Nuclear Reactions with No-Core Shell Model

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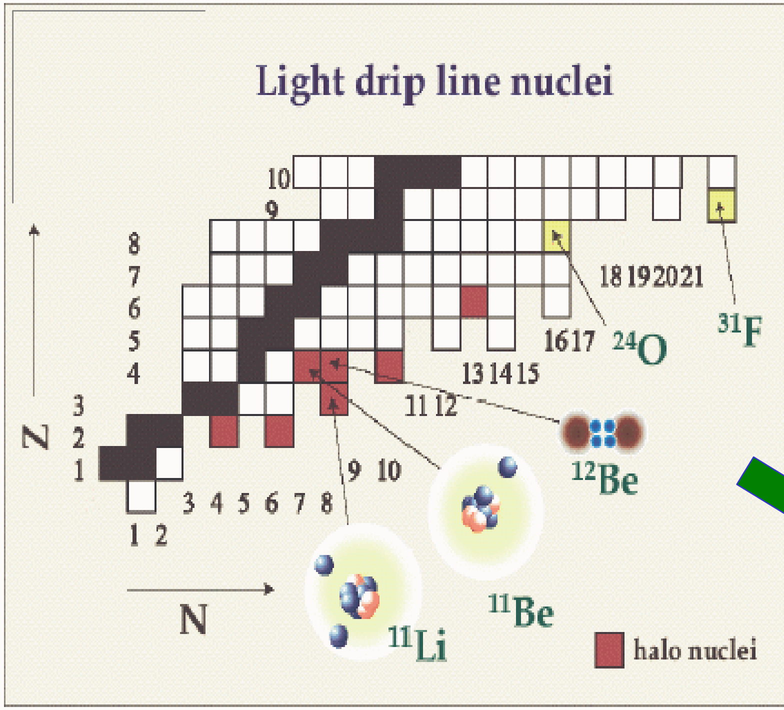
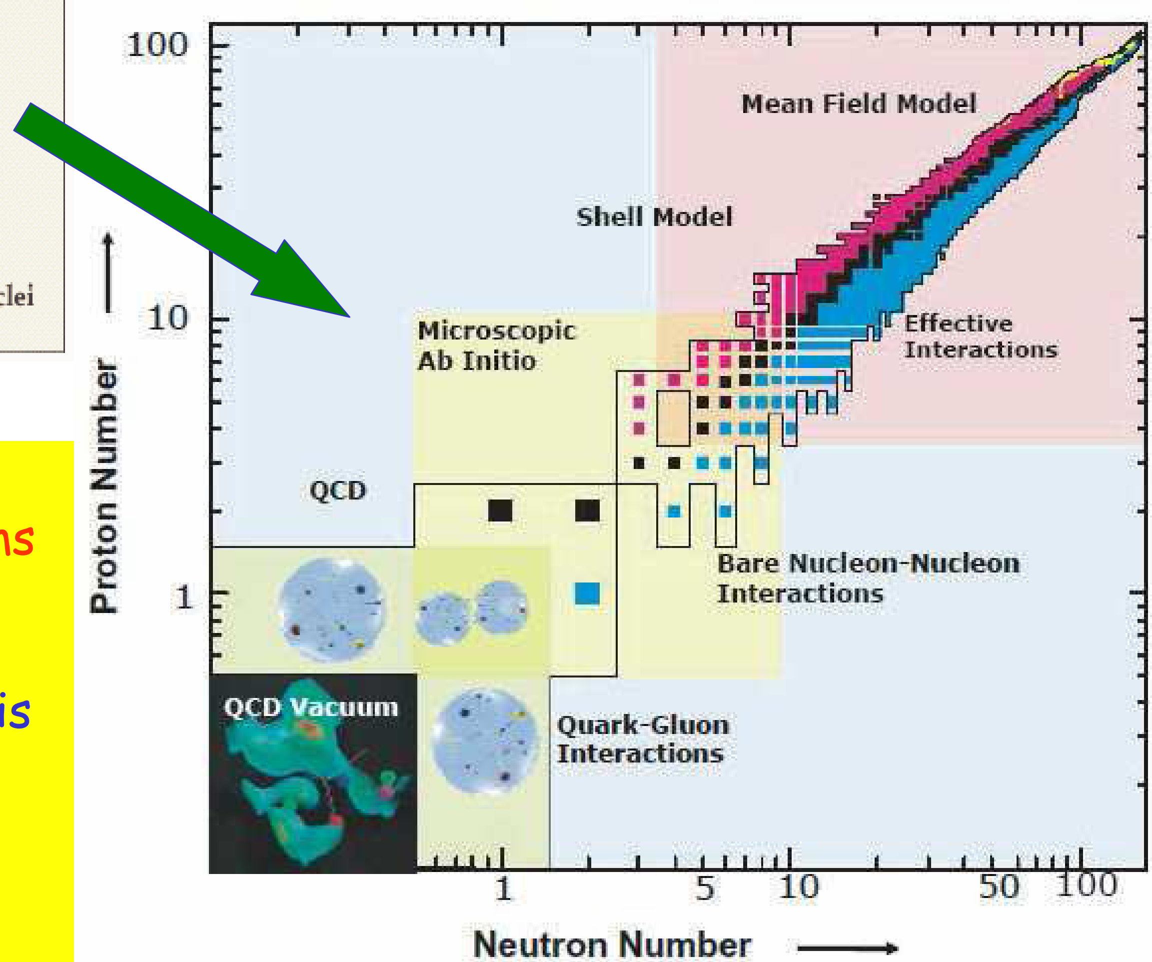
The nuclear structure problem





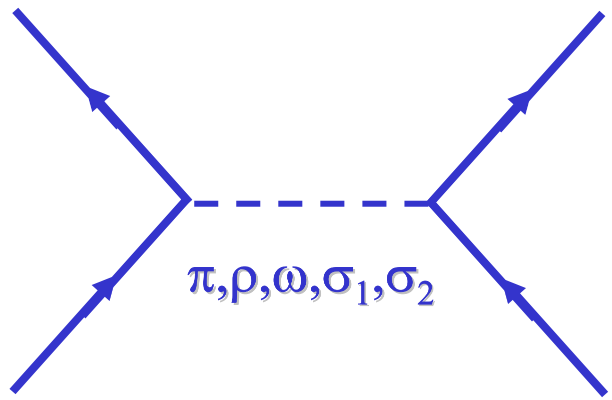
Nuclear many-body problem:
one of the hardest problems of all physics!

BECAUSE

- Nucleon-nucleon interaction is complicated
- Nucleons are composite particles
- Requires large computation

NN Potentials

Argonne potentials (AV18, AV8')
Wiringa, Stoks, Schiavilla, PRC **51**, 38 (1995)
- Electromagnetic + one pion exchange + intermediate- and short-range, local, in coordinates



Bonn potential (CD-Bonn 2000)
Machleidt, PRC **63**, 024001 (2001)
- Based on meson-exchange
- Nonlocal, in momentum space

Effective field theory
Ordóñez, Ray, van Kolck, PRC **53**, 2086 (1996)
Epelbaum, Glöckle, Meißner, NP ***A637***, 107 (1998)
Entem and Machleidt, PRC **68**, 041001(R) (2003)
- Based on Chiral Lagrangians
- Expansion in momentum relative to a cutoff parameter (~ 1 GeV)
- Generally has a soft core, nonlocal, in momentum space

Phenomenolgical nonlocal potential in coordinate space
P. Doleschall *et al.*, PRC **67**, 064005 (2003)
- INOY = Inside nonlocal, outside Yukawa
- Fits both two-nucleon and three-nucleon properties

Many-body nuclear problem with realistic NN forces

Green's Function Monte Carlo (GFMC)

Pieper, Wiringa, Carlson *et al.*

- Results published up to *A*=10, ^{12}C calculations under way

Coupled-Cluster Method (CCM), Unitary Model Operator Approach (UMOA)

Mihaila and Heisenberg, Dean and Hjort-Jensen (CCM)

K. Suzuki and R. Okamoto (UMOA)

Effective Interaction Hyperspherical Harmonic Method (EIHH)

N. Barnea, W. Leidemann, G. Orlandini

- Converged results for A=6,7, first results for ^{6}Li with AV8'
- Now in principle capable of using realistic three-body forces

Ab Initio No-Core Shell Model (NCSM)

Zheng, Barrett and Vary, 1993, G-matrix

Navratil and Barrett, 1996

- simple when just the two-body effective interaction considered
- much more complicated when three-body interaction included
- unitary transformation based effective interaction
- convergence to exact solution

Navratil and Ormand, 2003

- Three-body interaction included in *p*-shell nuclei calculations

The No-Core Shell Model (NCSM)

- Many-body Schroedinger equation
  A-nucleon wave function

$$H\Psi = E\Psi$$

$$H = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m_i} - \sum_{i<j} V_{NN}\left(\mathbf{r}_i - \mathbf{r}_j\right)$$

- Add the center-of-mass potential

$$H_{CM} = \frac{1}{2} A m \Omega^2 \mathbf{R}^2 = \sum_i \frac{1}{2} m \Omega \mathbf{r}_i^2 - \sum_{i<j} \frac{m\Omega^2}{2A}\left(\mathbf{r}_i - \mathbf{r}_j\right)^2$$

