Breakup and Transfer Reactions with Halo Nuclei

An old and new field

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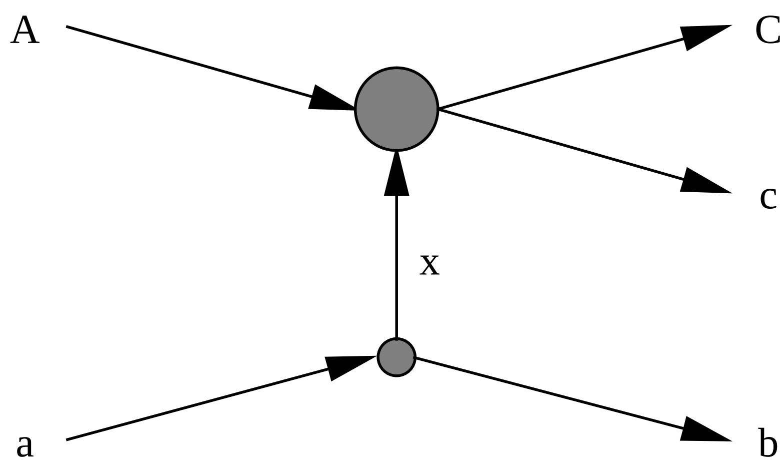
Trojan Horse Method

Transfer to the continuum: A method to study the reaction
$A + x \to C + c$
via the "surrogate reaction" $A + (b + x) \to C + c + b$
where b is a spectator
see S.Typel and G.Baur Ann. Phys. 305(2003)228, Progress of Theoretical Physics Supplement No. 154(2004)333 and further Refs. given there



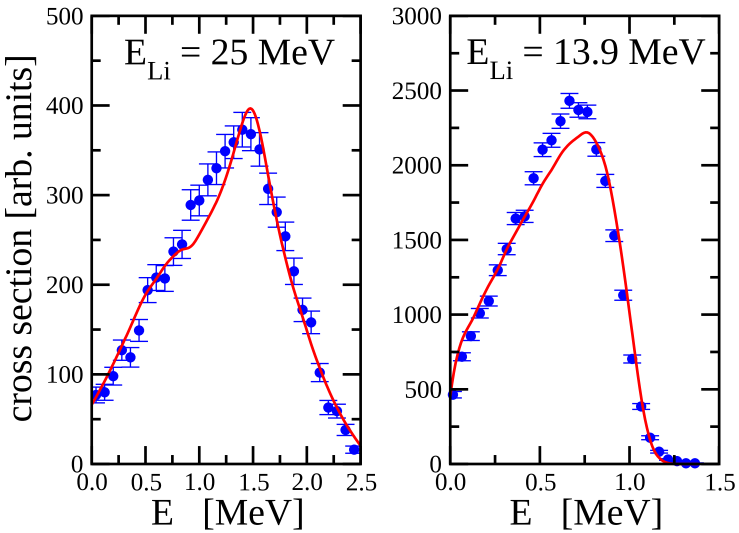
The energy $E_{Ax} \equiv E$ can be > 0 as well as < 0.

An example

$^6Li + p \rightarrow \alpha +^3 He$ studied via $^6Li + d \rightarrow \alpha +^3 He + n$
A.Tumino, C.Spitaleri, A.Di Pietro, P.Figuera, M.Lattuada, A.Musumarra, M.G.Pellegriti, R.G.Pizzone,S.Romano, C.Rolfs, S.Tudisco, and S.Typel
(Catania-Bochum-Darmstadt Collaboration)

Phys.Rev.C 67(2003)065803

Continuous Transition



see fig.7 of A.Tumino et al.PRC67(2003)065803

THM reaction:

$^{6}Li + d \rightarrow \alpha +^{3} He + n$

neutron is spectator

reaction to be studied:

$^{6}Li + p \rightarrow \alpha +^{3} He$

$E = p +^{6} Li$ relative energy, $Q = 4.02 MeV$

for $E = 0$ the cross section is finite

$\rightarrow$ continue experimentally as well as theoretically to $E < 0$!

