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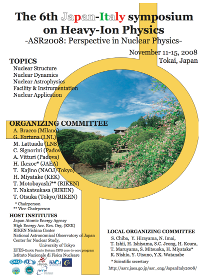

Unbound Nuclei Studied by Projectile Fragmentation

G. Blanchon * Ph.D. Thesis, Pisa July 2008
http://www.infn.it/thesis/

D.M. Brink and N. Vinh Mau

* now at DAM / Île de France

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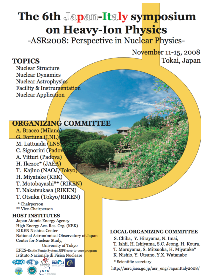

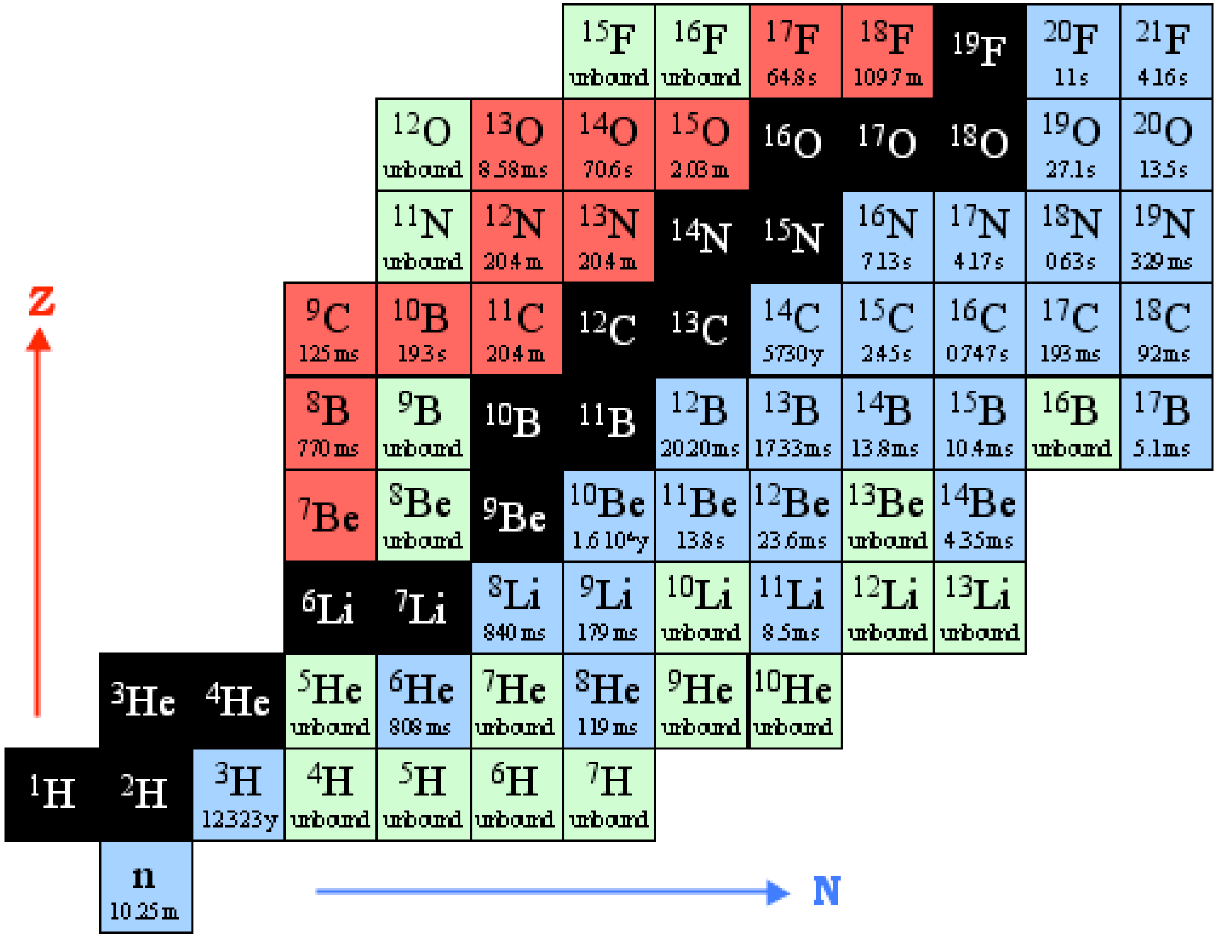

Some unbound nuclei are sub-costituents of two-halo bound nuclei

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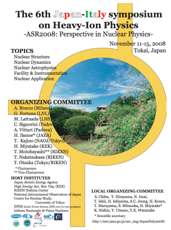

Introduction

- **Present and future most challenging problems in reaction theory for unstable beams**

- **Nuclear (both stripping and diffraction) and Coulomb breakup @ all energies.**
- Two nucleon breakup (two-n halo & proton radioactivity, pairing effects)
- **Elastic scattering including above channels**… optical potentials

Fundamental Problem

Determination of dripline position

Unbound Nuclei (...^{10}Li ^{13}Be)
http://www.df.unipi.it/~angela/Pisa08.html

- Transfer to the continuum vs. projectile fragmentation

Nuclear Physics A 784 (2007) 49–78

Unbound exotic nuclei studied by projectile fragmentation

G. Blanchon [a], A. Bonaccorso [a,*], D.M. Brink [b], A. García-Camacho [a], N. Vinh Mau [c]

Observables measured & calculated ---> structure information extracted.

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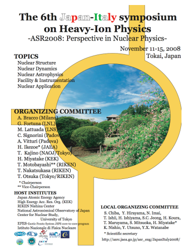
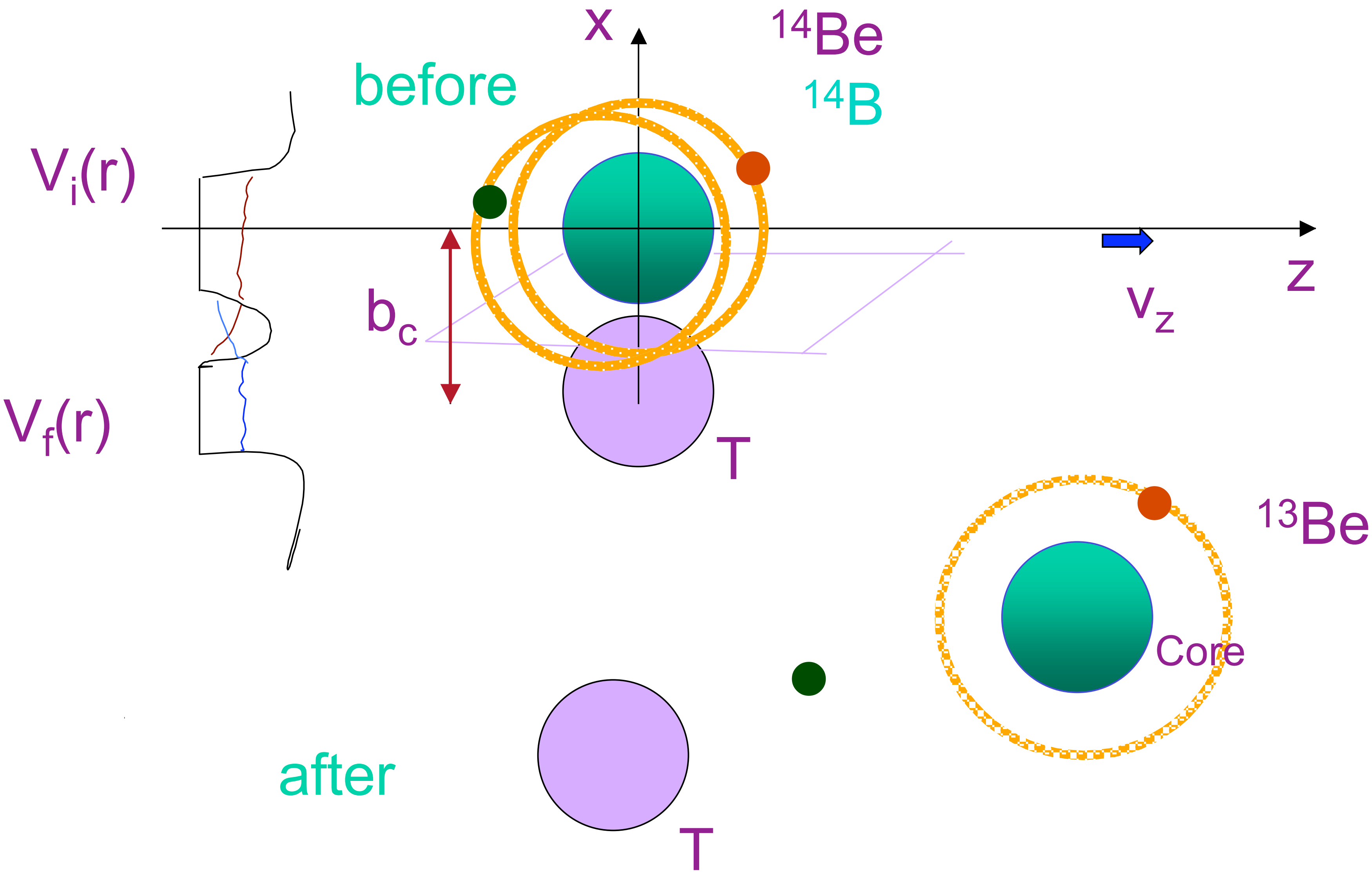