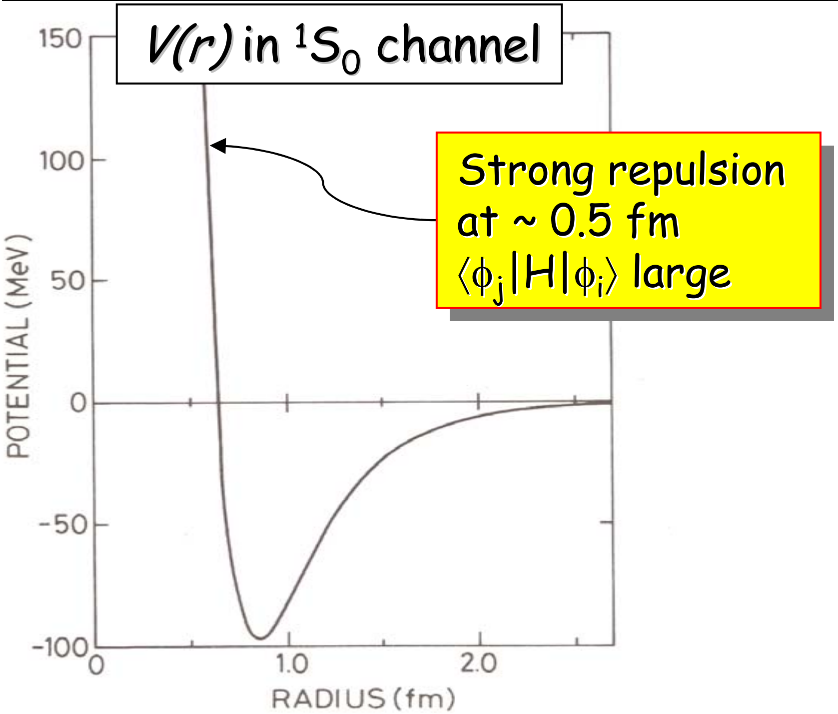
$$\Rightarrow \quad H = \sum_{i=1}^{A} \left[\frac{\mathbf{p}_i^2}{2m} + \frac{1}{2} m \Omega \mathbf{r}_i^2 \right] - \sum_{i<j} \left[V_{NN}\left(\mathbf{r}_i - \mathbf{r}_j\right) - \frac{m\Omega^2}{2A}\left(\mathbf{r}_i - \mathbf{r}_j\right)^2 \right]$$

Convenient to work with Harmonic oscillator basis

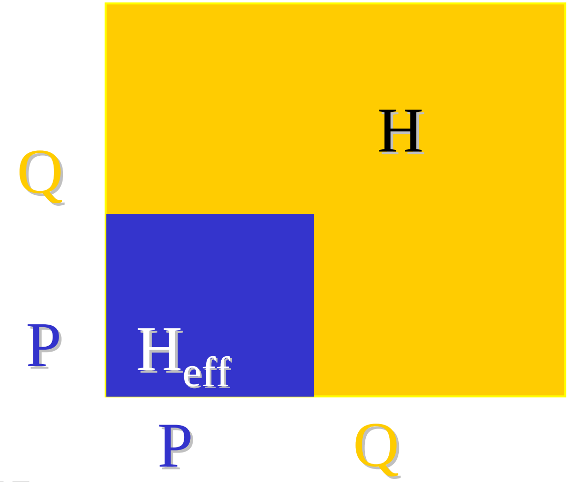
- **The Good:**
  - Does not affect the intrinsic motion
  - Exact separation between intrinsic and center-of-mass motion
- **The Bad:**
  - Radial behavior is not right for large **r**
  - Provides a confining potential, so all states are effectively bound

Nuclear structure with *NN*-interaction

Strong repulsion → Infinite space!



Solution: Effective interactions



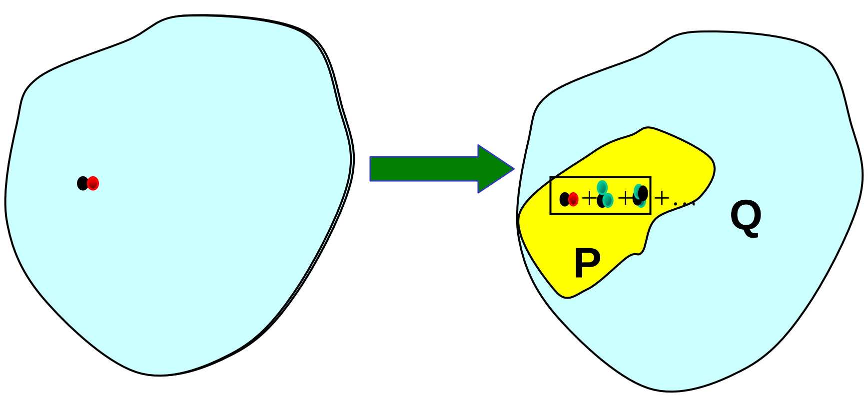
$$H \Rightarrow \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{nP}, \ldots, \varepsilon_\infty$$

$$H_{eff} \Rightarrow \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{nP}$$

Find H_{eff} with the decoupling condition (Lee-Suzuki):

$$QXHX^{-1}P=0$$

or $$H_{eff}=PXHX^{-1}P$$



Impossible problem → Difficult problem

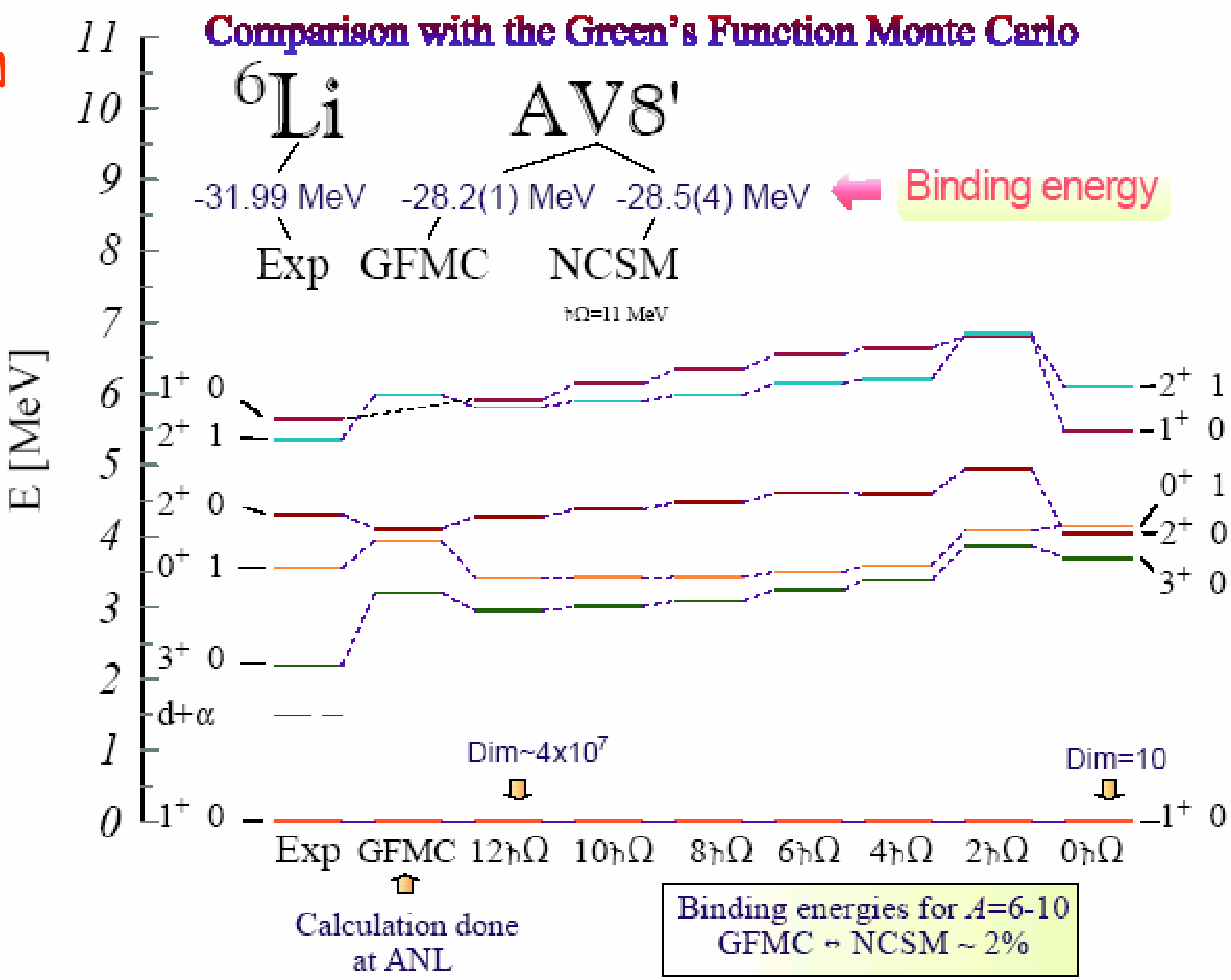
Two, three, four, … A-body operators

Some of the NCSM achievements

- Method for solving the nuclear structure problem for light nuclei ($A \lesssim 12$)
- Apart from the GFMC the only working method for $A>4$ at present

