D. BINOMIALE

- Esiti dicotomici per ogni singola prova : Sì o No
- la D.B. descrive le P di r successi su N prove, indipendentemente dall'ordine in cui capitano

N = prove Totali r = n.ro successi ciascuno con prob. p

D.B. $\rightarrow$ $P(r) = P(r, N, p) = \frac{N!}{r!\,(N-r)!}\, p^r (1-p)^{N-r} = \frac{N!}{r!\,(N-r)!}\, p^r q^{N-r}$

Nota : $\binom{N}{r} = \frac{N!}{r!\,(N-r)!}$ = coefficiente binomiale

Normalizzazione $\sum_{r=0}^{N} \frac{N!}{r!\,(N-r)!}\, p^r (1-p)^{N-r} = \sum_{r=0}^{N} \binom{N}{r} p^r q^{N-r} =$

$= (p+q)^N = 1$

Media $\mu = E[r] = \sum_{r=0}^{N} \binom{N}{r} r\, p^r q^{N-r} = Np$

Varianza $\sigma^2 = E[r^2] - E^2[r] = \sum_{r=0}^{N} \binom{N}{r} r^2 p^r q^{N-r} - N^2 p^2 = Npq$

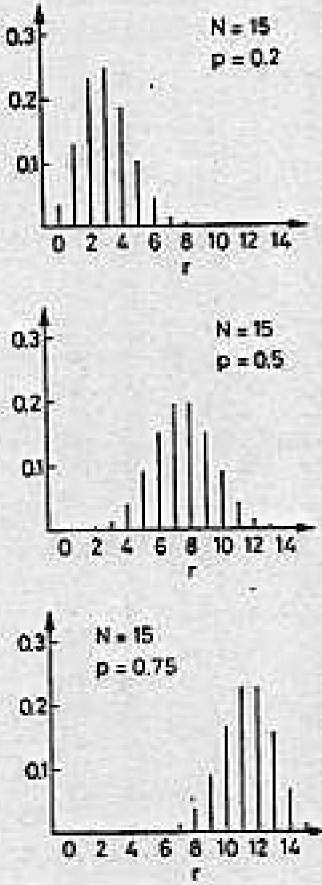


Fig. 4.1. Binomial distribution for various values of N and p

D. POISSON

- limite della DB se il numero medio successi $\mu \ll N$

$p \ll 1 \quad N \gg 1 \quad Np = \text{cost} = \mu$

$$\lim_{N\to\infty} P(r, N, p) = \lim_{N\to\infty} \binom{N}{r} p^r (1-p)^{N-r} = \lim_{N\to\infty} \frac{N!}{r!(N-r)!} \left(\frac{\mu}{N}\right)^r \left(1-\frac{\mu}{N}\right)^{N-r} =$$

$$= \lim_{N\to\infty} \underbrace{\boxed{\frac{N(N-1)\cdots(N-r+1)}{N^r}}}_{\to 1} \frac{\mu^r}{r!} \left(1-\frac{\mu}{N}\right)^N \underbrace{\boxed{\left(1-\frac{\mu}{N}\right)^{-r}}}_{\to 1} =$$

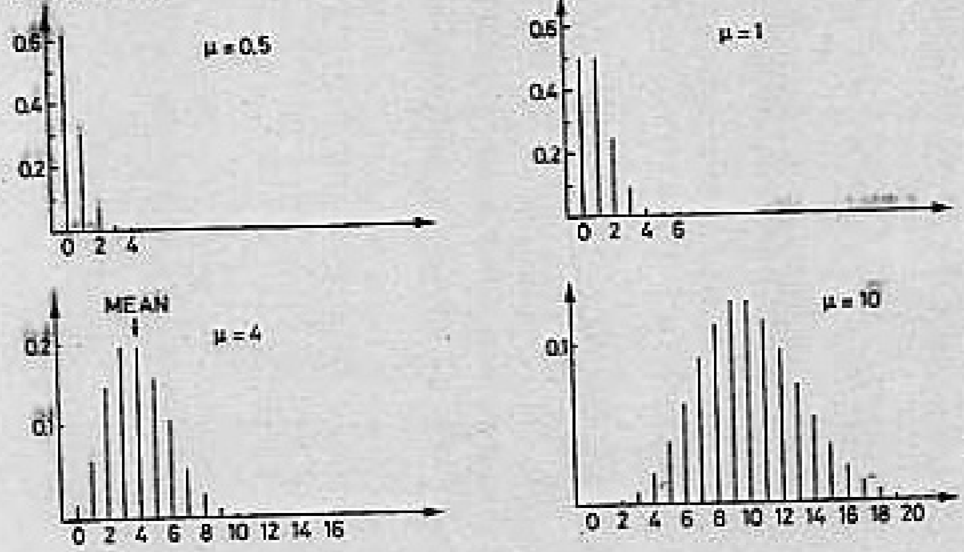
$$= \frac{\mu^r}{r!} e^{-\mu} = P(\mu, r)$$