Projectile fragmentation

n-core final state interaction



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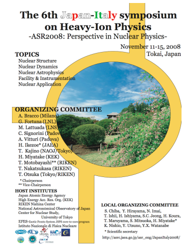

Projectile fragmentation

^{11}Be: a simple test case

from bound s-state to d-resonance: inelastic excitation to the continuum?

^{11}Be+^{12}C @ 67A.MeV

CAPEL, GOLDSTEIN, AND BAYE PHYSICAL REVIEW C 70, 064605 (2004)

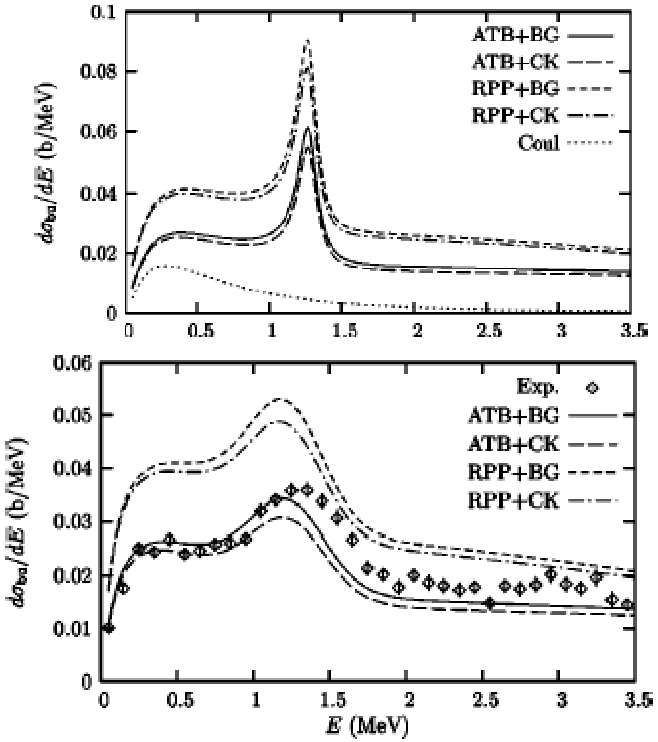

FIG. 4. Theoretical and experimental breakup cross sections as a function of the energy. The four curves correspond to the calculations performed with various combinations of the potentials of Table III convoluted with energy resolution. Experimental data are from Ref. [39].

[39] N. Fukuda *et al.*, Phys. Rev. C 70, 054606 (2004).

60 G. Blanchon et al. / Nuclear Physics A 784 (2007) 49–78

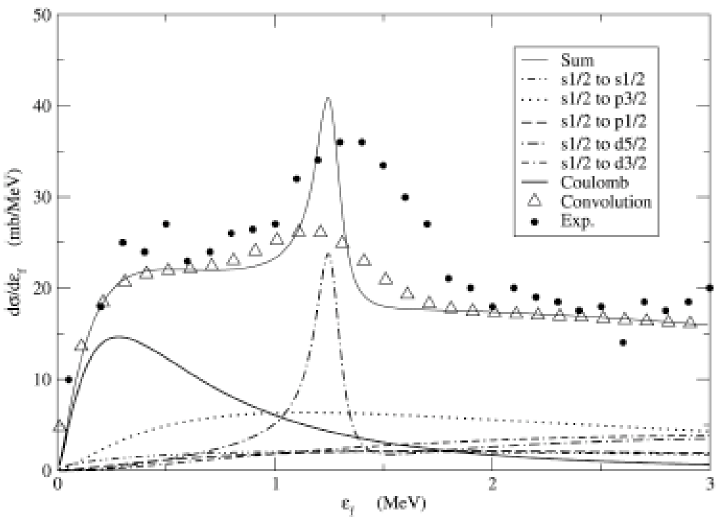

Fig. 3. n–^{10}Be relative energy spectrum, including Coulomb and nuclear breakup for the reac ^{11}Be + ^{12}C → n + ^{10}Be+X at 67A MeV. Only the contributions from an s initial state with spectroscopic tor C^2S = 0.84 are calculated. The triangles are the total calculated result after convolution with the experime resolution function. The dots are the experimental points from [1].

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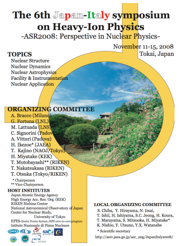

Transfer to the continuum vs. Projectile fragmentation: a model for diffractive breakup in which the observable studied is the n-core relative energy spectrum and its resonances

$$A_{fi} = \frac{1}{i\hbar}\int_{-\infty}^{\infty} dt \langle \psi_f(\mathbf{r},t)|V_2(\mathbf{r}-\mathbf{R}(t))|\psi_i(\mathbf{r},t)\rangle,$$

$$\begin{aligned}\frac{dP_t(b_c)}{d\varepsilon_f} &= \frac{1}{8\pi^3}\frac{mk_f}{\hbar^2}\frac{1}{2l_i+1}\Sigma_{m_i}|A_{fi}|^2 \\ &\approx \frac{4\pi}{2k_f^2}\Sigma_{j_f}(|1-S_{j_f}|^2 + 1 - |S_{j_f}|^2)(2j_f+1)(1+F_{l_f,l_i,j_f,j_i})B_{l_f,l_i} \\ &= \sigma_{nN}(\varepsilon_f)\mathcal{F},\end{aligned}$$

diffraction + *stripping* → inclusive breakup (

$$\frac{dP_{in}}{d\varepsilon_f} = \frac{2}{\pi}\frac{v_2^2}{\hbar^2 v^2}C_i^2\frac{m}{\hbar^2 k}\frac{1}{2l_i+1}\Sigma_{m_i,m_f}|1-\bar{S}_{m_i,m_f}|^2|I_{m_i,m_f}|^2,$$

→ exclusive breakup

diffraction

$$B_{l_f,l_i} \approx \frac{e^{-2\eta b_c}}{b_c}$$ **Knockout**

$$\bar{S} = Se^{2i\nu} = e^{2i(\delta+\nu)}$$

$\bar{S}$ *off-shell*
S *on shell*

$$I_{l_f,l_i} \approx \frac{e^{-2\gamma b_c}}{b_c^3}$$ **Fragmentation**

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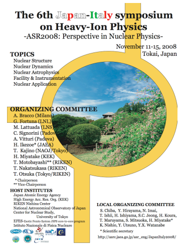


Initial states chosen to fit known separation energy,
s & p components for ^{11}Li with known spectroscopic factors.
For ^{14}Be s-p-d components with unit C^2S.
Final states determined by S-matrix via n-core interaction

