# Source Shapes from Low Relative Velocity Correlations

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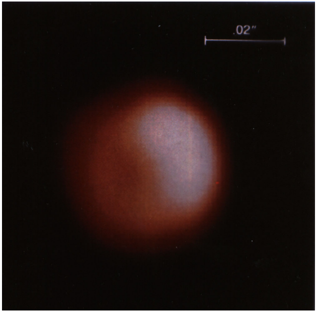
# Outline


1. Introduction
   - Imaging outside of Nuclear Physics
   - Heavy-Ion Collisions
   - Observed Asymmetries
2. Correlation Analysis
   - Multipole Decomposition & Imaging
   - Cartesian Harmonics
3. Illustration
   - Relative Source
   - Classical Coulomb Correlations
4. Summary

# Astronomy

Intensity/phase interferometry first used to assess sizes of astronomical objects. Astronomers have since moved to details:




red giant Betelgeuse

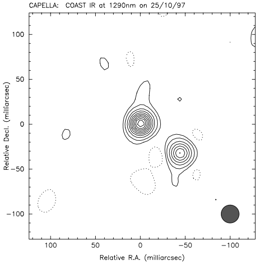



**Figure 5.7:** Reconstructed image of Capella, from data taken on 25 October 1997 at a wavelength of 1.3 µm. The contours are at -4, 4, 10, 20, 30, . . . , 90% of the peak ux. The map has been restored with a circular beam for clarity.

binary star Capella, Monnier Rep Prog Phy 66(03)789

Can we do comparably well?

# Imaging

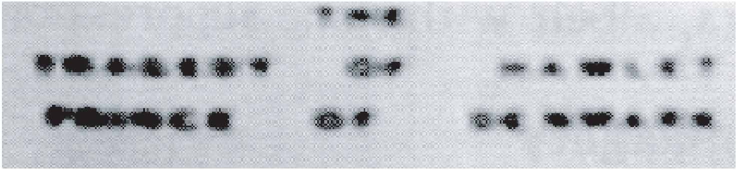
Geometric information from imaging. General task:

$$C(q) = \int dr\, K(q,r)\, S(r)$$

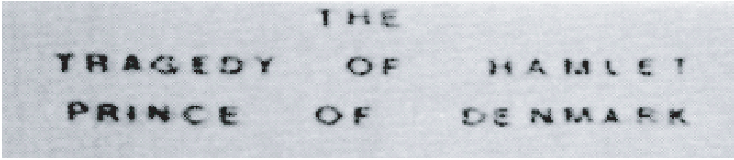
From data w/ errors, $C(q)$, determine the source $S(r)$.
Requires inversion of the kernel $K$.

Optical recognition: $K$ - blurring function, max entropy method

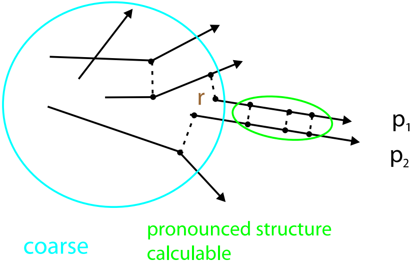
$C$:



$S$:



# Factorization of Final-State Amplitude in Reactions




2-ptcle inclusive cross section at low $|\mathbf{p}_1 - \mathbf{p}_2|$

$$\frac{d\sigma}{d\mathbf{p}_1\, d\mathbf{p}_2} = \int d\mathbf{r}\, S'_{\mathbf{P}}(\mathbf{r})\, |\Phi^{(-)}_{\mathbf{p}_1-\mathbf{p}_2}(\mathbf{r})|^2$$

data — source — 2-ptcle wf

$S'$: distribution of emission points in 2-ptcle CM

Normalizing with 1-ptcle cross sections yields correlation f:

$$C(\mathbf{p}_1 - \mathbf{p}_2) = \frac{\frac{d\sigma}{d\mathbf{p}_1\, d\mathbf{p}_2}}{\frac{d\sigma}{d\mathbf{p}_1}\,\frac{d\sigma}{d\mathbf{p}_2}} = \int d\mathbf{r}\, S_{\mathbf{P}}(\mathbf{r})\, |\Phi^{(-)}_{\mathbf{p}_1-\mathbf{p}_2}(\mathbf{r})|^2$$

Then the relative source is normalized to unity: $\int d\mathbf{r}\, S_{\mathbf{P}}(\mathbf{r}) = 1$.