Elastic Case:C+c=A+x

$E_{Ax} < 0$: stripping to bound states
asymptotic behaviour of radial wave function:$f_l(r) \to Bh_l(iqr)$
$l = 0$ is a special case $B \sim q^{3/2}$
whereas $B \sim q^{l+1} R^{l-1/2}$ for $l > 0$ S.Typel, G.Baur nucl-th/0411069
for $l = 0$ the cross section for stripping to a halo state
($q \to 0$)vanishes
but: the scattering length $a = 1/q$ tends to ∞
$\to$ contribution to continuum cross section
$\frac{d\sigma}{dE_b} \sim sin^2 \delta_0(k)/k$ with $\delta_0 = -ka$
see also G.Blanchon, A.Bonaccorso, N.Vinh Mau Nucl.Phys.A739(2004)259

in a Gedankenexperiment: decrease potential depth
the bound state disappears and reappears as a resonance in the continuum
$l > 0$: smooth transition from bound state stripping

to stripping to a resonance

Absorption at Zero Energy

Open channels $c + C \neq A + x$ at $E = 0$

E.P.Wigner Phys. Rev. 73(1948)1002

Following A. Kasano and M.Ichimura Phys. Lett. 115B(1982)81 (see also M.Ichimura, N.Austern, C.M. Vincent Phys. Rev. C 32(1985)431) we have for the inclusive $(b + x) + A \to b + X$ cross section in the surface approximation

$$\frac{d^2\sigma}{dE_b d\Omega_b} \propto k \int drW(r)|f_l|^2 \cdot |\int_R^\infty dru_l^+(kr) \cdots|^2$$

where

f_l= regular solution of radial Schrödinger equation for $A + x$-system in optical model potential with imaginary part W

$u_l^\pm \to \exp \pm i(kr - l\pi/2)$ (neutron case)

$$f_l \to \frac{1}{2i}(\exp 2i\delta_l u_l^+ - u_l^-)$$

Absorption at Zero Energy

$\frac{d^2\sigma}{dE_b d\Omega_b} \propto k \int drW(r)|f_l|^2 \cdot | \int_R^\infty dr u_l^+(kr) \cdots |^2$

Wronskian $w \equiv \frac{2m}{\hbar^2} \int_0^\infty drW(r)|f_l(r)|^2 = \frac{1}{2i}(f_l^* \frac{df_l}{dr} - f_l \frac{df_l^*}{dr})$

For $E = \frac{\hbar^2 k^2}{2m} > 0$

$w = k\text{Im}\delta_l$

For $E < 0$:$k = iq$ $E_b = \frac{\hbar^2 q^2}{2m}$

$w = q(-1)^l\text{Re}\delta_l$

Relate δ_l to interior logarithmic derivative ($L_i =$ complex)

$\delta_l = -(kR)^{2l+1}/((2l+1)!!(2l-1)!!)(1 - \frac{2l+1}{L_i+1})(E > 0)$

$\delta_l = i(-1)^l(qR)^{2l+1}/((2l+1)!!(2l-1)!!)(1 - \frac{2l+1}{L_i+1})(E < 0)$

- If L_i is continuous at $E = 0 \rightarrow \frac{d^2\sigma}{dE_b d\Omega_b}$ is continuous
- for k small:$u_l \propto (kr)^{-l-1}$ cross section $\frac{d^2\sigma}{dE_b d\Omega_b}$ finite for $E = 0$

Coulomb Dissociation

of Halo Nuclei

A Nuclear Physics Primakoff Effect
a (not so recent)review:
G.Baur, K.Hencken, D.Trautmann Prog.Part. Nucl. Phys. 51(2003)487

Nuclear Structure

GDR in Unstable Nuclei
low lying E1-strength

Nuclear Astrophysics

G.Baur, C.A. Bertulani, H.Rebel Nucl.Phys. A458(1986)
"Coulomb dissociation as a source of information on radiative capture processes of astrophysical interest"
e.g. $^{208}Pb +^8 B \rightarrow^{208} Pb +^7 Be + p$

determines $S_{17}(E)$ which determines high energy component of solar ν flux

General Properties

Method complementary to other methods like “direct” radiative capture experiments or excitation with real photons

There are a few selected examples
unstable nuclei become accessible
e.g. r- and rp-process nuclei
two-particle capture reactions become accessible in time-reversed process
Input from theory: QED (+some corrections from grazing nuclear interactions)

Important Parameters

With increasing beam energy the adiabaticity parameter
$\xi = \frac{\omega b}{\gamma v}$ gets smaller
$\xi < 1$: excitation
$\xi > 1$: no excitation
High lying states, like the giant dipole resonance
can be reached in intermediate energy Coulomb excitation
The strength parameter $\chi \sim 1/v$ gets smaller $\Rightarrow$