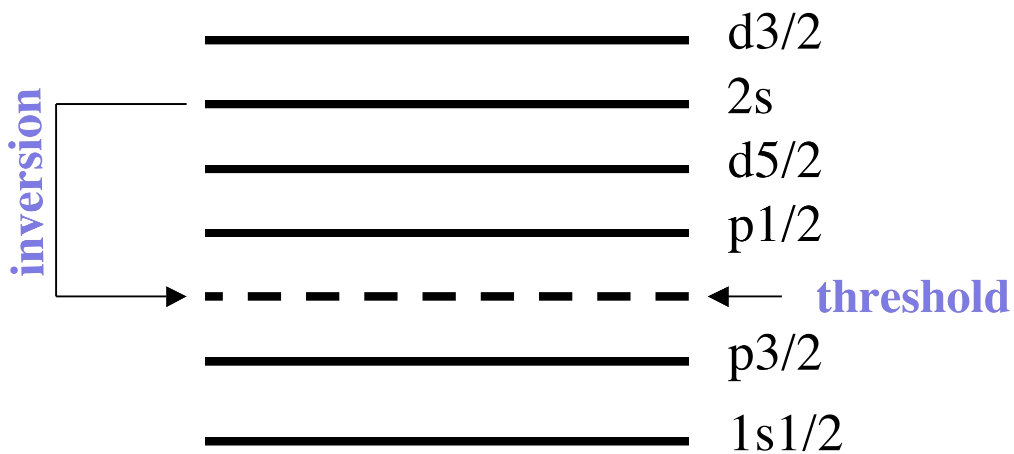
Potential corrections due to the particle-vibration coupling
(N. Vinh Mau and J. C. Pacheco, NPA607 (1996) 163.
also T. Tarutina, I.J. Thompson, J.A. Tostevin NPA733 (2004) 53)
…can be modeled as:

$$U(r) = V_{WS} + V_{so} + \delta V$$

$$\delta V(r) = 16\,\alpha\, e^{2(r-R)/a} / (1+e^{(r-R)/a})^4$$

n+^{12}Be:

	ε_{res} (MeV)	Γ (MeV)	α (MeV)
$1p_{1/2}$	0.67	0.28	8.34
$1d_{5/2}$	2.0	0.40	-2.36



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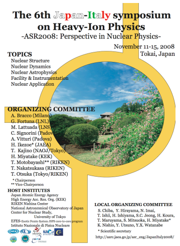


2. Effect of particle–vibration coupling on individual energies

Let us start with the Hartree–Fock(HF) potential for a neutron in the mean field of the core nucleus and calculate the correction to this HF potential due to the coupling of single particle states to the RPA collective one-phonon states of the core. Neglecting the antisymmetrisation between the single particle and the particles of the core, one can write this correction δV as [26,28]:

$$\delta V(\boldsymbol{r},\boldsymbol{r}';E) = \lim_{\eta\to+0}\sum_{N,\lambda}\left[\frac{1-n_\lambda}{E-\epsilon_\lambda-E_N+i\eta}+\frac{n_\lambda}{E-\epsilon_\lambda+E_N-i\eta}\right]$$
$$\times V_{0N}^*(\boldsymbol{r})V_{0N}(\boldsymbol{r}')\phi_\lambda^*(\boldsymbol{r}')\phi_\lambda(\boldsymbol{r}), \quad (1)$$

λ and N characterise respectively the HF single particle state of energy ϵ_λ and wave function ϕ_λ and the RPA collective state of the core of excitation energy E_N; n_λ is the occupation number of the state λ in the HF ground state of the core and V_{0N} the transition amplitude between the ground state and the excited state N. V_{0N} can be written as:

$$V_{0N}(\boldsymbol{r}) = \sum_{i,j}[n_i(1-n_j)+n_j(1-n_i)]x_{ij}^{(N)}\rho_{ij}(\boldsymbol{r})$$
$$= \langle\Psi_N|v|\Psi_0\rangle, \quad (2)$$

where $x_{ij}^{(N)}$ are the RPA amplitudes, $\rho_{ij}(\boldsymbol{r})$ the transition density for the unperturbed particle-hole state (ij), v the two body interaction and Ψ_N and Ψ_0 the RPA wave functions of the excited state and the ground state respectively. In the summation over N we keep only natural parity states with multipolarity L and define amplitudes $f_{NL}(r)$ such that:

$$V_{0N}(\boldsymbol{r}) = \frac{1}{\sqrt{2L+1}}f_{NL}(r)Y_L^{M*}(\hat{r}). \quad (3)$$

The correction of Eq. (1) to the HF potential induces a modification of the single particle energies which, in first approximation, can be calculated as the average of δV over the HF state of interest, replacing in Eq. (1) the energy E by the corresponding HF energy. For an HF state n of energy ϵ_n we get the modified energy e_n as:

$$e_n = \epsilon_n + \delta\epsilon_n, \quad (4)$$

$$\delta\epsilon_n = \sum_{N,\lambda}F_{N\lambda}\frac{2j_\lambda+1}{4\pi}\begin{pmatrix} j_n & L & j_\lambda \\ -1/2 & 0 & 1/2\end{pmatrix}^2\left|\int_0^\infty r^2\,\mathrm{d}r\,\mathcal{R}_\lambda(r)\mathcal{R}_n(r)f_{NL}(r)\right|^2 \quad (5)$$

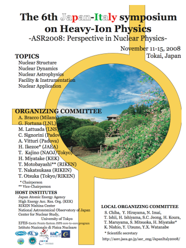
with

$$F_{N\lambda} = \frac{1-n_\lambda}{\epsilon_n-\epsilon_\lambda-E_N}+\frac{n_\lambda}{\epsilon_n-\epsilon_\lambda+E_N}. \quad (6)$$

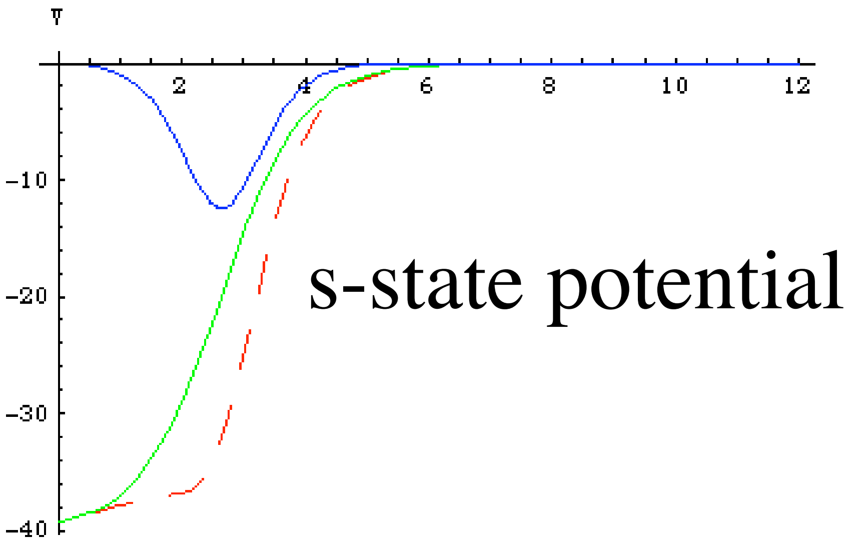
$\mathcal{R}_\lambda$ and $\mathcal{R}_n$ are the radial wave functions.