Note: $C$ may only give access to the density of relative emission points in 2-ptcle CM, integrated there over time

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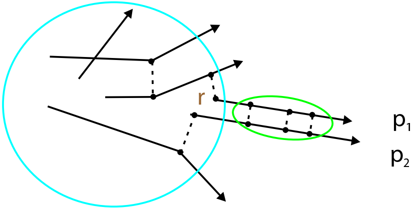


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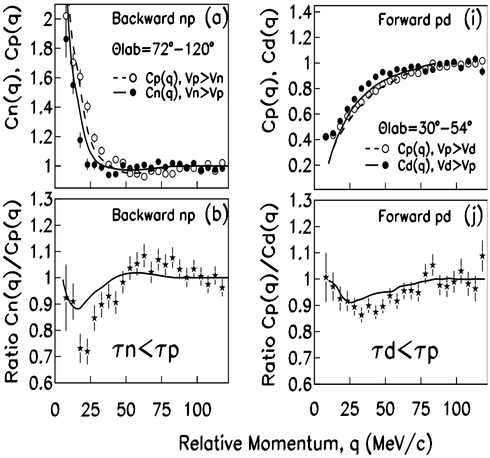
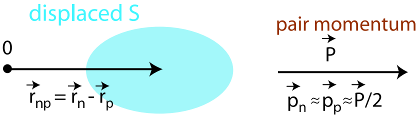
# Time Difference in Emission




Anisotropic $C$, dependent on orientation of **q**

Attributable to anisotropic $S$:

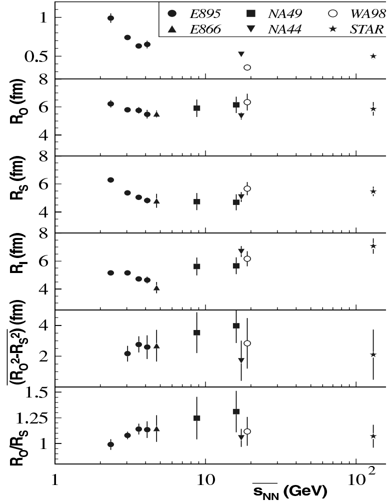


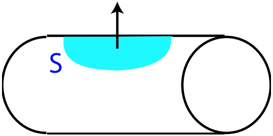

if n emitted earlier than p

Ghetti *et al.*, PRL91(03)0927011

Model fitted to data

# Geometry + Freezeout + Collective Motion




⇐ Fitted radii (longitudinal, outward & sideward) for an anisotropic Gaussian

Models fitted to data…

# Integral Relation

Of interest is the deviation of correlation function from unity:

$$\mathcal{R}(\mathbf{q}) = C(\mathbf{q}) - 1 = \int \mathrm{d}\mathbf{r} \left( |\Phi_{\mathbf{q}}^{(-)}(\mathbf{r})|^2 - 1 \right) S(\mathbf{r}) \equiv \int \mathrm{d}\mathbf{r}\, K(\mathbf{q}, \mathbf{r})\, S(\mathbf{r})$$

Learning on $S$ possible when $|\Phi_{\mathbf{q}}^{(-)}(\mathbf{r})|^2$ deviates from 1, either due to symmetrization or interaction within the pair.