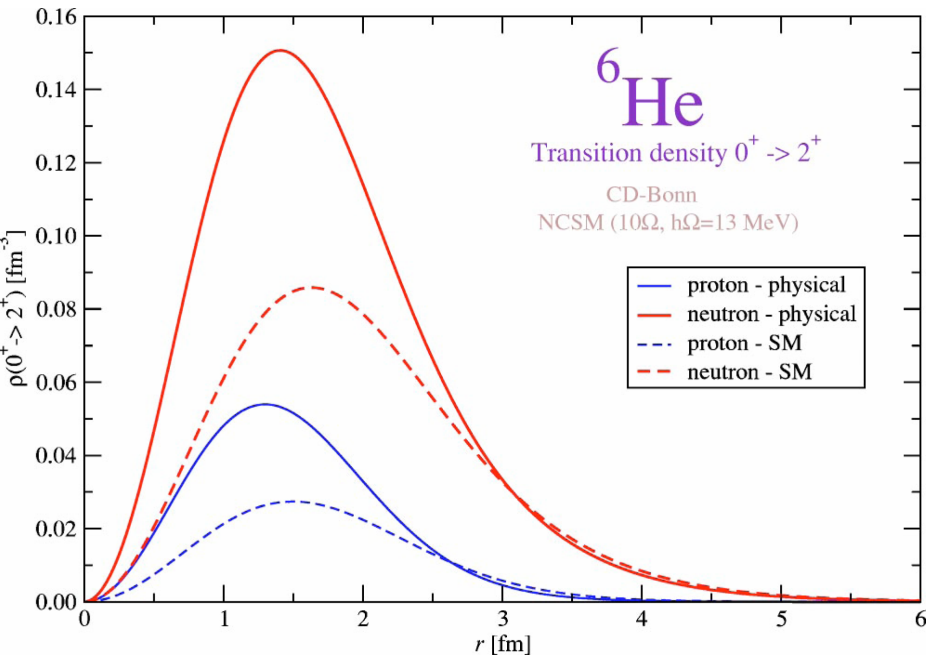
Advantages

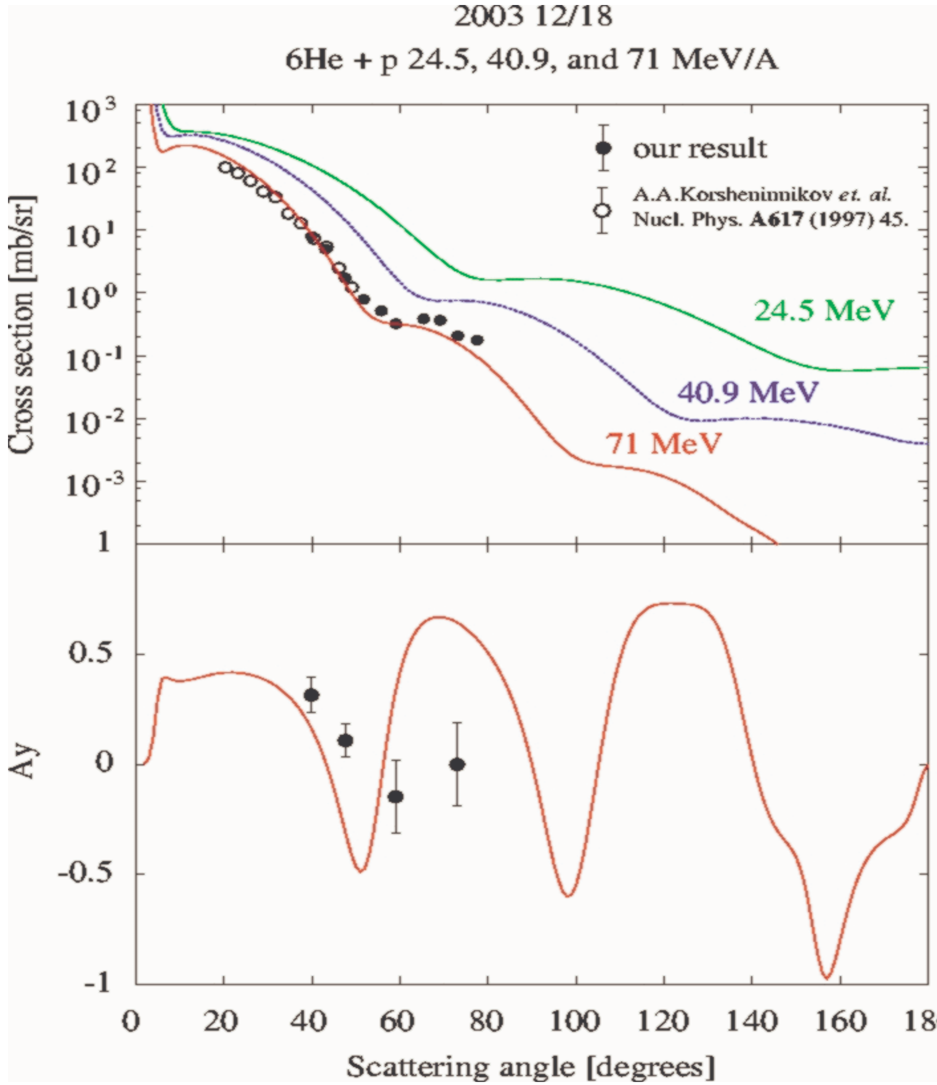
- applicable for any NN potential (e.g. effective field theory)
- Easily extendable to heavier nuclei
- Determination of the structure of the three-body force



NCSM and reactions with light nuclei

Navratil, PRC 70, 014317 (2004): Translationally invariant densities





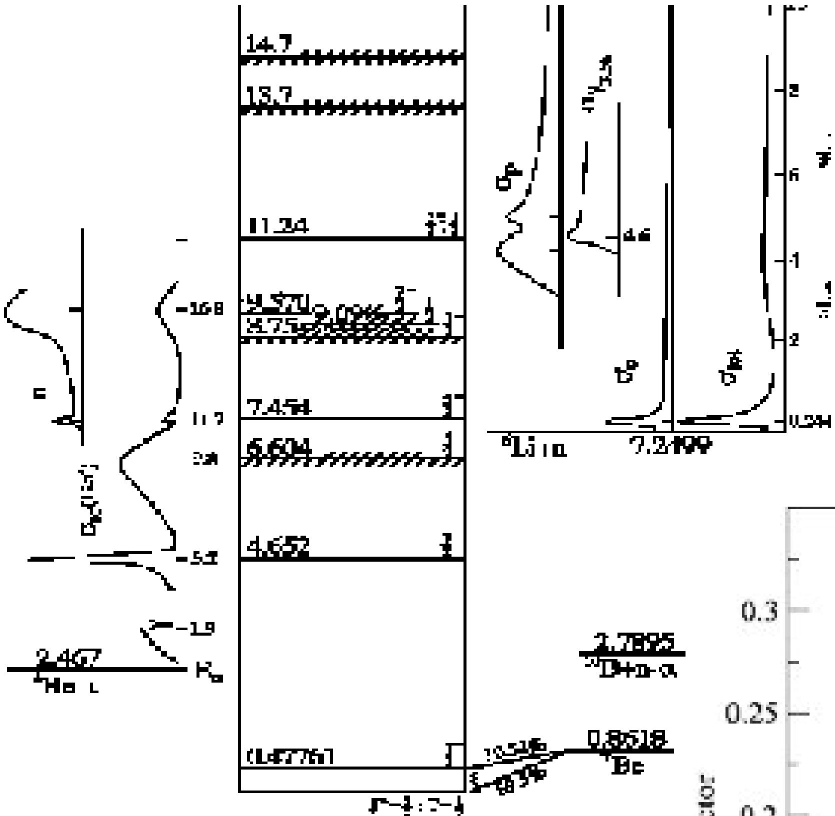
NCSM ^{6}He ground state density

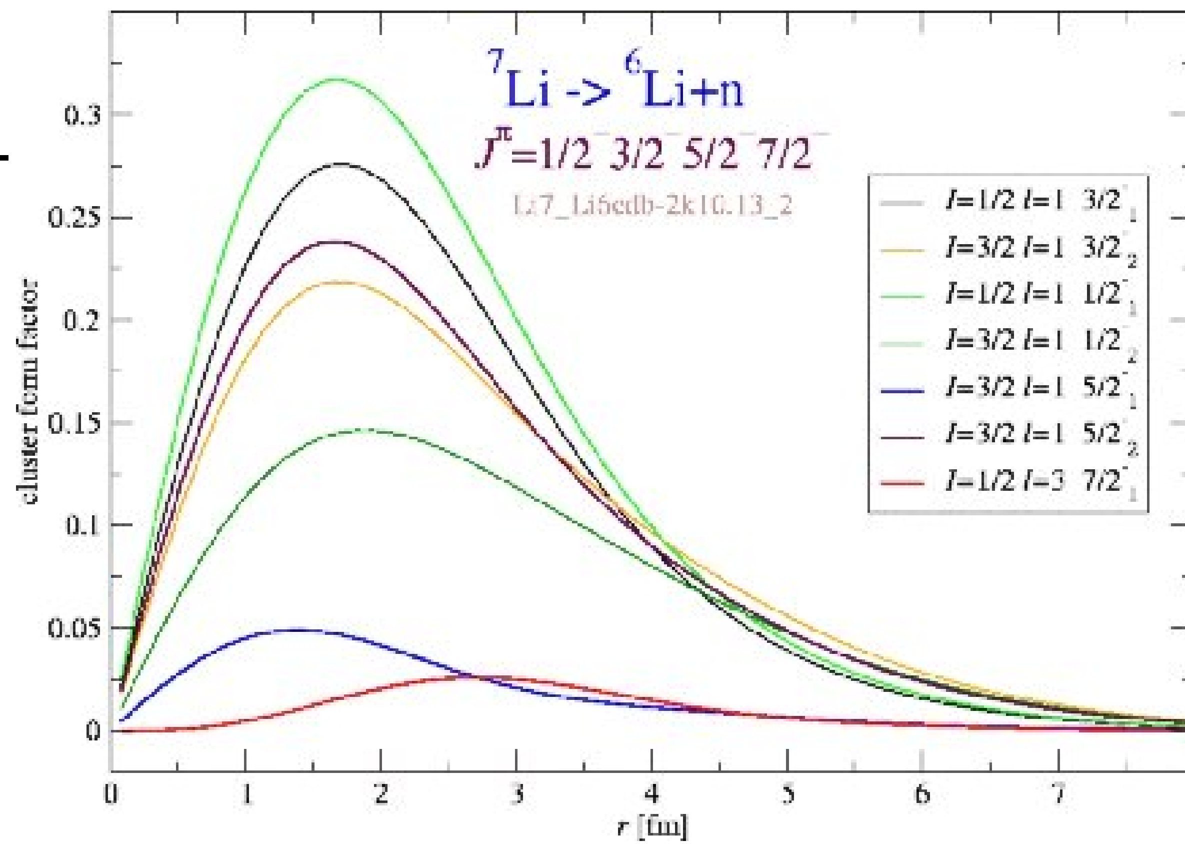
JLM folding optical potential

Direct reaction code FRESCO (I. Thompson)