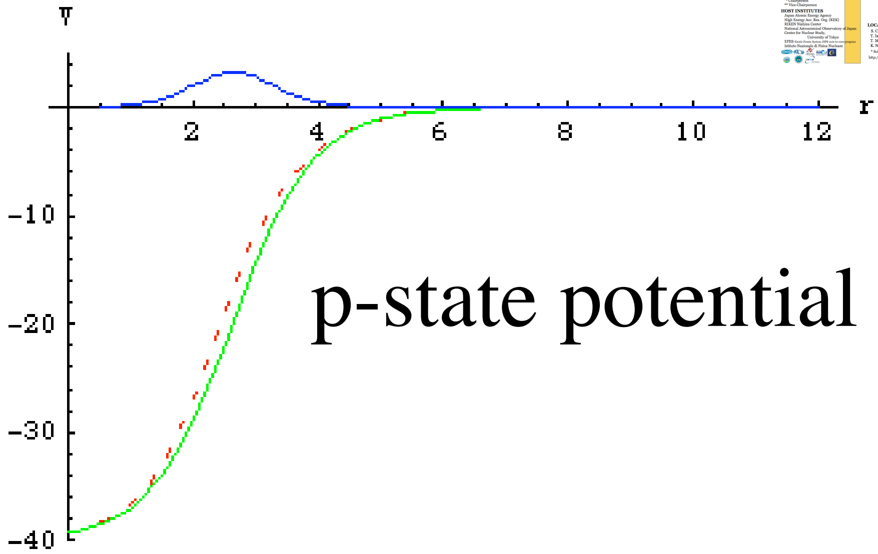
$$f_{NL}(r) = \beta_{NL}R_0\frac{\mathrm{d}U(r)}{\mathrm{d}r},$$

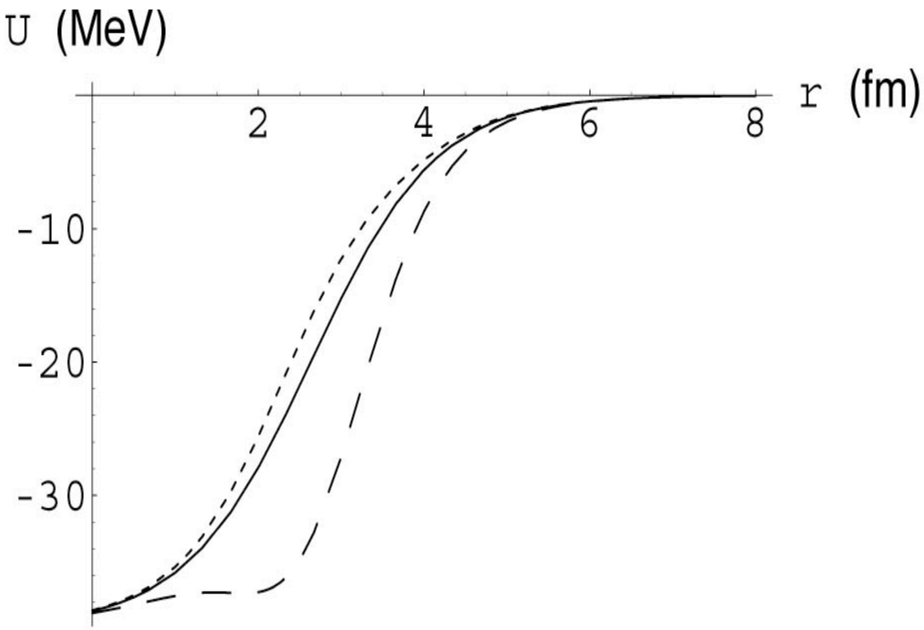
Pacheco & N. Vinh Mau,
PRC 65 (2002) 044004
Labiche et al. PRC 60 027303 (1999)



$$U(r) = V_{WS} + \delta V$$







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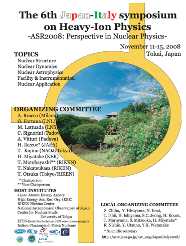
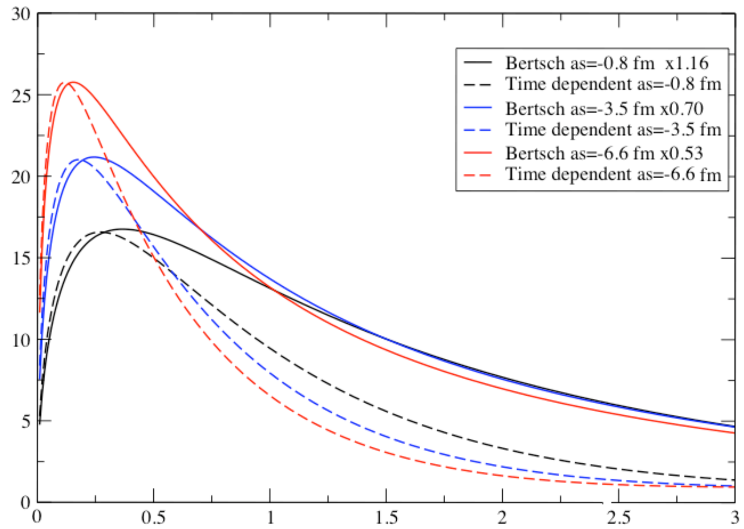


Sudden vs. time dependent



sudden for s-states →

$$\frac{d\sigma}{d\varepsilon_f} \approx \frac{1}{k}\left(\frac{k\cos\delta - \gamma\sin\delta}{\gamma^2+k^2}\right)^2$$

$$\approx \frac{1}{k}|\sin(\delta+\beta)|^2$$

$|1-\bar{S}|^2$

see G.F. Bertsch, K. Hencken & H. Esbensen et al: PRC 57, 1366 (1998)

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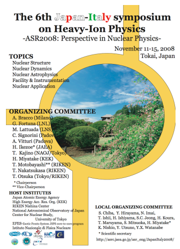
ELSEVIER Nuclear Physics A 791 (2007) 303–312

^{10}Li spectrum from ^{11}Li fragmentation

G. Blanchon [a], A. Bonaccorso [a,*], D.M. Brink [b], N. Vinh Mau [c]



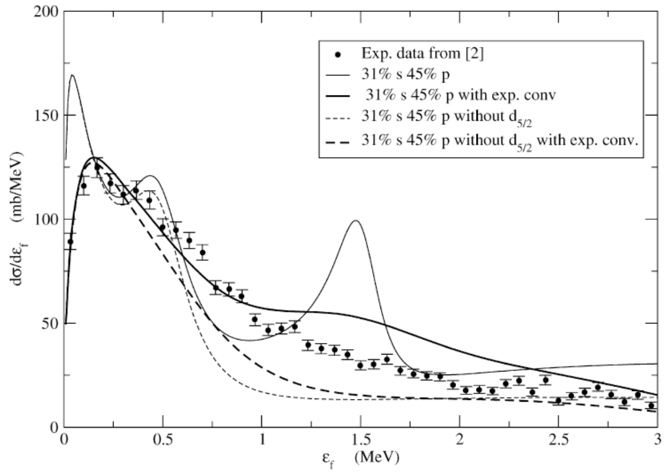
^{10}Li:

1) virtual state: sudden approach OK;
2) evidence for a $d_{5/2}$ resonance

Table 3 Scattering length of the 2s continuum state, energies and widths of the *p*-and *d*-resonances in 10Li and corresponding strength parameters for the δVpotential

	εres (MeV)	Γj(MeV)	as(fm−1)	α(MeV)
$2s1/2$			−17.2	−10.0
$1p1/2$	0.63	0.35		3.3
$1d5/2$	1.55	0.18		−9.8

*



a_s=-17.4fm

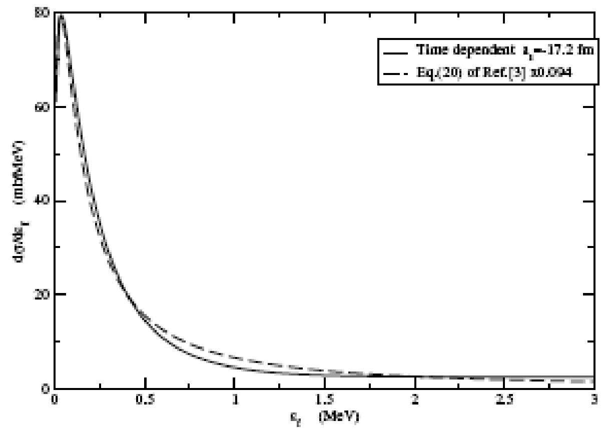


Figure 4: Comparison for the s to s transition of our calculation (solid curve) with that according to the sudden formula Eq. (20) of [3]. Both calculations use the *exact* phase shifts.