The spin-averaged kernel $K$ depends only on the relative angle between $\mathbf{q}$ and $\mathbf{r}$. This facilitates the angular decomposition. With

$$K(\mathbf{q}, \mathbf{r}) = \sum_{\ell} (2\ell + 1)\, K_\ell(q, r)\, P^\ell(\cos\theta)\,, \qquad \text{and}$$

$$\mathcal{R}(\mathbf{q}) = \sqrt{4\pi} \sum_{\ell m} \mathcal{R}^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{q}})\,, \qquad S(\mathbf{r}) = \sqrt{4\pi} \sum_{\ell m} S^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{r}})$$

we reduce the 3D relation to a set of 1D:

$$\mathcal{R}^{\ell m}(q) = 4\pi \int \mathrm{d}r\, r^2\, K_\ell(q, r)\, S^{\ell m}(r)$$

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# $\ell = 0$

Different multipolarities of deformation for the source and correlation functions are directly related to each other.

The $\ell = 0$ version:

$$\mathcal{R}(q) = 4\pi \int \mathrm{d}r \, r^2 \, K_0(q, r) \, S(r)$$

where $\mathcal{R}(q)$, $K_0$ and $S(r)$ – angle-averaged correlation, kernel and source, respectively.

For pure interference, $\pi^0$'s or $\gamma$'s, $\Phi_{\mathbf{q}}^{(-)}(\mathbf{r}) = \frac{1}{\sqrt{2}} \left( e^{i\mathbf{q}\cdot\mathbf{r}} + e^{-i\mathbf{q}\cdot\mathbf{r}} \right)$, the kernel $K = |\Phi|^2 - 1$ results from the interference term in $|\Phi|^2$ and the correlation-source relation is just the FT:

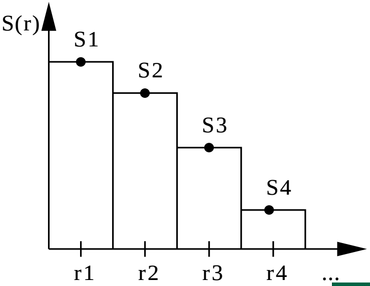
$$\mathcal{R}_0(q) = \frac{2\pi}{q} \int dr \, r \, \sin(2qr) S_0(r)$$

# Discretization & Imaging

Source discretization w/ $\chi^2$ fitting applies to <u>any</u> pair:

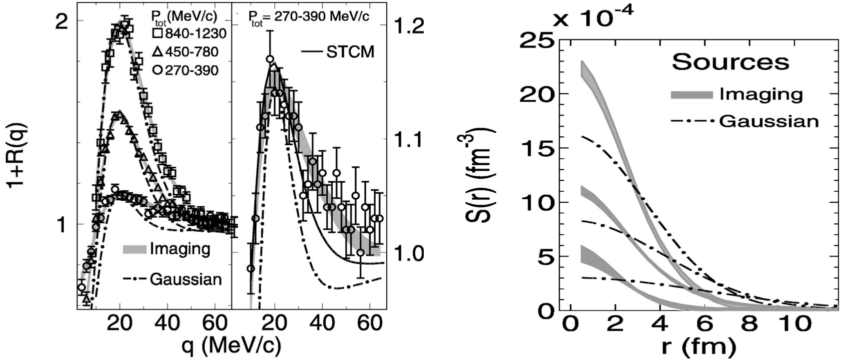
1. Discretize integral
$$\mathcal{R}_i = \sum_j 4\pi \, \Delta r \, r_j^2 \, K_0(q_i, r_j) \, S(r_j) \equiv \sum_j K_{ij} \, S_j$$
2. Vary $S(r_j)$ to minimize $\chi^2$:
$$\chi^2 = \sum_i \frac{(\sum_j K_{ij} \, S_j - \mathcal{R}_i^{exp})^2}{\sigma_i^2}$$
3. $S_j$-derivative of $\chi^2$ yields:
$$\sum_{ij} \frac{1}{\sigma_i^2} (K_{ij} \, S_j - \mathcal{R}_i^{exp}) \, K_{ij} = 0$$

with solution in a mtx form:
$$S = (K^\top K)^{-1} \, K^\top \, \mathcal{R}^{exp}$$



# pp Imaging

Imaging impacted interpretation of $C_{pp}$, Verde PRC65(02)054609




Gauss par: quickly changing radii. Imaging: quickly changing preequilibrium fraction, non-Gaussian source shapes!