Higher order effects tend to become less important for higher beam velocities v

Neutron Halo Nuclei

A separation of scales: $E_b << E_{core-excitation}$

There are a few basic parameters

$\gamma = qR$

R= range of core-halo-nucleon interaction

binding energy $E_b = \frac{\hbar^2 q^2}{2\mu}$

$R_{halo} = 1/q$ ...extension of nuclear halo wave function

halo nucleus is characterized by $\gamma < 1$

$\gamma = \frac{R}{R_{halo}}$ is a suitable expansion parameter

Effective Range Theory

of Halo Nuclei

The whole dynamics in encoded in a few low energy constants

S.Typel and G.Baur Phys.Rev.Lett. 93(2004)142502 and nucl-th/0411069, submitted to Ann. Phys.
Analytical formulae for low lying dipole strength in neutron halo nuclei
transitions $l_i \rightarrow l_f$
initial bound state characterized by γ

and ANC(spectroscopic factor C^2S)

Low Energy Constants

final continuum state: low relative energy: $E = \frac{\hbar^2 k^2}{2\mu}$ effective range expansion

$tan(\delta_l) = -(x c_l \gamma)^{2l+1}$

$x = \frac{k}{q}$

$c_l =$"reduced scattering length"

Natural value of c_l is $O(1)$

(unnaturally large e.g. for an $l = 0$ halo state:

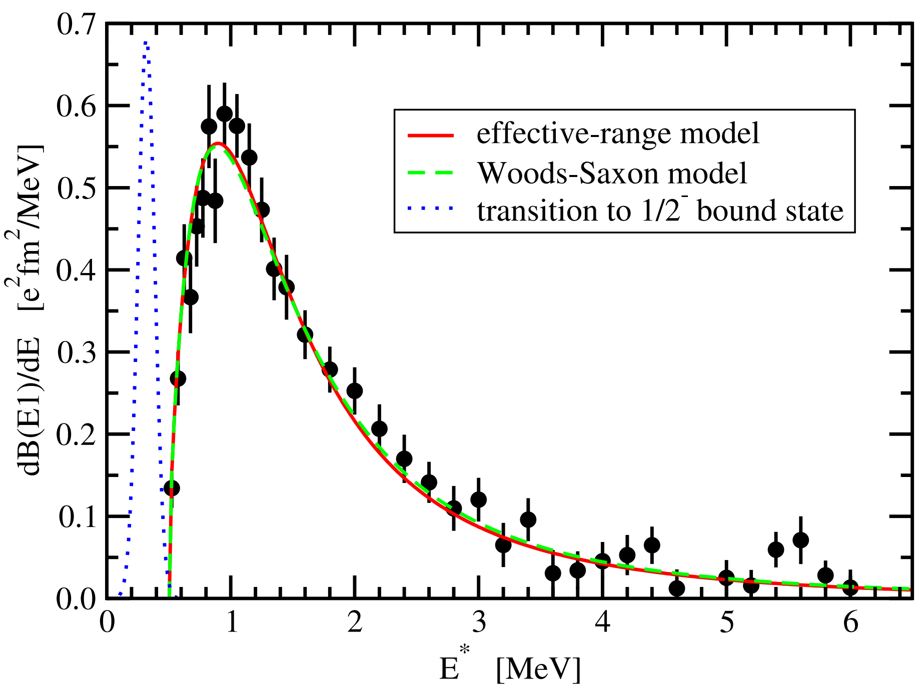
$c_0 = a_0/R = 1/\gamma$)

shape function $S_{l_i}^{l_f}$ determines $B(E1)$-strength distribution

$S_0^1 = \frac{4x^3}{(1+x^2)^4}(1 - c_1^3(1+3x^2)\gamma^3 + ...)$

Electromagnetic Strength

of Halo Nuclei
^{11}Be as an Example



Comparison to GSI data R.Palit et al. Phys.Rev. C68(2003)034318

The low energy constants extracted from the fit are (with R=2.78fm)

$\gamma = 0.4132$

spectroscopic factor

$C^2 S = 0.704(15)$

p-wave scattering lengths $c_1^{3/2} = -0.41(86, -20)$ and $c_1^{1/2} = 2.77(13, -14)$

Due to the $p_{1/2}$-bound state in ^{11}Be the scattering length is (un-naturally) large in this channel