see also Progress of Theoretical Physics 119 (2008) 561-581

Systematic study of ^{9,10,11}Li with the tensor and pairing correlations

Takayuki Myo *,[1] Yuma Kikuchi †,[2] Kiyoshi Katō ‡,[2] Hiroshi Toki §,[1] and Kiyomi Ikeda ¶[3]
[1] Research Center for Nuclear Physics (RCNP), Ibaraki, Osaka 567-0047, Japan
[2] Division of Physics, Graduate School of Science, Hokkaido University, Sapporo 060-0810, Japan.
[3] RIKEN Nishina Center, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan

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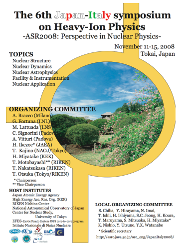
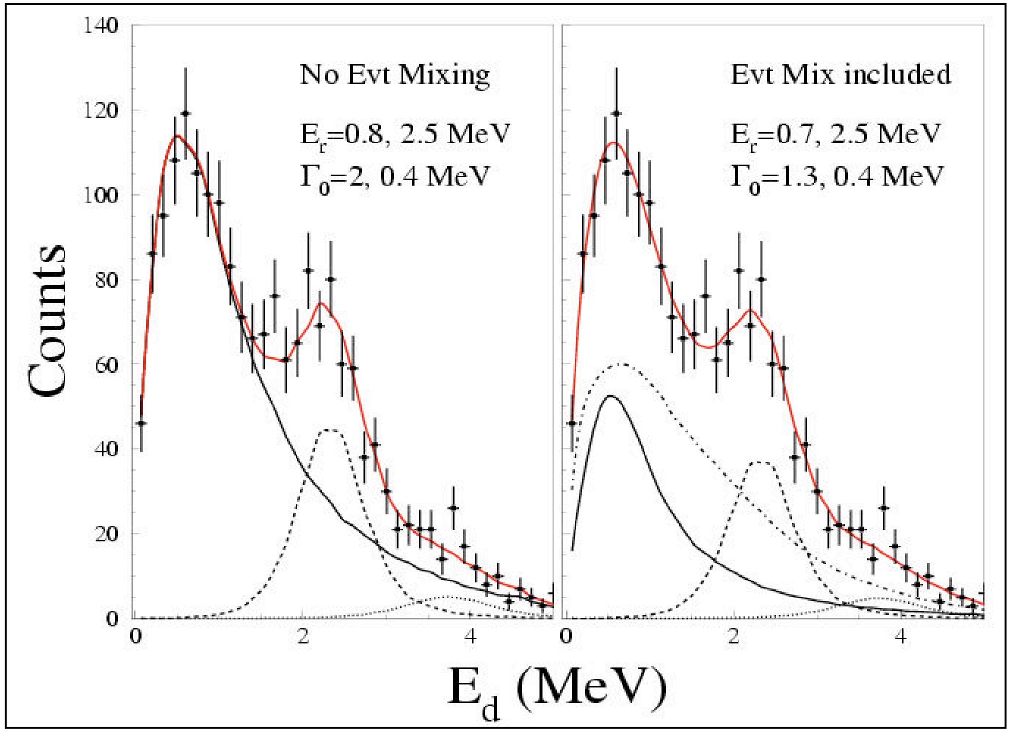


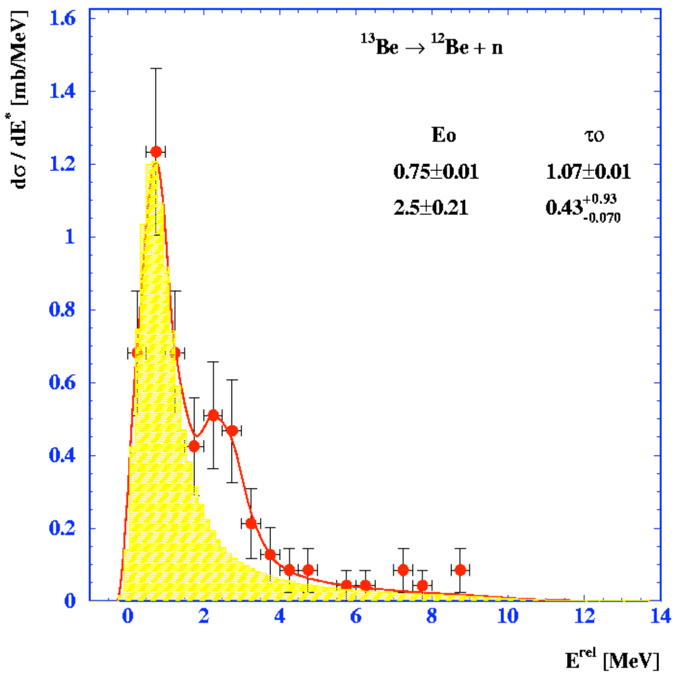
• ^{14}B **fragmentation on C**

JL Lecouey LPCC 02-03; Few-Body Systems 34 (2004) 21



• **GSI (U. Datta Pramanik)(2004).**

• **Unpublished**



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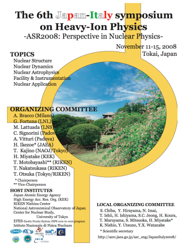

Resumee:

^{13}Be has been obtained from:

- transfer to the continuum: ^{12}Be (d,p) RIKEN (**Korsheninnikov**) (1995).
- ^{14}C+ ^{11}B multinucleon transfer: (Berlin Group ,1998).
- ^{14}Be nuclear and Coulomb breakup: GANIL

  (K. Jones thesis, 2000).
- ^{18}O fragmentation MSU (Thoennessen, 2001) n-core relative velocity spectra.
- ^{14}B fragmentation: GANIL (**Lecouey**, Orr) (2002).
- GSI (U. Datta Pramanik)(2004).
- ^{14}Be frgmentation , GSI (**Simon, 2007**), 287AMeV, n-core angular correlations
- ^{14}Be nuclear breakup: RIKEN (Kondo,Nakamura et al.) (2008).

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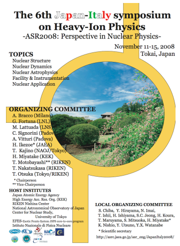