$S(r \rightarrow 0)$: preequilibrium fraction, entropy, freeze-out $\rho$...

# Anisotropies??

As far as anisotropies are concerned, with

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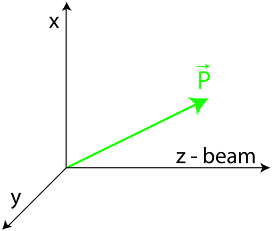


A set of 1D integral relations

Problem: Why turning real quantities, $R$ & $S$, into imaginary, $R^{\ell m}$ & $S^{\ell m}$? Other basis than $\mathrm{Y}^{\ell m}$??

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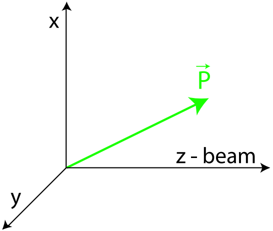


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# Cartesian Basis

Take the direction vector: $\hat{n}_\alpha = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$

Rank-$\ell$ tensor product:

$$(\hat{n}^\ell)_{\alpha_1\ldots\alpha_\ell} \equiv \hat{n}_{\alpha_1}\,\hat{n}_{\alpha_1}\ldots\hat{n}_{\alpha_\ell} = \sum_{\ell'\leq\ell,m} c_{\ell' m}\,\mathrm{Y}^{\ell' m}$$

$\mathcal{D}^{(\ell,\ell)}$ projection operator that, within the space of rank-$\ell$ cartesian tensors, removes $\mathrm{Y}^{\ell' m}$ components with $\ell' < \ell$:

$$(\mathcal{D}\hat{n}^\ell)_{\alpha_1\ldots\alpha_\ell} = \sum_m c_{\ell m}\,\mathrm{Y}^{\ell m}$$

The components $\mathcal{D}\hat{n}^\ell$ are real and can be used to replace $\mathrm{Y}^{\ell m}$.

# Low-$\ell$ Cartesian Harmonics

$$
\begin{aligned}
\mathcal{D}\hat{n}^0 &= 1 \\
(\mathcal{D}\hat{n}^1)_\alpha &= \hat{n}_\alpha \\
(\mathcal{D}\hat{n}^2)_{\alpha_1\,\alpha_2} &= \hat{n}_{\alpha_1}\,\hat{n}_{\alpha_2} - \frac{1}{3}\delta_{\alpha_1\,\alpha_2} \\
(\mathcal{D}\hat{n}^3)_{\alpha_1\,\alpha_2\,\alpha_3} &= \hat{n}_{\alpha_1}\,\hat{n}_{\alpha_2}\,\hat{n}_{\alpha_3} - \frac{1}{5}(\delta_{\alpha_1\,\alpha_2}\,\hat{n}_{\alpha_3} + \delta_{\alpha_1\,\alpha_3}\,\hat{n}_{\alpha_2} + \delta_{\alpha_2\,\alpha_3}\,\hat{n}_{\alpha_1}) \\
&\vdots
\end{aligned}
$$

$\mathcal{D}$ can be called a detracing operator as

$$
\sum_\alpha (\mathcal{D}\hat{n}^\ell)_{\alpha\,\alpha\,\alpha_3\ldots\alpha_\ell} = 0
$$

# Decomposition with Cartesian Harmonics

Completeness relation ($\mathcal{D} = \mathcal{D}^\top = \mathcal{D}^2$):

$$\begin{aligned}\delta(\Omega' - \Omega) &= \frac{1}{4\pi} \sum_\ell \frac{(2\ell+1)!!}{\ell!} \sum_{\alpha_1 \ldots \alpha_\ell} (\mathcal{D}\hat{n}'^\ell)_{\alpha_1 \ldots \alpha_\ell} (\mathcal{D}\hat{n}^\ell)_{\alpha_1 \ldots \alpha_\ell} \\ &= \frac{1}{4\pi} \sum_\ell \frac{(2\ell+1)!!}{\ell!} \sum_{\alpha_1 \ldots \alpha_\ell} (\mathcal{D}\hat{n}'^\ell)_{\alpha_1 \ldots \alpha_\ell} \hat{n}_{\alpha_1} \ldots \hat{n}_{\alpha_\ell}\end{aligned}$$