Other Examples

low lying strength in ^{23}O as determined from Coulomb dissociation at GSI C.Nociforo et al.(LAND-FRS Collaboration) Phys.Lett.B605(2004)79
application of effective range model to proton halo nuclei: see S.Typel, G.Baur nucl-th/0411069

methods hopefully useful also for 2,...-nucleon-halo nuclei: evidently much more involved

Various Philosophies

Study of Halo Nuclei: A low energy phenomenon: one is insensitive to details of the potential which only become relevant at higher energies(shorter wave-lengths)
Same philosophy as in effective field theories:
Compare:
(i) “Reality”
a Woods-Saxon potential model
(ii)Model:
e.g. an analytically solvable square well model
(iii) Effective Range Approach:
for low energies(these are the ones relevant for halo nuclei): all methods should agree with each other
at which energies will the differences show up?

model studies can be very well performed, analytical results are available.

A Toy Model

for Coulomb Dissociation of Neutron Halo Nuclei

A 3-body Hamiltonian:

$H = T + \frac{ZZ_c e^2}{r_c} + V_{nc}$ breakup process:$Z + (c + n) \rightarrow Z + c + n$

$V_{nc} =$ a (very)short range potential which supports one s-wave bound state $a = (c + n)$ at $E_b = \frac{\hbar^2 q^2}{2\mu}$

e.g. a square well with $VR^2 = const., R \rightarrow 0$

Proposal: Use this model as a basis to compare different calculational methods

The different results will be due to the differences of the methods, and not to the differences of the model Hamiltonians

advantage: interesting limits are known analytically and can serve as "benchmarks"

A Theoretical Laboratory

to study

- Higher order effects like postacceleration
- quantal and semiclassical methods
- relativistic effects of projectile motion

kinematics: $\vec{q_a} \longrightarrow \vec{q_{cm}} + \vec{q_{rel}}$

Analytical results

Born approximation: $T_{Born} = 4\pi \frac{ZZ_c e^2}{q_{coul}^2} a(\vec{\Delta})$

with "Coulomb push" $\vec{q_{coul}} = \vec{q_a} - \vec{q_{cm}} = \vec{\Delta}(m_n + m_c)/m_n$ and

$$a = \sqrt{8\pi q}\left(\frac{1}{(\vec{q_{rel}} - \vec{\Delta})^2 + q^2} + \frac{i(q + iq_{rel})}{2|\Delta|(q^2 + q_{rel}^2)} ln \frac{q + i(q_{rel} + \Delta)}{q + i(q_{rel} - \Delta)}\right)$$

For $\Delta << q_{rel}$ dipole approximation

$$a^{dipole} = \sqrt{8\pi q}\frac{2\vec{q_{rel}} \cdot \vec{\Delta}}{(q_{rel}^2 + q^2)^2}$$

High energies

Strong Coulomb field: Coulomb parameter $\eta_a = \frac{ZZ_c e^2}{\hbar v_a} >> 1$

Straight line (impact parameter b) ,electric dipole and sudden approximation: S.Typel and G.Baur Nucl.Phys.A573(1994)486

excitation amplitude is found to be

$a(\Delta = \frac{2\eta_a m_n}{m_a b})$

same formula as for Born approximation, although different region of η- values

Expansion in strength parameter

S.Typel and G.Baur Phys. Rev. C64(2001)024601

expand, for $\xi = \frac{\omega b}{v_a} << 1$

in strength parameter $y = \frac{m_n \eta_a}{m_a b q}$ $\quad x = \frac{k}{q}$

$\frac{dP_{LO}}{dq_{rel}} = \frac{16}{3\pi q} y^2 \frac{x^4}{(1+x^2)^4}$, (see above)

next to leading order: $\frac{dP_{NLO}}{dq_{rel}} = \frac{16}{3\pi q} y^4 \frac{x^2(5-55x^2+28x^4)}{15(1+x^2)^6}$