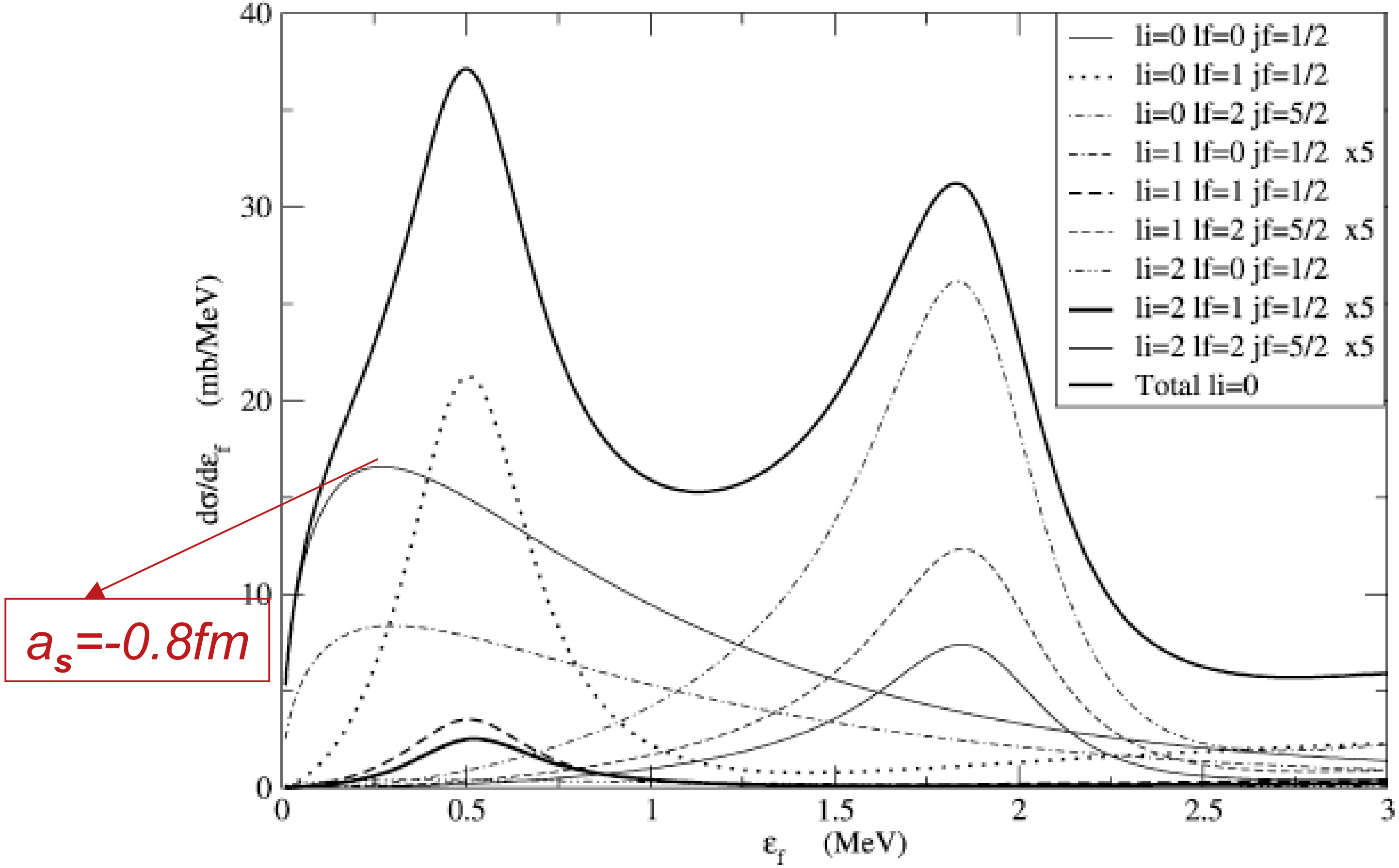
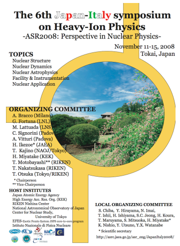


Fig. 8. Results obtained including the s, p and d states. Each curve corresponds to just one transition as indicated. The solid curve is the sum of all transitions from the s-bound state. To make them visible some curves have been multiplied by a factor of five as indicated in the legend.



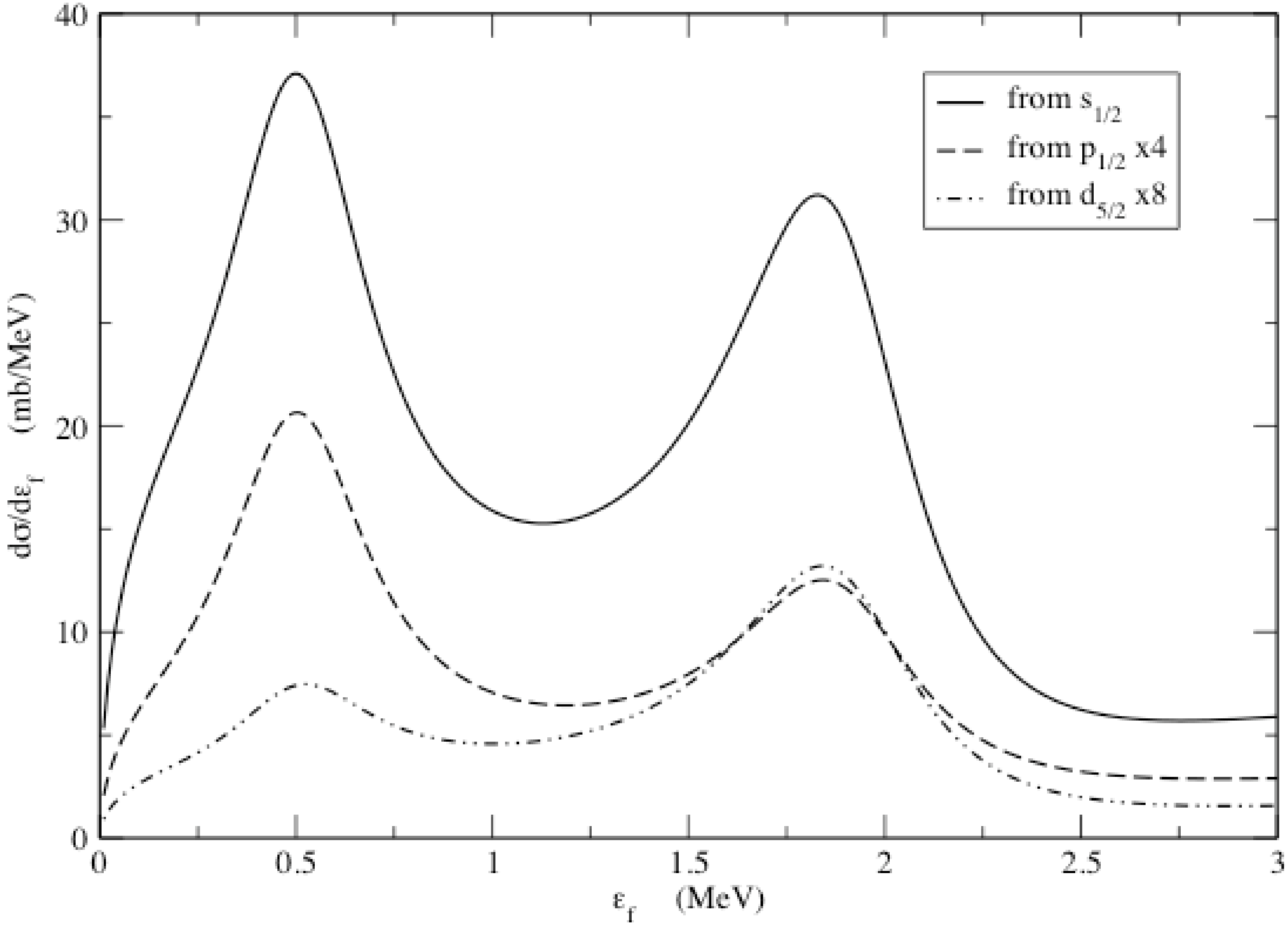


Figure 11: Check of the dependence from the initial state angular momentum. Full curve: sum of transitions from s-initial state. Dashed and dotdashed lines: sum of transitions from p and d-initial states respectively.