In consequence

$$\mathcal{R}(\mathbf{q}) = \int \mathrm{d}\Omega' \, \delta(\Omega' - \Omega) \, \mathcal{R}(\mathbf{q}') = \sum_\ell \sum_{\alpha_1 \ldots \alpha_\ell} \mathcal{R}^{(\ell)}_{\alpha_1 \ldots \alpha_\ell}(q) \, \hat{q}_{\alpha_1} \ldots \hat{q}_{\alpha_\ell}$$

where coefficients are angular moments

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# Consequences

Cartesian coefficients for $\mathcal{R}$ & $S$ directly related to each other:

$$\mathcal{R}^{(\ell)}_{\alpha_1 \cdots \alpha_\ell}(q) = 4\pi \int \mathrm{d}r \, r^2 \, K_\ell(q,r) \, \mathcal{S}^{(\ell)}_{\alpha_1 \cdots \alpha_\ell}(r)$$

For weak anisotropies, only lowest-$\ell$ matter:

$$\mathcal{R}(\mathbf{q}) = \mathcal{R}^{(0)}(q) + \sum_\alpha \mathcal{R}^{(1)}_\alpha(q) \, \hat{q}_\alpha + \sum_{\alpha_1 \, \alpha_2} \mathcal{R}^{(2)}_{\alpha_1 \alpha_2}(q) \, \hat{q}_{\alpha_1} \, \hat{q}_{\alpha_2} + \ldots$$

$\mathcal{R}^{(0)}$ - angle-averaged correlation

$\mathcal{R}^{(1)}_\alpha \equiv R^{(1)} \, e^{(1)}_\alpha$ - dipole distortion, magnitude + direction vector

$\mathcal{R}^{(2)}_{\alpha\beta}(q) = R^{(2)}_1 \, e^{(2)}_{1\alpha} \, e^{(2)}_{1\beta} + R^{(2)}_3 \, e^{(2)}_{3\alpha} \, e^{(2)}_{3\beta} - \left( R^{(2)}_1 + R^{(2)}_3 \right) \, e^{(2)}_{2\alpha} \, e^{(2)}_{2\beta}$
- quadrupole distortion, 2 magnitude values + 3 orthogonal direction vectors

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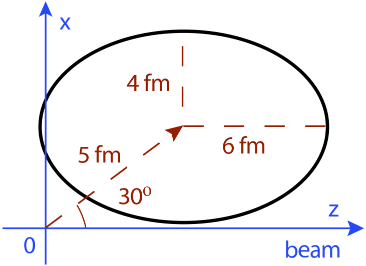
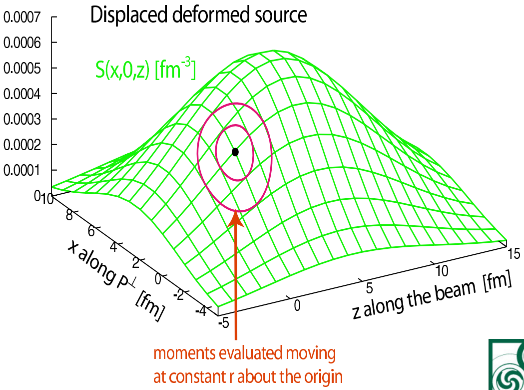
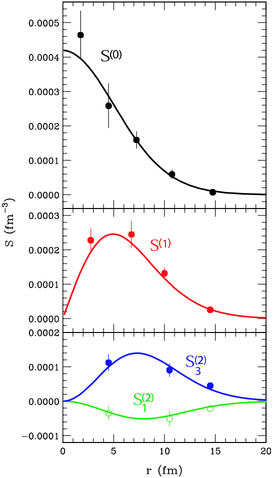
# Sample Relative Source

