[\eta_E^+(t,\dot{\varepsilon})\right] = \eta_E^+(t) = 3\eta^+(t) ,$$

$$\lim_{\dot{\varepsilon} \to 0} \left[\eta_E(\dot{\varepsilon})\right] = 3\eta_0 .$$

Note 2: Some researchers prefer to use the strain ε as an independent variable in place of time when presenting tensile stress growth results.

2. Cessation of steady tensile extension

A material that has been subjected to steady tensile extension at a strain rate $\dot{\varepsilon}$ until the net tensile stress is constant in time is brought suddenly to rest at time $t = 0$, and the net tensile stress is monitored as a function of time.

Tensile stress decay coefficient:

$$\eta_E^-(t,\dot{\varepsilon}) = \sigma_E / \dot{\varepsilon} .$$

Note: If the material of interest tends toward linear viscoelastic behavior at small strain rates:

$$\lim_{\dot{\varepsilon} \to 0} \left[\eta_E^-(t,\dot{\varepsilon})\right] = \eta_E^-(t) = 3\eta^-(t) .$$

3. Tensile creep

A sample initially at rest is subjected to a constant net tensile stress σ_E, at time $t = 0$, and the true strain ε is monitored as a function of time.

Tensile creep compliance:

$$D(t,\sigma_E) \equiv \varepsilon/\sigma_E$$

Alternatively, one can monitor the extensional strain rate as a function of time.

Tensile creep rate decay function:

$$\dot{\varepsilon}^-(t,\sigma_E) \equiv \dot{\varepsilon}$$

The relationship between the stress and strain rate is defined as the tensile creep rate coefficient:

$$\eta^+_{E,c}(t,\sigma_E) \equiv \sigma_E/\dot{\varepsilon}^- \,.$$

The tensile creep rate coefficient is the quantity determined in creep which is analogous to the tensile stress growth coefficient η^+_E determined in a constant strain rate test.

If the material of interest is a fluid, then, after the strain becomes linear with time, the tensile creep rate coefficient becomes constant, and the tensile creep compliance is given by

$$D(t,\sigma_E) = D_s(\sigma_E) + t/\eta_E \,,$$

where D_s is the steady-state tensile compliance, and $\eta_E \equiv \eta^+_{E,c}(\sigma_E)$ is evaluated at the steady strain rate corresponding to σ_E.

Note: If the behavior of the material of interest tends toward linear viscoelasticity at small values of stress, we have:

$$\lim_{\sigma_E \to 0}\left[D(t,\sigma_E)\right] = D(t) = \tfrac{1}{2}J(t)$$

and

$$\lim_{\sigma_E \to 0}\left[\eta^+_{E,c}(t,\sigma_E)\right] = \eta^+_{E,c}(t) = 3\eta^+_c(t)\,,$$

and, after the strain becomes a linear function of time:

$$D(t) = D^0_s + t/\eta_{E,0} \,,$$

where D^0_s is the steady-state tensile compliance in the limiting case of small stress and $\eta_{E,0} \equiv \eta^+_{E,c}$ is the limiting tensile viscosity at zero extension rate.

4. Tensile recoil

A material in which the stresses and the strain rate $\dot{\varepsilon}$ are steady in time has its tensile stress σ_E reduced suddenly to zero at time $t = 0$, and the recoil strain ε_r is monitored as a function of time.

Tensile recoil strain (a positive quantity):

$$\varepsilon_r \equiv \varepsilon(0) - \varepsilon(t)$$

Tensile recoil function:

$$S(t,\sigma_E) \equiv \varepsilon_r/\sigma_E$$

Ultimate tensile recoil:

$$\varepsilon_\infty(\sigma_E) \equiv \lim_{t\to\infty}\left[\varepsilon_r(t,\sigma_E)\right]$$

Ultimate tensile recoil function:

$$S_\infty(\sigma_E) = \varepsilon_\infty(\sigma_E)/\sigma_E$$

Note 1: Since $\sigma_E = \eta_E\dot{\varepsilon}$ at t = 0, the strain rate can be used in place of the stress as an independent variable.

Note 2: If the material of interest tends toward linear viscoelastic behavior at small stresses:

$$\lim_{\sigma_E\to 0}\left[S(t,\sigma_E)\right] = S(t) = \tfrac{1}{3}R(t)\,.$$

Note 3: Recoil experiments are sometimes performed by releasing the stress in a creep experiment at some time t_0, *before* the strain has become linear in time. In this case, the functions defined above can still be used, but the new independent variable must be noted. For example,

$$\varepsilon_r(t-t_0,t_0,\sigma_E) \equiv \varepsilon(t_0) - \varepsilon(t)\,.$$

5. Tensile step strain

A material initially in its rest state is subjected to a sudden strain of magnitude ε at time t = 0, and the net tensile stress is observed as a function of time.