Reasonable agreement with experiment also for other energies (24.5 MeV - Dubna, 40.9 MeV - Ganil

Clustering in light nuclei



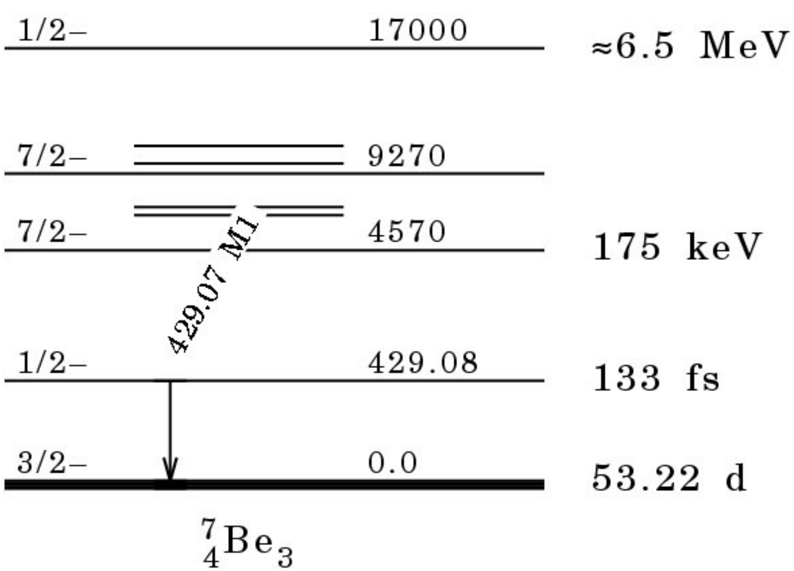


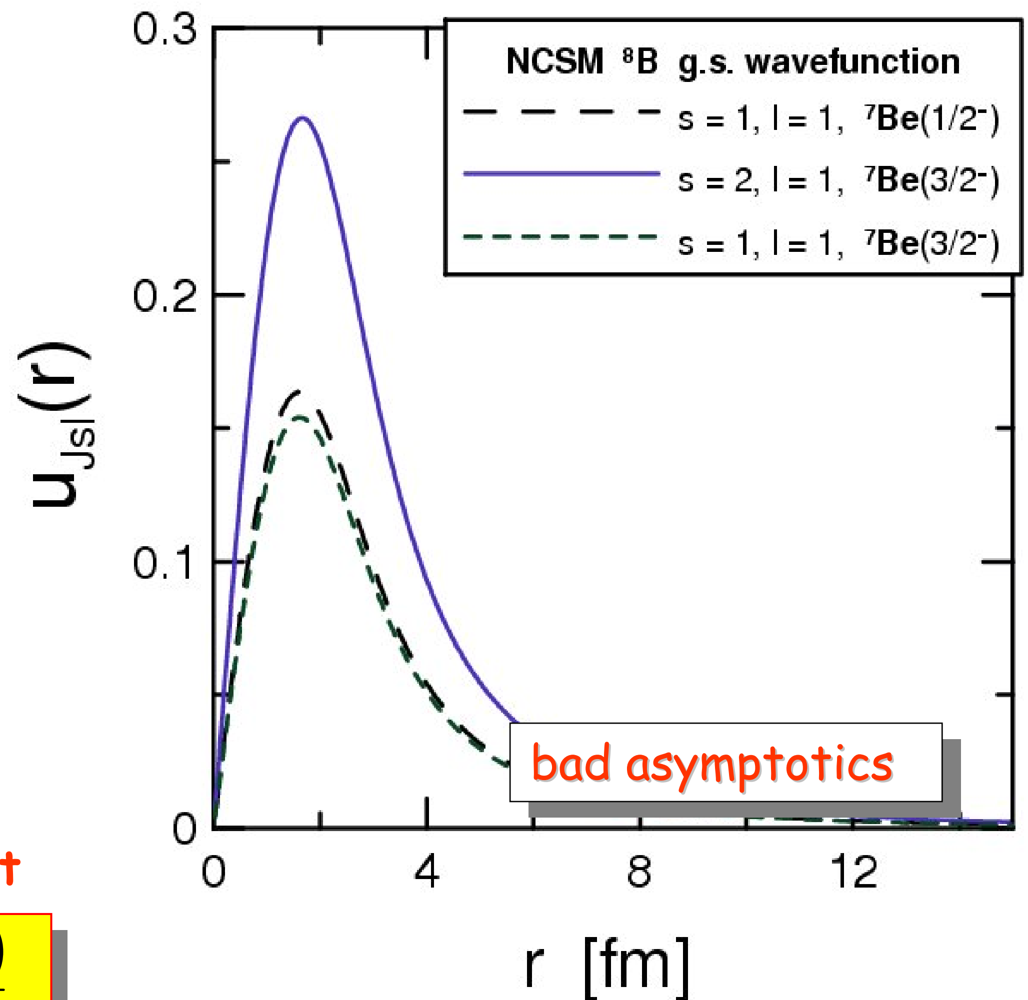
In weakly-bound nuclei:

Harmonic oscillator basis

asymptotics not correct

Clustering in weakly-bound nuclei





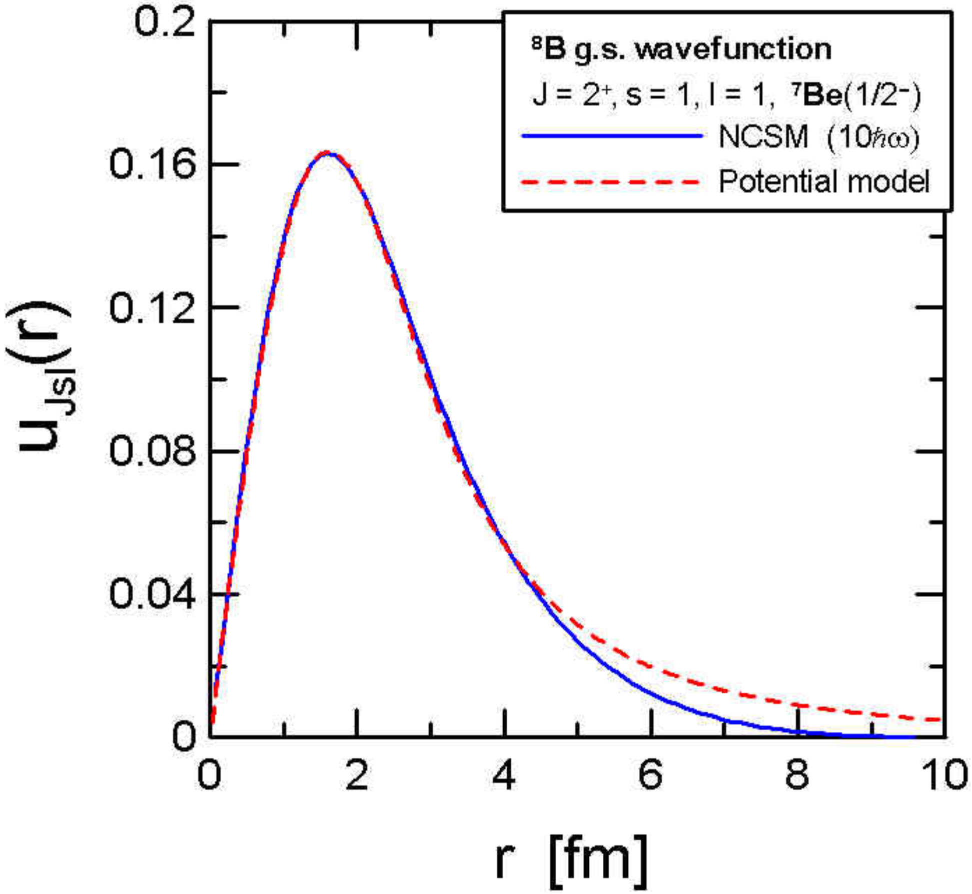
How to fix it?