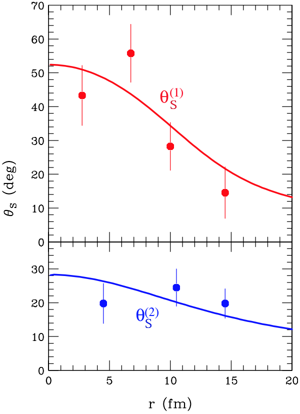
Anisotropic Gaussian, elongated along the beam axis, displaced along the pair momentum



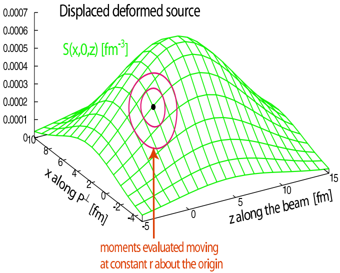



# Low-$\ell$ Characteristics




Values + Angles




$$S^{(\ell)} \propto r^{\ell}$$

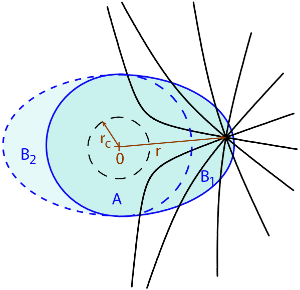
# Classical Coulomb Correlations

Coulomb kernel is a function of $\theta_{\mathbf{qr}}$ and $r/r_c$, where $r_c$ distance of closest approach in head-on collision, $\frac{q^2}{2m_{ab}} = \frac{Z_a Z_b e^2}{4\pi\epsilon_0 r_c}$:

$$|\phi|^2 = \frac{\mathrm{d}^3 q_0}{\mathrm{d}^3 q} = \frac{\Theta\left(1 + \cos\theta_{\mathbf{qr}} - 2r_c/r\right)\left(1 + \cos\theta_{\mathbf{qr}} - r_c/r\right)}{\sqrt{(1+\cos\theta_{\mathbf{qr}})^2 - (1+\cos\theta_{\mathbf{qr}})\, 2r_c/r}}$$

$$K_0 = \Theta(r - r_c)\sqrt{1 - r_c/r} - 1$$

Correlation reflects the distribution of relative Coulomb trajectories emerging from an anisotropic source.



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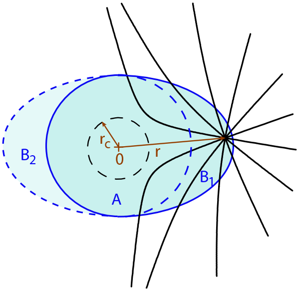
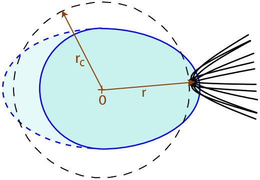


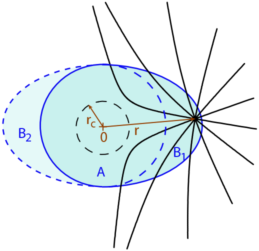
# Momentum from Spatial Anisotropy: Evolution with $r_c$