For $\xi =$ finite: analytical results available for order y^2

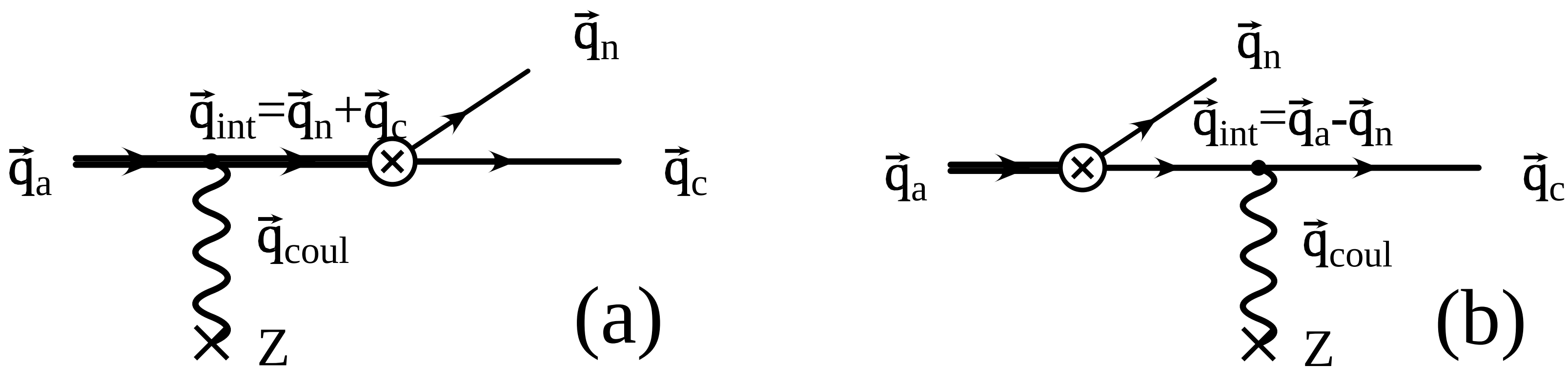
For $\xi >> 1$: exponential suppression of excitation amplitude

Post Form DWBA

analytical solution: “Sommerfeld Bremsstrahlung integral”
final state: plane-wave neutron$*$ Coulomb wave of core c
This approach is- in contrast to the prior form DWBA- very successful for Coulomb breakup around the Coulomb barrier (see Prog.Part.Nucl.Phys.51(2003)487 and further refs.)
$\Rightarrow$ “postacceleration” is taken into account
P. Banerjee et al. Phys.Rev.C65(2002)064602: numerical studies show that postacceleration is important for low energies. postacceleration tends to vanish for high energies. Analytical studies: CWBA approaches Born approximation for high energies, even for $\eta_a >> 1$

A related result is known for Bremsstrahlung (see e.g. Landau and Lifshitz Vol.4)

low η limit



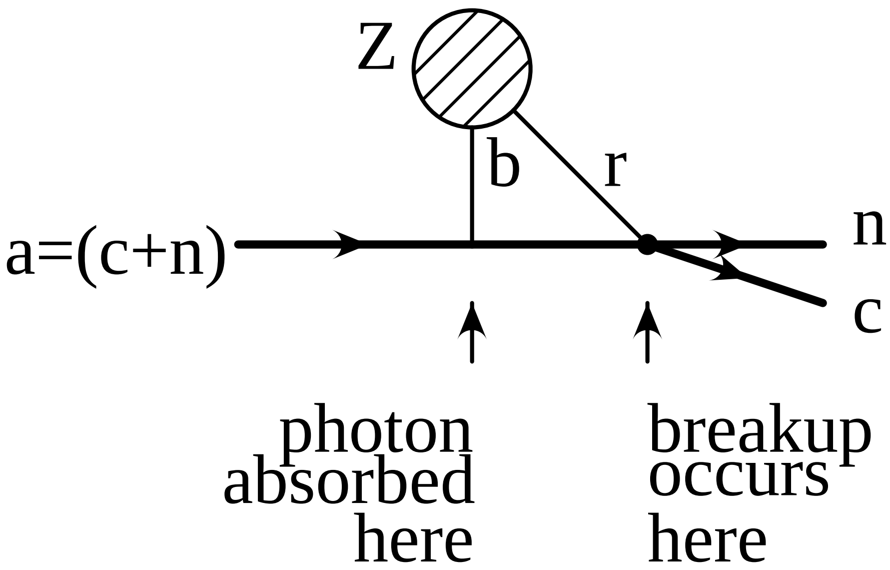
$$T \sim \frac{1}{q_a^2 - (\vec{q_n} + \vec{q_c})^2} + \frac{m_c}{m_a} \frac{1}{q_c^2 - (\vec{q_a} - \vec{q_n})^2}$$

cf. Bremsstrahlung:neutron, with mass m_n is “radiated off”