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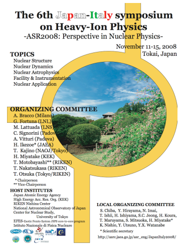
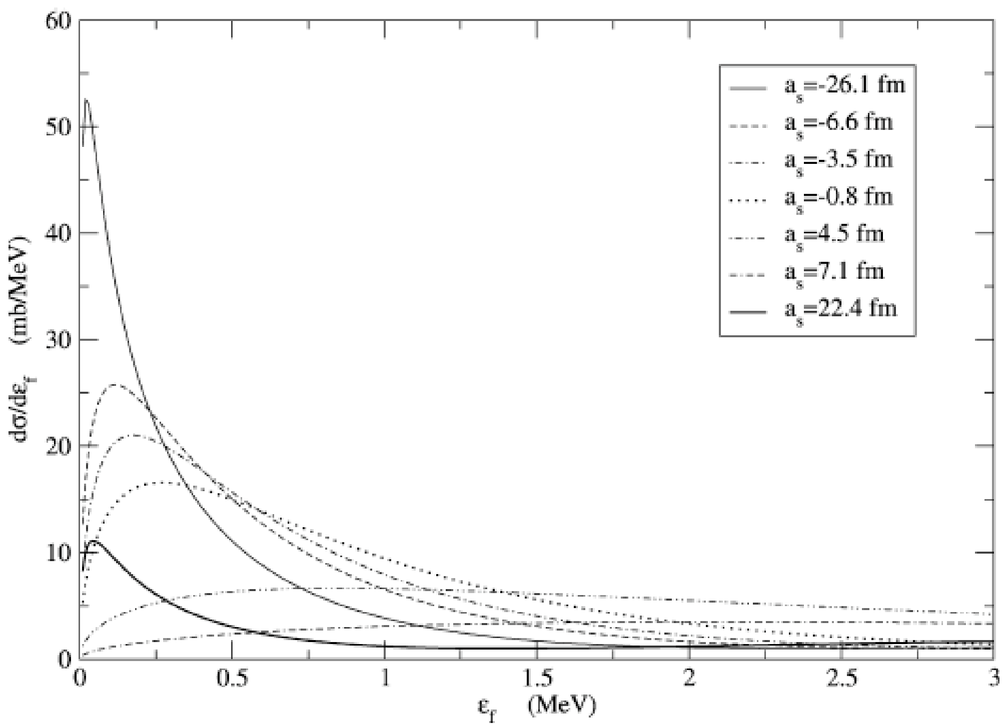


Strengths of the s-state δV potential in Eq. (40) and corresponding scattering lengths, effective range parameter and *energy parameter* ϵ. The strength of the central Woods–Saxon part is $V_0 = -39.8$ MeV in all cases (cf. Table 3)

α (MeV)	a_s (fm)	r_e (fm)	$\lvert\epsilon\rvert$ (MeV)
8.0	−0.8	117.0	
4.0	−3.5	17.9	
2.0	−6.6	11.8	
−1.0	−26.1	7.58	
−5.0	22.4	5.9	0.06
−15.0	7.1	3.8	1.34
−35.0	4.5	2.7	6.49



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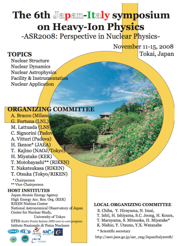


Peak positions of possible continuum s-states in ^{13}Be are not close enough to thereshold to make accurate predictions by the

effective range theory to 1^0 order: higher orders needed or **not a simple single particle state**

$$k \cot an\, \delta = -\frac{1}{a_l} + \frac{1}{2} r_l k^2 - P_l r_l^3 k^4 + O(k^6).$$

α (MeV)	a_s (fm)	r_e (fm)	$\lvert\epsilon\rvert$ (MeV)
8.0	−0.8	117.0	
4.0	−3.5	17.9	
2.0	−6.6	11.8	
−1.0	−26.1	7.58	
−5.0	22.4	5.9	0.06
−15.0	7.1	3.8	1.34
−35.0	4.5	2.7	6.49

In Progress

a_s & r_s from fit to exact phase shifts

Angela Bonaccorso

$^{14}Be\ T_{1/2}=4.35ms\pm0.17$

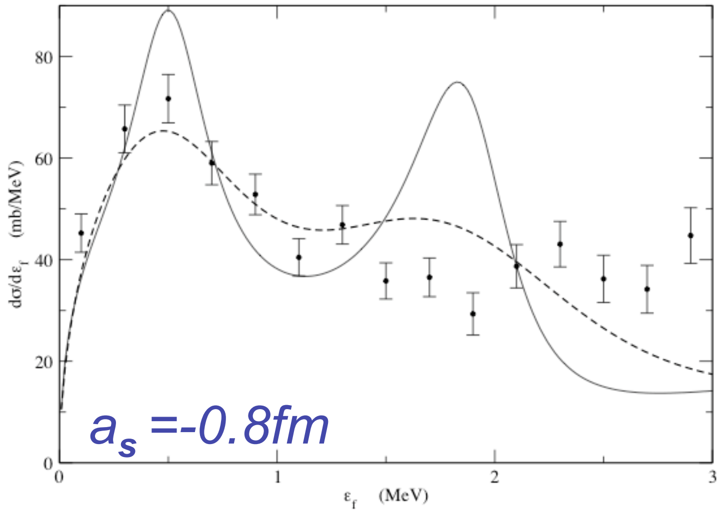


Figure 12: Sum of all transitions from the s initial state with ε_f=-1.85 MeV (solid line). Experimental points from L. Chulkov et al. [56]. Dashed line is the folding of the calculated spectrum with the experimental resolution curve.

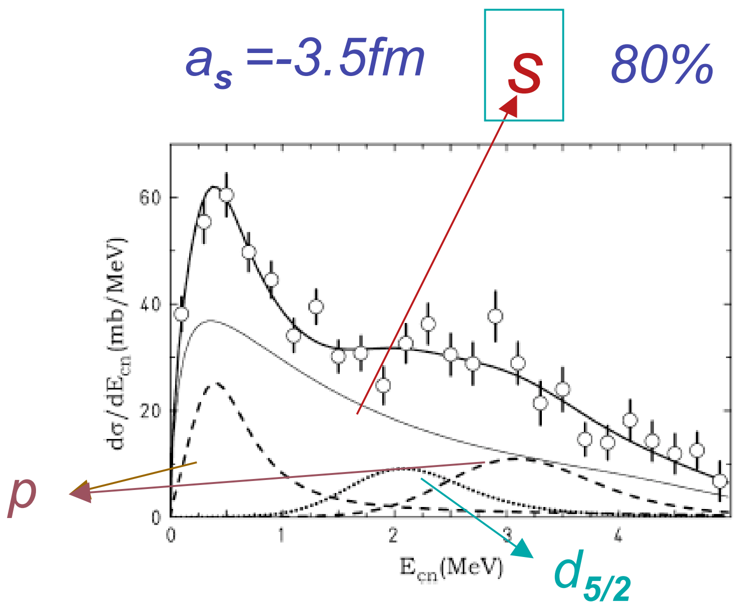
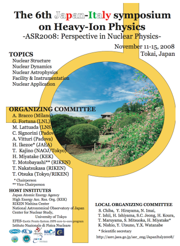


Fig. 10. Differential cross section as a function of the relative energy between ^{12}Be and a neutron. The data points are the same as those in Fig. 4, panel 3. The thin-solid line corresponds to the $1/2^+$ virtual state, the dotted line is the contribution from the $5/2^+$ state. Dashed lines display the $1/2^-$ state and its satellite at low energy. The thick-solid line is the sum of the reaction branches.



Systematic investigation of the drip-line nuclei ^{11}Li and ^{14}Be and their unbound subsystems ^{10}Li and ^{13}Be.

Nucl. Phys. A791 (2007) 267.

H. Simon [a,b], M. Meister [a,c], T. Aumann [b,d], M.J.G. Borge [e], L.V. Chulkov [b,f], Th. W. Elze [g], H. Emling [b], C. Forseén [h,c], H. Geissel [b], M. Hellström [b], B. Jonson [c], J. V. Kratz [d], R. Kulessa [i], Y. Leifels [b], K. Markenroth [c], G. Münzenberg [b], F. Nickel [b], T. Nilsson [a,c], G. Nyman [c], A. Richter [a], K. Riisager [j], C. Scheidenberger [b], G. Schrieder [a], O. Tengblad [e] and M. V. Zhukov [c]

see also Kondo , Nakamura et al, in preparation

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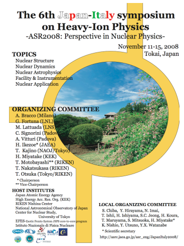
G. Blanchon, N. Vinh Mau and A.B. in preparation



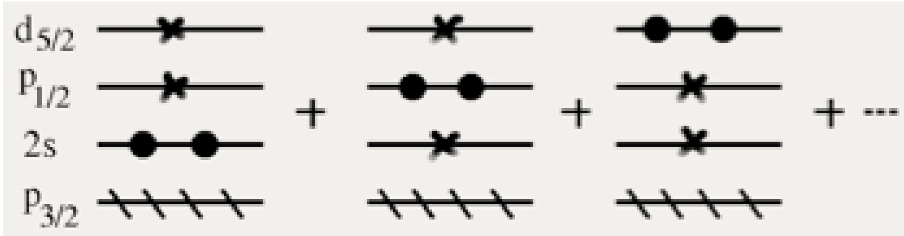


Figure 3.5: Formation of ^{13}Be with one neutron (crosses) added to an open shell ^{12}Be. Only one neutron is added in each configuration, but the cross indicates states which can be populated.