No trajectories can contribute from $r < r_c$

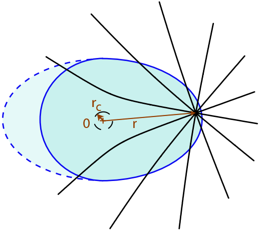



$r - r_c \ll r_c$

*C* directly reflects anisotropies of *S*-margins



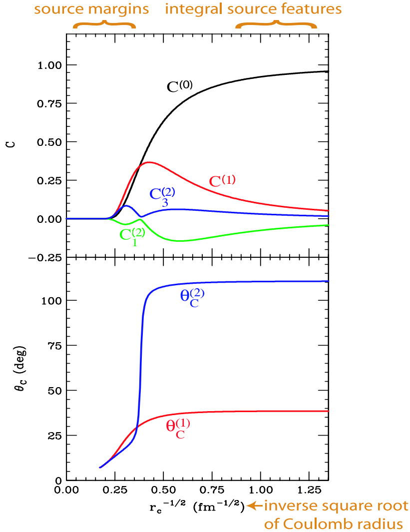

$r \gtrsim r_c$




$r_c \ll r$

$\mathcal{R}$ reflects integral characteristics of *S*

# Coulomb Correlation




$$r_c^{-1/2} \propto q$$

For more schematic sources, one or more correlation values vanish and/or angles exhibit less variation.

90° jump associated with $K_2$ sign change and prolate-oblate transition

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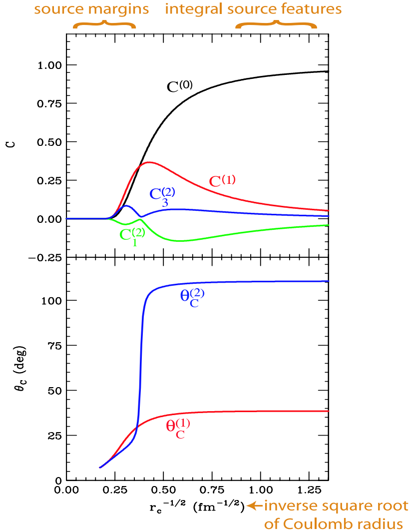


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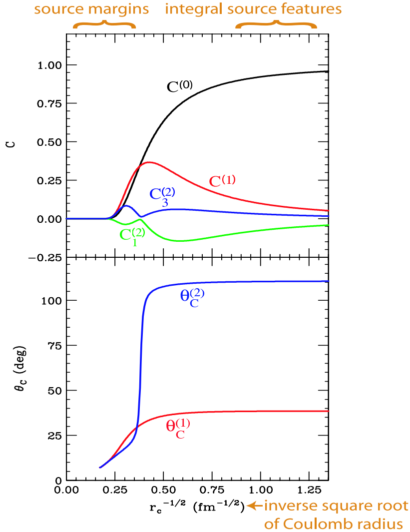


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# Imaging

Assumption: $\ell \leq 2$ cartesian coefficients measured at 80 values of $r_c^{-1/2}$ subject to an r.m.s. error of 0.015.

No of restored values, with the region of $r < 17$ fm: 5 for $\ell = 0$, 4 for $\ell = 1$ and 3 for $\ell = 2$.

Return to source

$r < 17$ fm Source Characteristics:

| | Unit | Restored | Original |
|---|---|---|---|
| $4\pi \int \mathrm{d}r\, r^2\, S^{(0)}$ | | 0.99±0.05 | 1.00 |
| $\langle x \rangle$ | fm | 2.47±0.11 | 2.45 |
| $\langle z \rangle$ | fm | 4.25±0.13 | 3.90 |
| $\langle (x - \langle x \rangle)^2 \rangle^{1/2}$ | fm | 3.80±0.24 | 3.90 |
| $\langle y^2 \rangle^{1/2}$ | fm | 3.81±0.22 | 3.91 |
| $\langle (z - \langle z \rangle)^2 \rangle^{1/2}$ | fm | 5.54±0.19 | 5.60 |
| $\langle (x - \langle x \rangle)(z - \langle z \rangle) \rangle$ | fm$^2$ | 2.23±1.49 | -0.41 |

# Summary

- Relative correlations give access to space-time geometry of emission.
- Cartesian harmonic coefficients allow for a systematic quantification of anisotropic correlation functions.
- The correlation coefficients are directly related to the analogous respective coefficients for the relative source.
- Features of the source anisotropies may be, to an extent, read off straight from the correlation anisotropies. Otherwise, they can be imaged.

nucl-th/0501003

Collaborators: S. Pratt, D. Brown, G. Verde. . .

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