NOTA: $e^{-\mu} = \lim_{N\to\infty} \left(1-\frac{\mu}{N}\right)^N = \sum_{k=0}^{\infty} \frac{(-\mu)^k}{k!}$

Media $\quad E[r] = \sum_{r=0}^{\infty} r \frac{\mu^r}{r!} e^{-\mu} = \mu$

Varianza $\quad E[r^2] - E^2[r] = \sum_{r=0}^{\infty} r^2 \frac{\mu^r}{r!} e^{-\mu} - \mu^2 = \mu$

Fig. 4.2. Poisson distribution for various values of μ



D. GAUSSIANA (O NORMALE)

Simmetrica, continua, su tutto $\mathbb{R}$ $\quad G: x \to \mathbb{R}$

$$G(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{1}{2}\,\frac{(x-\mu)^2}{\sigma^2}\right)$$

È simmetrica $\Rightarrow$ $\mu = E[x]$ (media, mediana e valore + prob. coincidono)

$G = \max$ se $\frac{x_m - \mu}{\sigma} = 0$ $\Rightarrow$ $x_m = \mu$

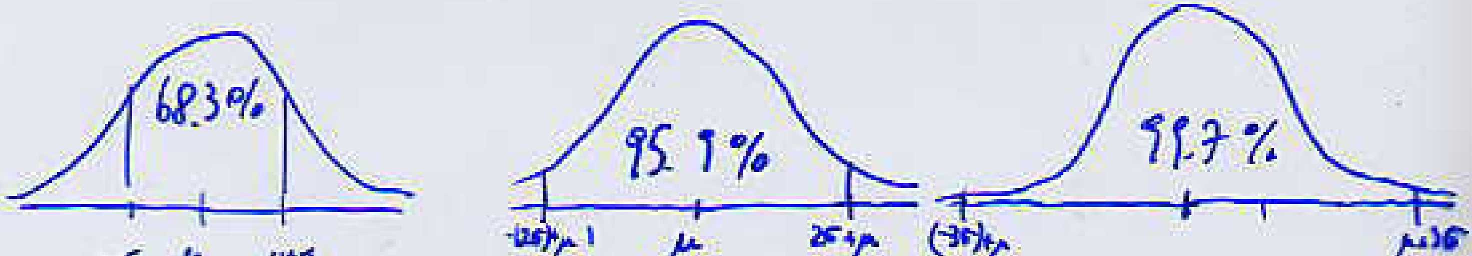
Semilarghezza a metà altezza

$\frac{x-\mu}{\sqrt{2}\,\sigma} = t \qquad \exp(-t^2) = \frac{1}{2} \qquad t^2 = \ln 2$

$\Rightarrow \quad x - \mu = \sigma\sqrt{2\ln 2} \sim 1.2\,\sigma$

Media μ $\qquad$ Varianza σ^2

IN PRATICA



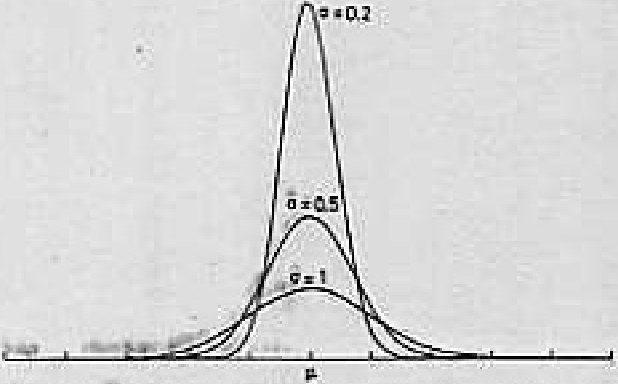


Fig. 4.3. The Gaussian distribution for various σ. The standard deviation determines the width of the distribution