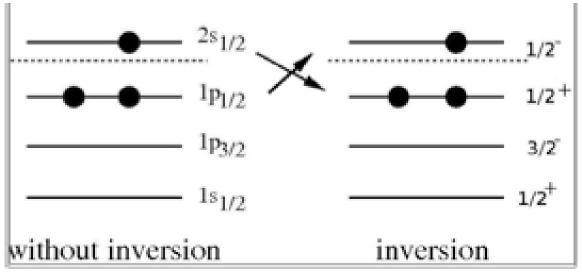


Table 3.3: ^{13}Be neutron spectrum with and without inversion with the corresponding α_l from Eq.(3.102). The continuum is discretized in a 20 fm box.

Inversion				Without Inversion			
Core							
l	j	ε (MeV)	α_l	l	j	ε (MeV)	α_l
0	1	-28.00	0	0	1	-28.00	0
1	3	-6.58	0	1	3	-6.58	0
0	1	-3.15	-23.3	1	1	-3.03	0
Continuum							
l	j	ε (MeV)	α_l	l	j	ε (MeV)	α_l (MeV)
1	1	0.67	8.9	0	1	0.09	-4.4
1	3	1.20	0	1	3	1.20	0
1	1	1.27	0	1	1	1.27	0
2	3	1.83	0	2	3	1.83	0
0	1	1.97	0	0	1	1.97	0
2	5	2.00	-2.4	2	5	2.00	-2.4

G. Blanchon, Ph.D. Thesis, Pisa July 2008, http://www.infn.it/thesis/

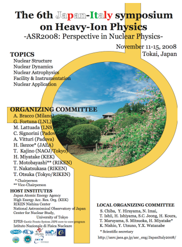


Table 3.2: Theoretical results vs. experimental data. E and S_{2n} are respectively the energy of the excited states and the two-neutron separation energy in MeV. rms, $\sqrt{\langle\rho^2\rangle}$ and $\sqrt{\langle\lambda^2\rangle}$ are respectively the root mean squared radius, the average distance between the two halo neutrons and the average distance between the c.m. of the core and the c.m. of the two neutrons out of the core (see Fig. 3.2) in fm. B(E1) is the total dipole strength. The sum rule is calculated with Eq.(3.101) in e.fm^2. B(E2) is the total quadrupole strength in e^2.fm^4.

		RPA ^{10}Be core	Experiment	RPA ^{12}Be core Inversion	RPA ^{12}Be core Without Inversion
^{8}Be	E_{2+}	3.1	3.04		
^{10}Be	S_{2n}	8.29	8.5		
	E_{0+}		6.18	5.4	7.2
	E_{1-}		5.96	4.8	
^{12}Be	S_{2n}	3.63	3.67±0.015	3.83	1.16
	rms	2.76	2.59±0.06		
	E_{0+}	2.47	2.25		
	E_{1-} (1)	2.57	2.71		
	E_{1-} (2)	4.25			
	E_{1-} (3)	4.47			
	E_{2+} (1)		2.10		
	E_{2+} (2)	3.72	3.37		
^{14}Be	S_{2n}		1.34±0.11	1.33	0.54
	rms		3.1±0.4	2.90	3.51
	$\sqrt{\langle\rho^2\rangle}$		5.4±1.0	4.6	8.7
	$\sqrt{\langle\lambda^2\rangle}$		4.5±1.0	4.0	5.6
	E_{0+} (1)		2.56	2.74	1.71
	E_{0+} (2)			3.11	2.71
	E_{0+} (3)				2.83
	E_{1-} (1)		3.14	2.89	1.79
	E_{1-} (2)			3.38	1.86
	E_{1-} (3)			3.50	3.52
	E_{2+} (1)		1.59		
	E_{2+} (2)			3.16	2.03
	E_{2+} (3)			3.67	2.45

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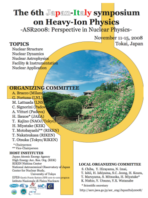
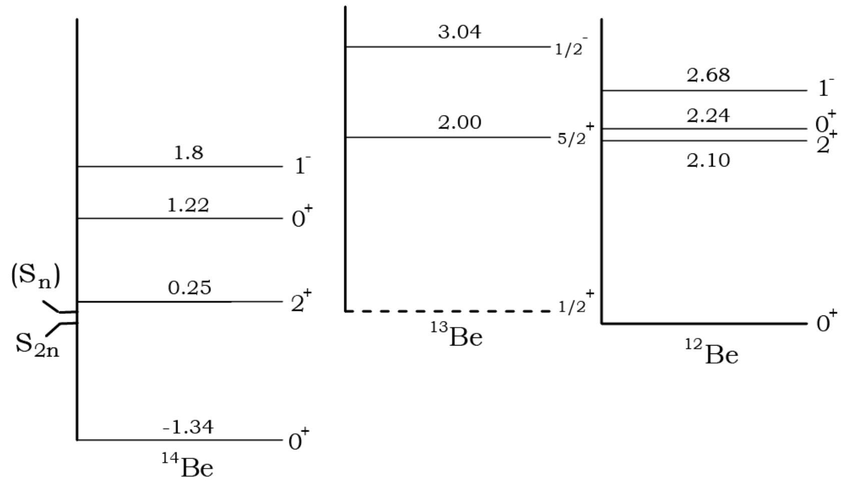
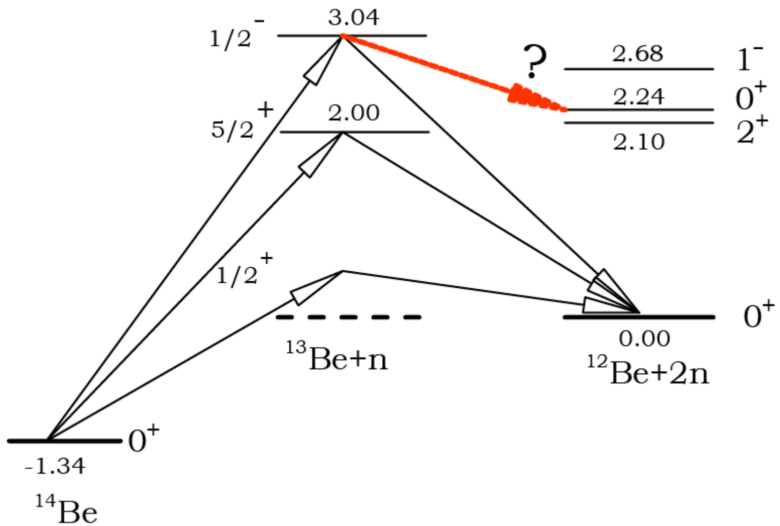


H. Simon et al. Nucl. Phys. A791 (2007) 267.





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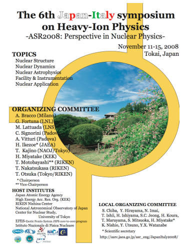

Conclusions

- A time dependent theory for projectile fragmentation reaction has been established which contains the sudden approximation and R-matrix theories as limiting cases.

- 2p correlations and particle-vibration couplings play a fundamental role.

- Importance of coupling to core excited states.

- For the ***first time the shell ordering of ^{13}Be has been established*** **theoretically** on a firm basis and ***parity inversion across threshold has been proved to persist for N=9 isotones***

- Much care is needed in analyzing experimental results in order not to draw misleading or unphysical conclusions: sudden approximation vs.time dependent, use of Breit-Wigner resonance form (vs. exact S-matrix or R-matrix),importance of various initial state components.

Japan is playing a leading role in this type of research