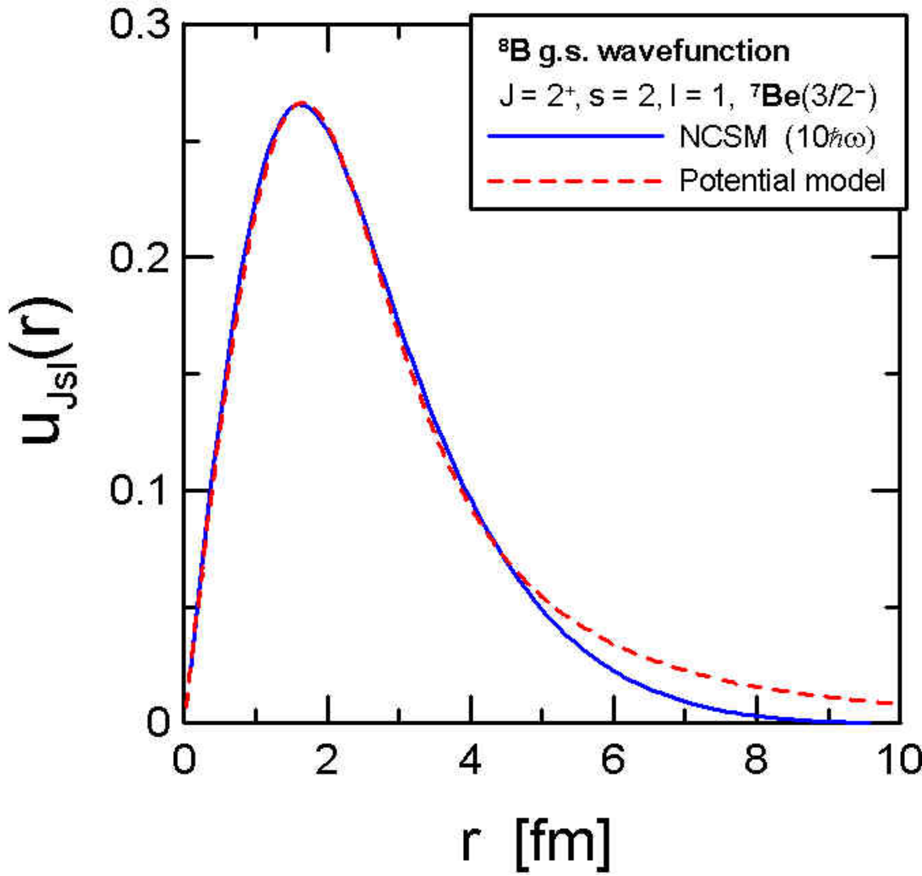
- go over correct Coulomb tail at large *r*'s

$$u_{klsj}(r) \rightarrow C_{km}\frac{W_{km}(2\gamma r)}{r}$$

$$\gamma^2 = \frac{2\mu_{17}B_{17}}{\hbar^2}, \quad m = l + \frac{1}{2}$$

Clustering in weakly-bound nuclei: fixing the asymptotics

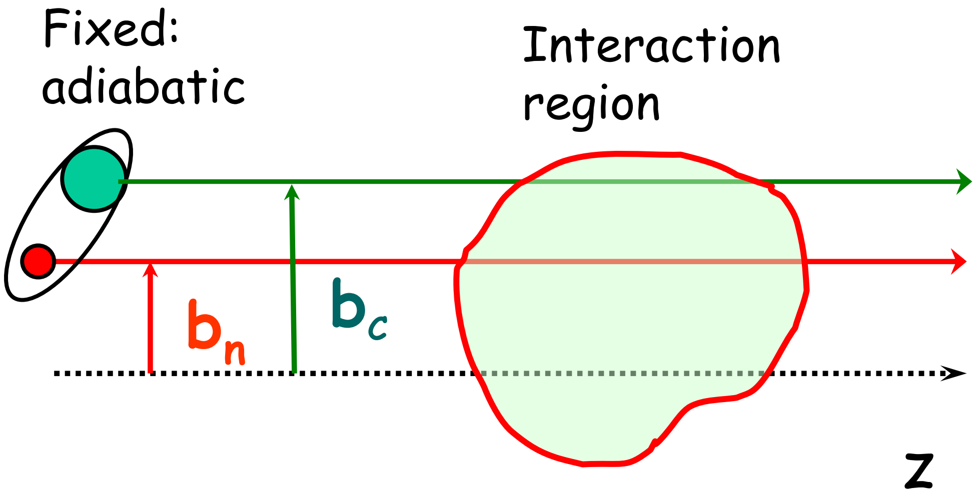




Fits obtained with a potential model:
adjusting Woods-Saxon+Coulomb+spin-orbit

- spectroscopic factors from NCSM
- internal part from NCSM
- correct asymtotic part

Breakup Reactions Review: (a) elastic



Elastic:

including breakup effects

$$\Psi^{eik}(\mathbf{r}) = S_C(\mathbf{b}_C)\, S_n(\mathbf{b}_n)\, e^{i\mathbf{k}\cdot\mathbf{r}}\, \phi_0$$

$$S_{elast}(\mathbf{b}) = \langle \phi_0 | S_C(\mathbf{b}_C)\, S_n(\mathbf{b}_n) | \, \phi_0 \rangle$$

Best possible wfs:

(Spectroscopy)

Survival amplitude

for projectile at impact parameter b

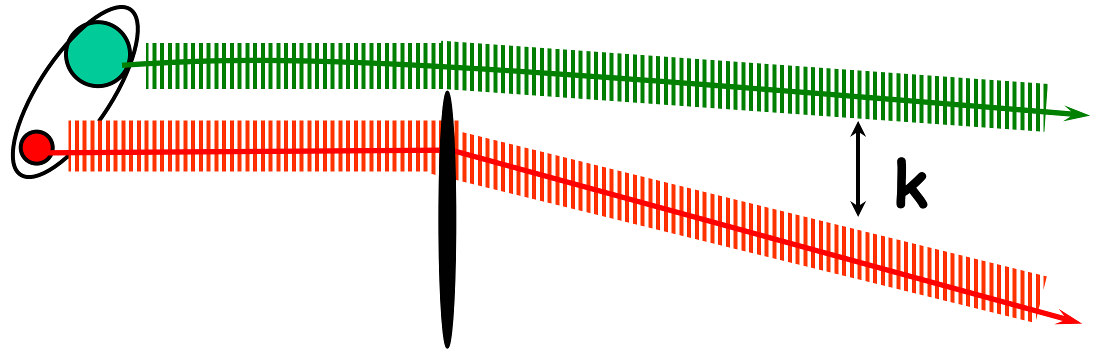
Survival amplitudes

for particles C and n at impact parameters b_C and b_n

(Dynamics)

(b) Diffraction Dissociation:

Breakup amplitude:
to state $\phi_{\mathbf{k}}$



$$1 - S_{dif.dis.}(\mathbf{b}) = 1 - S_C(\mathbf{b}_C) + 1 - S_n(\mathbf{b}_n) - [1 - S_C(\mathbf{b}_C)][1 - S_n(\mathbf{b}_n)]$$

$\phi_{\mathbf{k}} \perp \phi_0$ ⟹

$$S_{dif.dis.}(\mathbf{b}) = \langle \phi_{\mathbf{k}} | S_C(\mathbf{b}_C) S_n(\mathbf{b}_n) | \phi_0 \rangle$$

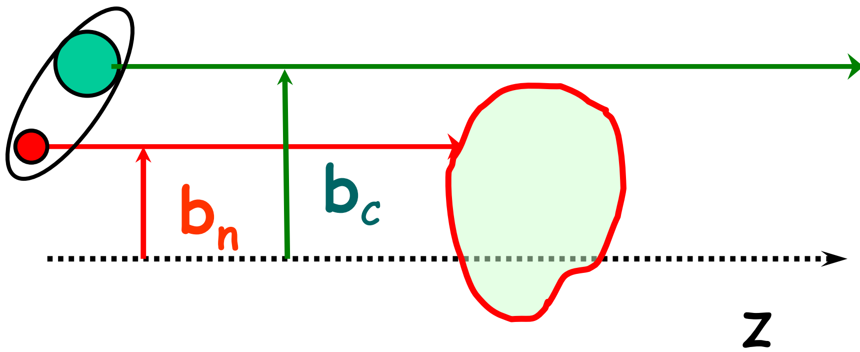
Closure:

$$\int d\mathbf{k} |\phi_{\mathbf{k}}\rangle\langle\phi_{\mathbf{k}}| = 1 - |\phi_0\rangle\langle\phi_0| - |\phi_1\rangle\langle\phi_1| - \cdots$$

⟹ Breakup X-section (only one bound state):

$$\sigma_{dif.dis.}(\mathbf{b}) = \int d\mathbf{b} \left[\langle \phi_0 | |S_C S_n|^2 | \phi_0 \rangle - |\langle \phi_0 | S_C S_n | \phi_0 \rangle|^2 \right]$$

(c) Stripping:

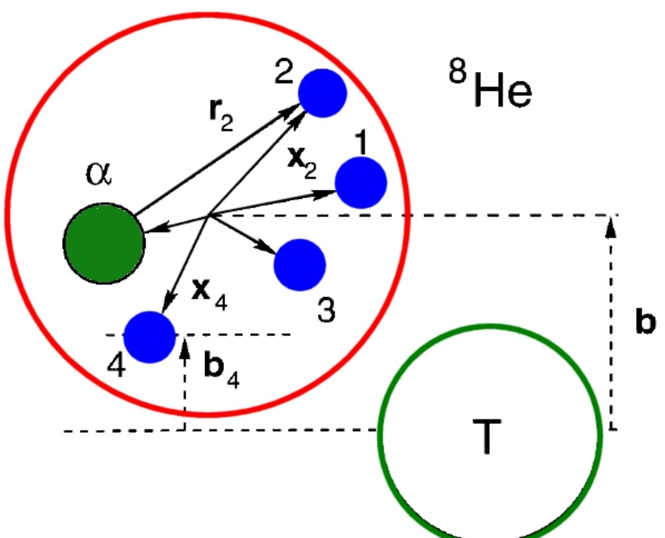


$$|S_C(\mathbf{b}_C)|^2\left(1-|S_n(\mathbf{b}_n)|^2\right)$$

C survives, n absorbed

$$\sigma_{strip}(\mathbf{b}) = \int d\mathbf{b} \left\langle \phi_0 \middle| \, |S_C|^2\left(1-|S_n|^2\right) \middle| \, \phi_0 \right\rangle^2$$