Tensile stress relaxation modulus:

$$E(t,\varepsilon) \equiv \sigma_E/\varepsilon$$

Note: If the material of interest tends toward linear viscoelastic behavior at small values of strain:

$$\lim_{\varepsilon\to 0}\left[E(t,\varepsilon)\right] = E(t) = 3G(t)\,.$$

B. Biaxial extensional flow (axisymmetric)

This flow is defined as follows for a fluid with constant density:

$$v_1 = \dot{\varepsilon}_B x_1\,,$$

$$v_2 = \dot{\varepsilon}_B x_2\,,$$

$$v_3 = -2\dot{\varepsilon}_B x_3\,,$$

where $\dot{\varepsilon}_B \geq 0$.

Note: This flow is axisymmetric and can be described in a cylindrical coordinate system as follows:

$$v_r = \dot{\varepsilon}_B r\,,$$

$$v_z = -2\dot{\varepsilon}_B z\,.$$

1. Biaxial start-up flow

A sample initially in its rest state is subjected to a constant biaxial strain rate $\dot{\varepsilon}_B$ at time t = 0, and the net stretching stress σ_B is monitored as a function of time.

$$\sigma_B \equiv \sigma_{11} - \sigma_{33} = \sigma_{22} - \sigma_{33} = \sigma_{rr} - \sigma_{zz}\,.$$

Biaxial stress growth coefficient:

$$\eta_B^+(t,\dot{\varepsilon}_B) \equiv \sigma_B / \dot{\varepsilon}_B$$

Biaxial extensional viscosity:

$$\eta_B(\dot{\varepsilon}_B) = \lim_{t \to \infty} \left[\eta_B^+(t,\dot{\varepsilon}_B)\right]$$

Note 1: If the material of interest tends toward linear viscoelastic behavior at small strain rate:

$$\lim_{\dot{\varepsilon}_B \to 0} \left[\eta_B^+(t,\dot{\varepsilon}_B)\right] = \eta_B^+(t) = 6\,\eta^+(t)\,,$$

$$\lim_{\dot{\varepsilon}_B \to 0} \left[\eta_B(\dot{\varepsilon}_B)\right] = 6\,\eta_0\,.$$

Note 2: Those calculating the predictions of constitutive equations may prefer to show results for both steady tensile flow and steady biaxial extension on a single plot. This can be accomplished by noting that axisymmetric biaxial extensional flow can be considered to be tensile flow at a negative strain rate. For example, let $\overline{\eta}(\dot{\varepsilon})$ be an extensional viscosity defined only for axisymmetric flows, where $\dot{\varepsilon}$ can take on both positive and negative values. This material function is then defined as follows:

$$\overline{\eta} \equiv (\sigma_{zz} - \sigma_{rr}) / \dot{\varepsilon}\,.$$

The following correspondences can then be made with the functions described above:

$$\overline{\eta}(\dot{\varepsilon}) = \eta_E(\dot{\varepsilon}),\ \text{ for }\ \dot{\varepsilon} \geq 0\,,$$

$$\overline{\eta} = \tfrac{1}{2}\,\eta_B \ \text{ with }\ \dot{\varepsilon} = -2\,\dot{\varepsilon}_B,\ \text{ for }\ \dot{\varepsilon} \leq 0\,.$$

Note that $\overline{\eta}$ is positive for all values of $\dot{\varepsilon}$.

C. Extensional flow (general)

Tensile extension and biaxial extension are two special cases of a class of flows for which the components of the rate of deformation tensor have been shown by Stevenson, Chung, and Jenkins[3] to have the following form:

$$\dot{\varepsilon}\begin{bmatrix} 1 & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & -(1+m) \end{bmatrix}$$

Meissner *et al*.[4] have pointed out that if $\dot{\varepsilon}$ is interpreted as the largest principal strain rate, and m does not vary with time, then every possible extensional flow corresponds to some value of m between –0.5 and +1.0.

Unless the flow is axisymmetric, two independent normal stress differences can, in principle, be measured:

$$\sigma_1 \equiv \sigma_{11} - \sigma_{22}\,,$$

$$\sigma_2 \equiv \sigma_{22} - \sigma_{33}\,.$$

Thus, two independent material functions can be defined for a given value of *m* and a given strain or stress history. For example, if $\dot{\varepsilon}$ is constant, two extensional viscosities can be defined:

$$\eta_1^{(m)} \equiv \sigma_1 / \dot{\varepsilon} ,$$

$$\eta_2^{(m)} \equiv \sigma_2 / \dot{\varepsilon} .$$

We note that $m = -1/2$ corresponds to simple extension, while $m = 1$ corresponds to biaxial extension. However, in these cases there is only one independent normal stress difference, and the special symbols proposed for these flows should be used rather than the ones defined above. Another special case of interest is "planar" extension, for which $m = 0$. The most easily measured stress difference is σ_1, and a planar extensional viscosity can be defined as follows:

$$\eta_p(\dot{\varepsilon}) \equiv \eta_1^{(0)}(\dot{\varepsilon}) .$$

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