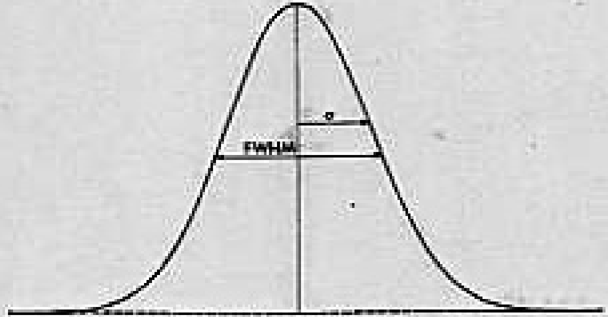


Fig. 4.4. Relation between the standard deviation σ and the full width at half-maximum (FWHM)

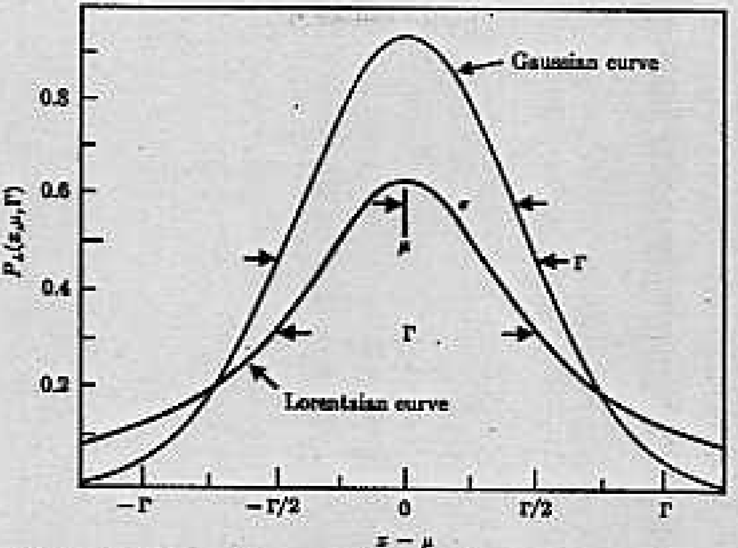


FIGURE 3-6 Lorentzian probability distribution $P_L(x,\mu,\Gamma)$ vs. $x-\mu$, compared with Gaussian function $P_G(x,\mu,\sigma)$ where $\sigma = \Gamma/2.345$. Higher narrower peak is Gaussian curve.

$$G(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$$

$$\mathcal{L}(x) = \frac{1}{\pi} \frac{\Gamma/2}{(x-\mu)^2 + (\Gamma/2)^2}$$

D. χ^2

ν = numero variabili aleatorie indipendenti x_i gaussiane con medie μ_i e σ_i dev. standard

$$u = \sum_{i=1}^{\nu} \left(\frac{x_i - \mu_i}{\sigma_i} \right)^2$$ è noto come χ^2

$\chi^2 = u$ nuova v. casuale

Si dimostra che

$$P(u) = \frac{(u/2)^{(\nu/2)-1} \exp(-u/2)}{2^{\nu/2} \, \Gamma(\nu/2)}$$

$\mu = n$

$\sigma^2 = 2n$

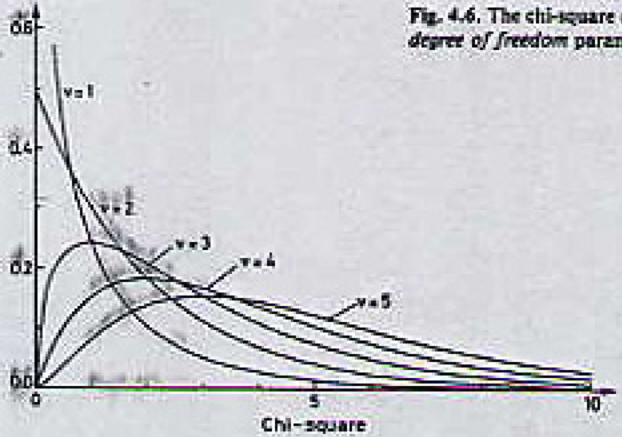


Fig. 4.6. The chi-square distribution for various values of the *degree of freedom* parameter ν