(d) Composite particles:

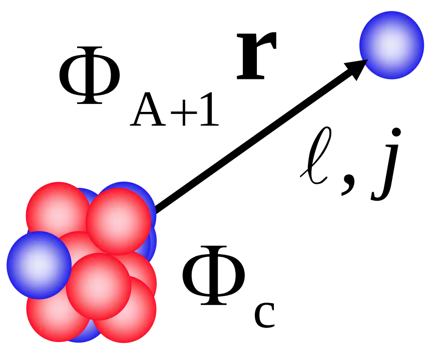


$$S_{dif.dis.}(\mathbf{b}) = \left\langle \phi_8 \middle| S_\alpha(\mathbf{b}_\alpha) \prod_{i=1}^{4} S_i(\mathbf{b}_i) \middle| \phi_8 \right\rangle$$

$$\prod_{j\ survive} |S_j(\mathbf{b}_j)|^2 \prod_{k\ absorbed} \left(1-|S_k(\mathbf{b}_k)|^2\right)$$

Spectroscopic factors

Nucleon removal from Φ_{A+1} will leave mass A residue in the ground or an excited state - amplitude for finding nucleon with sp quantum numbers ℓ,j, about core state Φ_c in Φ_{A+1} is



$$F^{c}_{\ell j}(\mathbf{r}\ ) = \langle \mathbf{r}, \Phi_c \,|\, \Phi_{A+1} \rangle, \ \ S_N = E_{A+1} - E_c$$

$$|\, I_{A+1} - j \,| \le I_c \le I_{A+1} + j$$

Usual to write

$$\int d\mathbf{r}\, |\, F^{c}_{\ell j}(\mathbf{r})\,|^2 = C^2 S(\ell j)$$

Spectroscopic factor - occupancy of the state

$$F^{c}_{\ell j}(\mathbf{r}) = \sqrt{C^2 S(\ell j)}\ \phi_0(\mathbf{r})\,; \qquad \int d\mathbf{r}\, |\, \phi_0(\mathbf{r})\,|^2 = 1$$

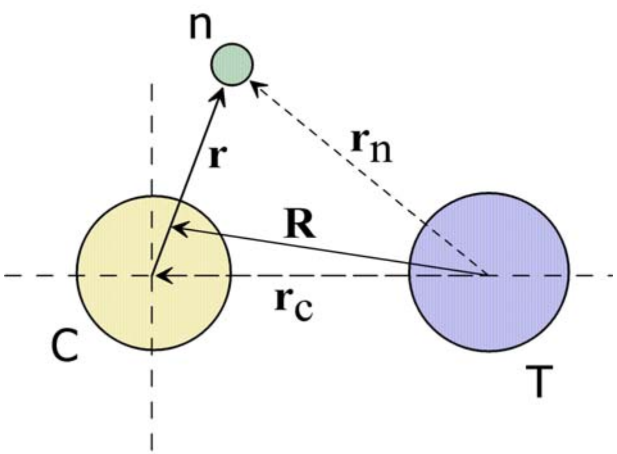
Momentum Distributions: (a) Stripping

C scatters elastically and C+n breaks up:

$$\left|\left\langle \phi_{Continuum}(\mathbf{r}) \right| S_C(\mathbf{b}_C)\, \phi_{lm}(\mathbf{r}) \right\rangle\right|^2$$

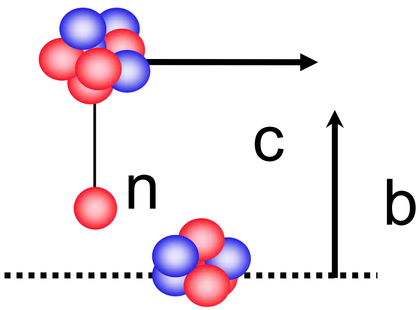
n is absorbed:

$$1-\left|S_n(\mathbf{b}_n)\right|^2$$



$$\mathbf{r}_n = \mathbf{R} + \frac{m_C}{m}\mathbf{r}, \qquad \mathbf{r}_C = \mathbf{R} - \frac{m_n}{m}\mathbf{r}$$

$$\mathbf{K} = \mathbf{k}_C - \mathbf{k}_n, \qquad \mathbf{k} = \frac{m_n}{m}\mathbf{k}_C - \frac{m_C}{m}\mathbf{k}_n$$



$$\phi_{Continuum}(\mathbf{r}) \sim e^{i\mathbf{k}.\mathbf{r}}$$

$$\frac{d\sigma_{strip}}{d^3k_C} = \frac{1}{(2\pi)^3}\sum_m \frac{C^2 S_{lj}}{(2l+1)} \int d^2 b_n \left[1-\left|S_n(\mathbf{b}_n)\right|^2\right] \left| \int d^3 r\, e^{-i\mathbf{k}_C\cdot\mathbf{r}}\, S_C(\mathbf{b}_C)\, \phi_{jlm}(\mathbf{r}) \right|^2$$

(b) Diffraction dissociation

C and n scatters elastically:

$$S_C(\mathbf{b}_C)\,S_n(\mathbf{b}_n)\,\phi_{jlm}(\mathbf{r})$$

Project onto continuum CM and relative coordinates:

$$\left\langle e^{-i\mathbf{K}_\perp\cdot\mathbf{b}}\phi_{\mathbf{k}}^{*}(\mathbf{r})\right| S_C(\mathbf{b}_C)\,S_n(\mathbf{b}_n)\,\phi_{jlm}(\mathbf{r})\Big\rangle$$

CM

Relative motion

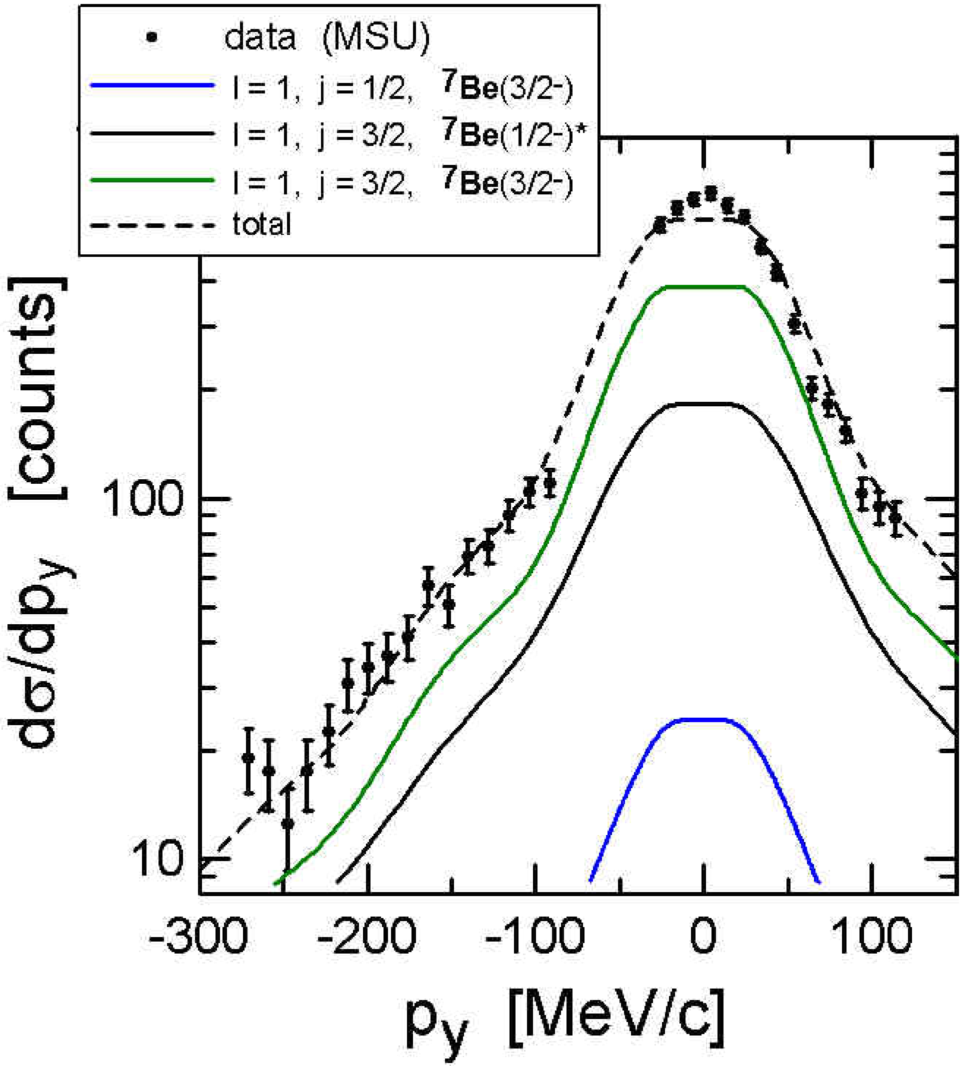
$$\frac{d\sigma_{dif.dis.}}{d^2K_\perp\, d^3k}=\frac{1}{(2\pi)^5}\sum_m \frac{C^2 S_{lj}}{(2l+1)}\left|\int d^3r\, d^2b\, e^{-i\mathbf{K}_\perp\cdot\mathbf{b}}\phi_{\mathbf{k}}^{*}(\mathbf{r})\,S_C(\mathbf{b}_C)\,S_n(\mathbf{b}_n)\,\phi_{jlm}(\mathbf{r})\right|^2$$