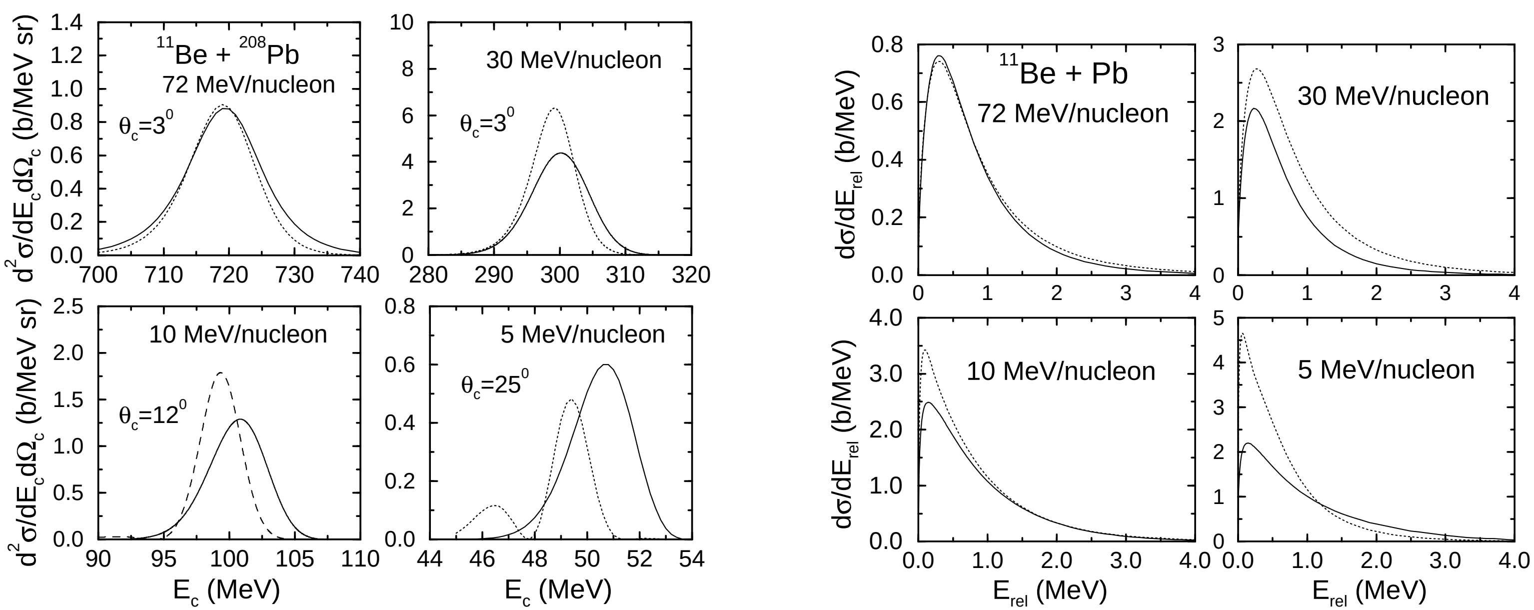
1^{st} order in Coulomb push q_{coul} : agrees with Born approximation (prior form)

Differences for higher orders in the Coulomb push q_{coul}

This picture is too classical:



(Fully quantal)Post form CWBA calculations: P.Banerjee et al. Phys.Rev.C65(2002)064602
for small beam energies: postacceleration; for large beam energies: postacceleration tends to vanish
continuous line: CWBA; dotted line: first order semiclassical



Further Features

- Apply CDCC-methods, relativistic models,...
- numerical methods... is the singular nature of the potential V_{nc} a problem for the numerical studies?
- interaction between c and n only in the $l = 0$-state
- Compare fully quantal results(known analytically for various limits) to semiclassical methods

cf. : Relativistic Coulomb excitation of the giant dipole resonance in nuclei: a straightforward approach
C.H.Dasso, M.I.Gallard, H.M.Sofia, A. Vitturi
Phys.Rev.C70(2004)044903

Compare various theoretical methods to a given model Hamiltonian

Conclusion

- Analysis of Coulomb Dissociation data: based on QED
- and some small nuclear effects
- The deuteron remains the prototype of a halo nucleus
- effective range methods very suited for low energy halo phenomena

- transition from stripping to bound states and unbound states is continuous
- $l = 0$ is a special case
- reactions with particles with negative energy(closed channels) are accessible