Solving these integrals: Gaussian expansion method

C.B., Hansen, PRC C70, 034609 (2004)

Transverse Momentum Distributions with NCSM

$$\frac{d\sigma_{strip}}{d^2k_C^{\perp}} = \frac{1}{2\pi}\sum_m \frac{C^2S_{lj}}{(2l+1)}\int d^2b_n \left[1-\left|S_n(\mathbf{b}_n)\right|^2\right]\int dz \left|\int d^2\rho\, e^{-i\mathbf{k}_C^{\perp}\cdot\mathbf{r}}\, S_C(\mathbf{b}_C)\,\phi_{NCSM}^{ljm}(\mathbf{r})\right|^2$$



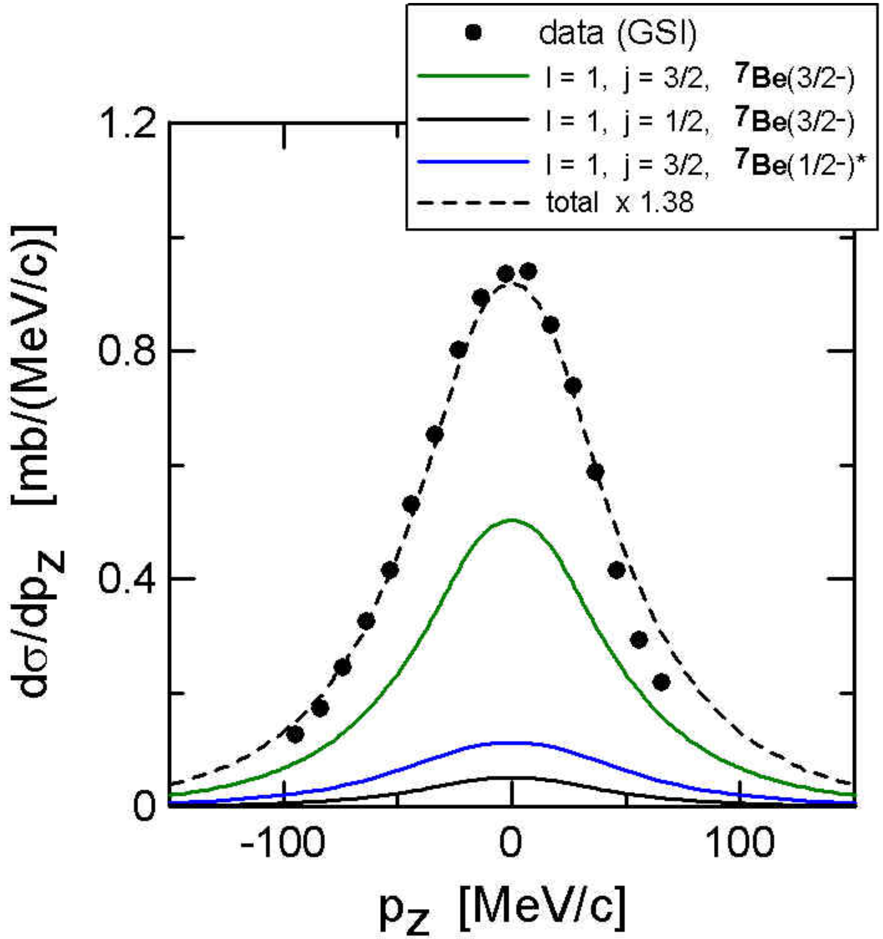
^{8}B (41 MeV/nucleon) + ^{9}Be → 7Be + X

With NCSM wavefunctions (10 ħω)

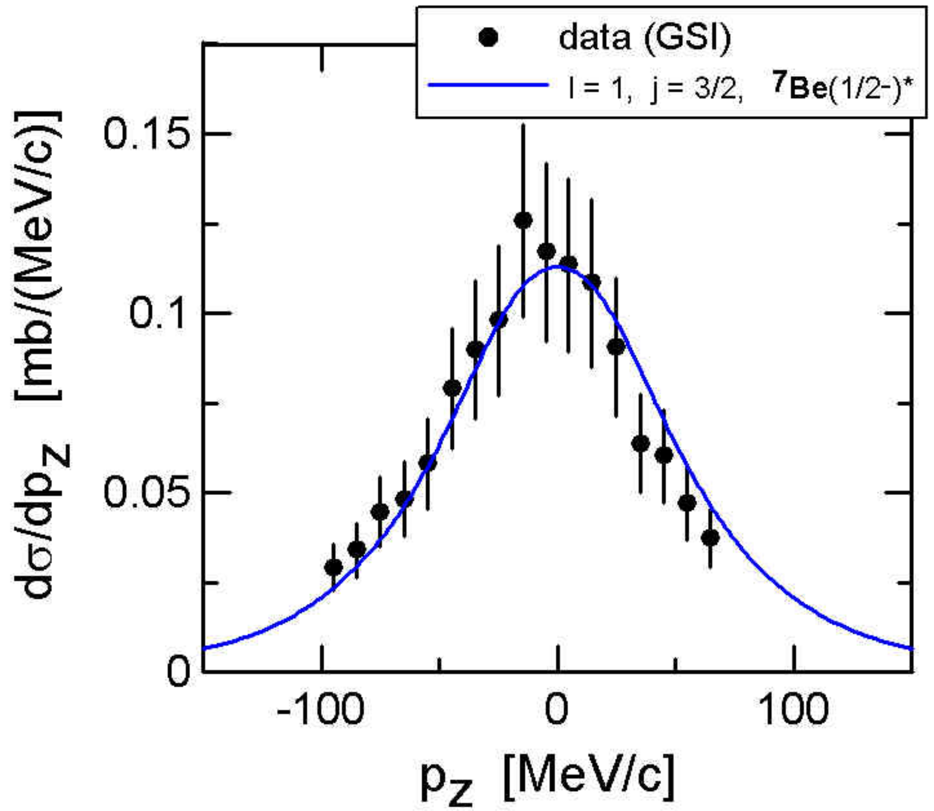
- l=1, j=1/2, I_{7Be}=3/2: C^2S=0.085
- l=1, j=3/2, I_{7Be}=1/2: C^2S=0.280
- l=1, j=3/2, I_{7Be}=3/2: C^2S=0.958

Longitudinal Momentum Distributions with NCSM

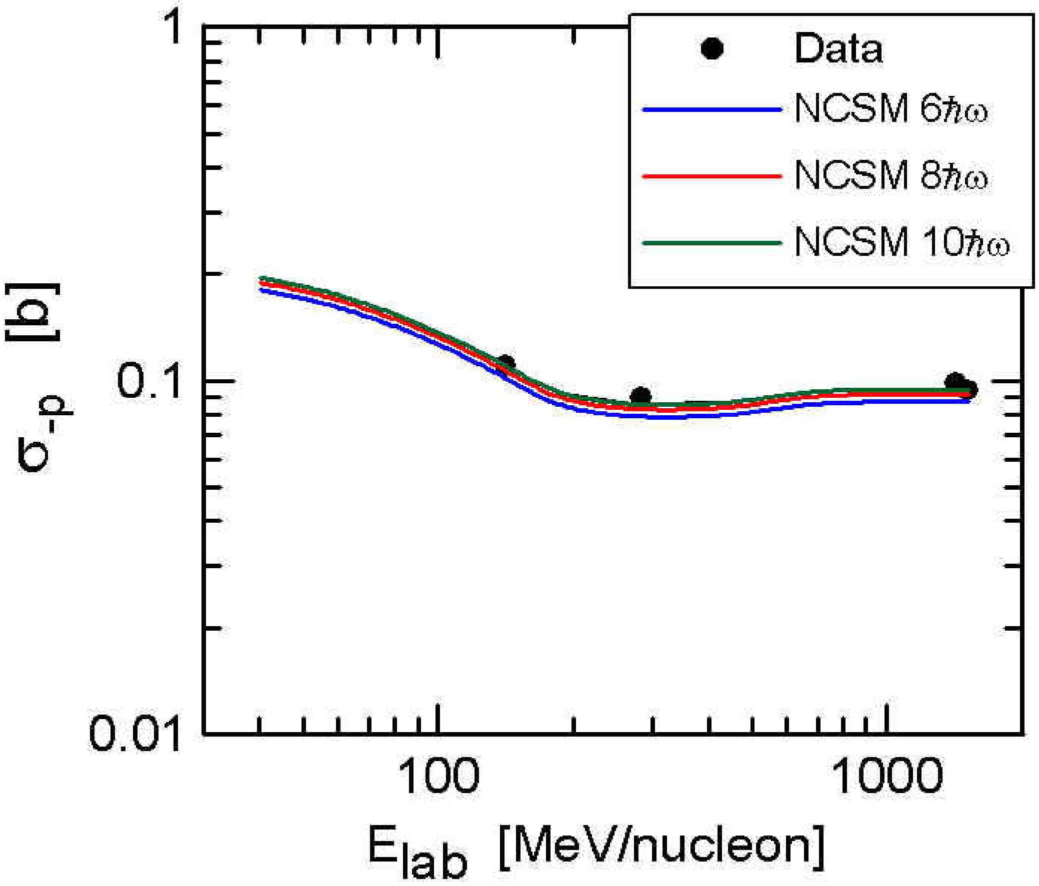
$$\frac{d\sigma_{strip}}{dk_C^z} = \frac{1}{2\pi}\sum_m \frac{C^2 S_{lj}}{(2l+1)} \int d^2 b_n \left[1 - |S_n(\mathbf{b}_n)|^2\right] \int d^2\rho \, |S_C(\mathbf{b}_C)|^2 \left| \int dz \, e^{-ik_C^z \cdot z} \; \phi_{NCSM}^{jlm}(\mathbf{r}) \right|^2$$



^8B (938 MeV/nucleon) + ^{12}C → ^7Be + X



Proton removal cross sections: sensitivity to basis size



6 ħΩ
8 ħΩ
10 ħΩ

Why is ^{8}B interesting?

Understanding our Sun

p-p chain

$p(p,e^+\nu)d$

$d(p,\gamma)^3He$

86% | 14%

$^3He(^3He,2p)^4He$

$^3He(\alpha,\gamma)^7Be$

14% | 0.02%

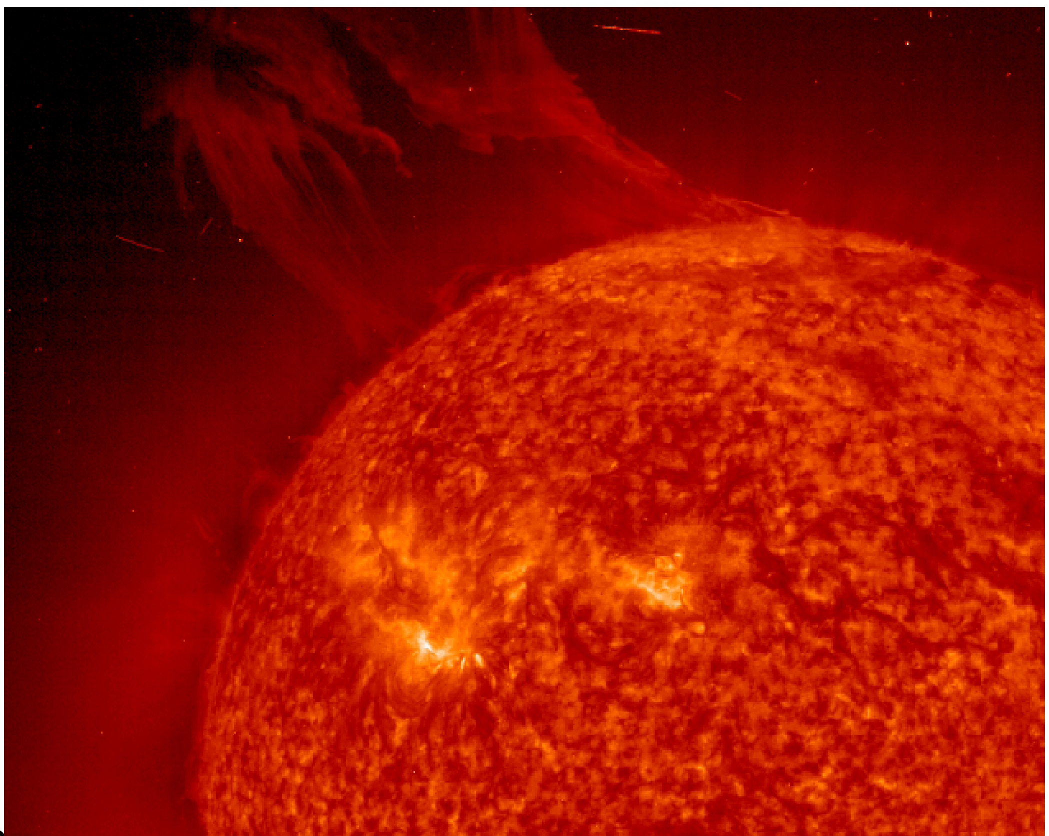
$Be(e^-,\nu)^7Li$

$^7Li(p,\alpha)^4He$

$^7Be(p,\gamma)^8B$

$^8B(e^+,\nu)^8Be$

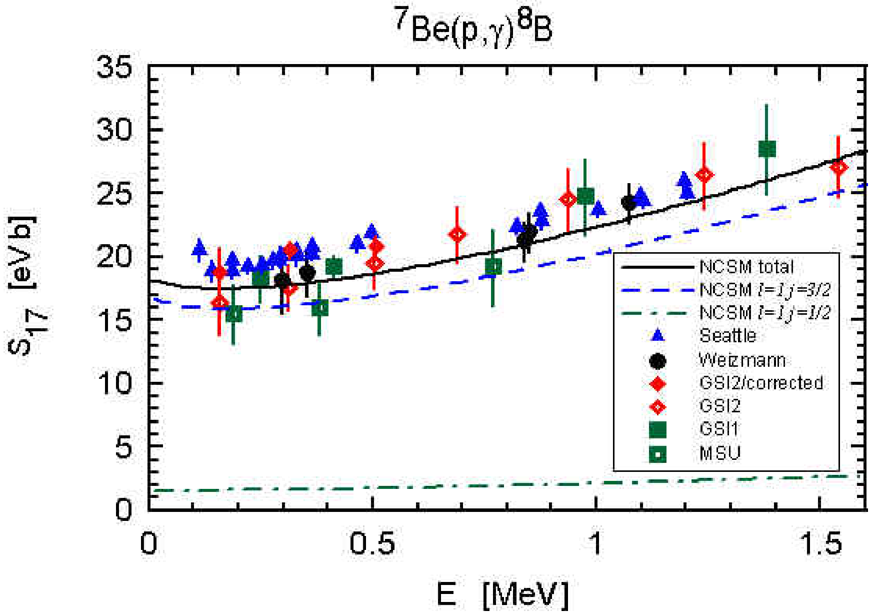
$^8Be(\alpha)^4He$

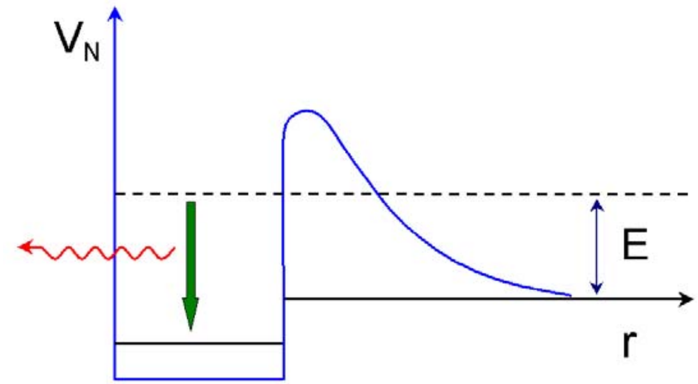


Solar neutrinos

E_ν < 15 MeV

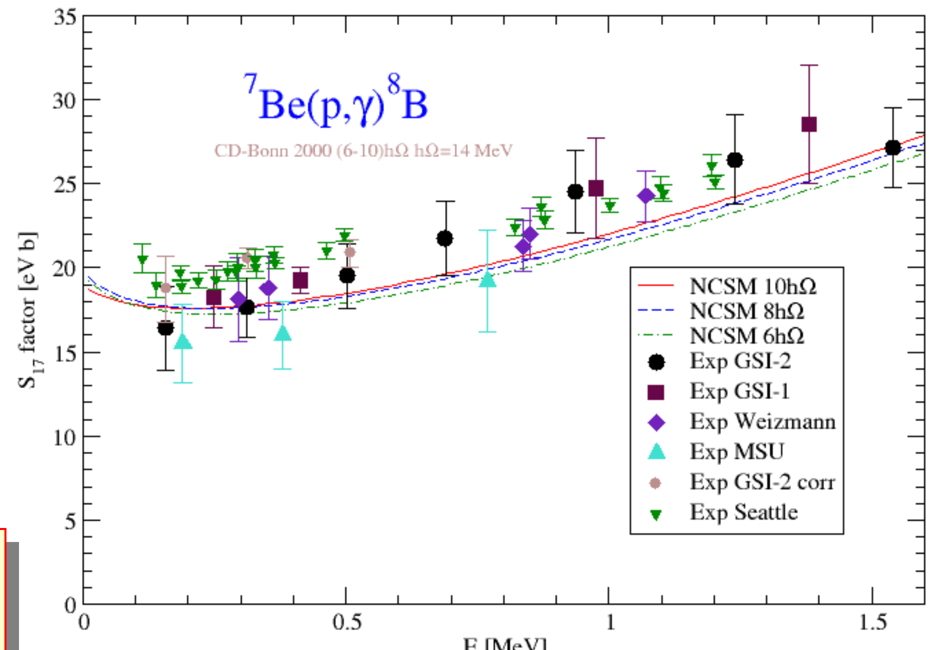
NCSM prediction for the 8B S-factor





$$\sigma(E) = \frac{1}{E} S(E) \exp\left[-2\pi \frac{Z_1 Z_2 e^2}{\hbar v} \right]$$

$$S(E) = C^2 S_{lj}\, f(\lambda, E) \left| \left\langle \Psi_f \left\| r^\lambda Y_\lambda \right\| \Psi_i \right\rangle \right|^2$$



Conclusions

My italian connection

- Angela Bonaccorso, Pisa
- Claudio Spitaleri, Catania
- Giuseppe Verde, Catania