

DIRECT REACTIONS With EXOTIC NUCLEI

References, Collaborators

Recent Developments in Electromagnetic Excitation
With Fast Heavy Ions

G.Baur, K.Hencken, D.Trautmann, S.Typel
nucl-th/0402012, Hirschegg2004

Theory of the Trojan Horse Method
G.Baur, S.Typel nucl-th/0401054, Fusion03

Trento March 9, 2004

General

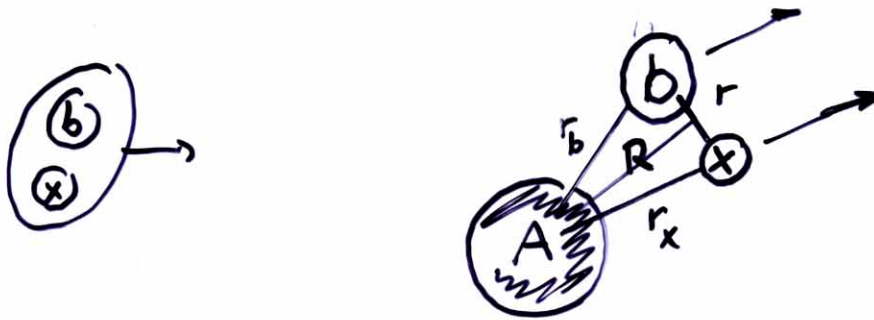
Direct reactions have been studied for decades

low energies: DWBA, coupled channels

simplification for high energies: Glauber(Eikonal) Methods

Reaction theories for elastic scattering, inelastic scattering ,
transfer reactions

breakup: inelastic scattering? or transfer reaction?



$$\psi_{\text{final}} : \text{ more like } \sim \chi_b(r_b) \chi_x(r_x)$$

or

$$\sim \chi_{(bx)}(R) \psi(r) \quad ?$$

Content

1. Transfer Reactions

Trojan Horse Method

Transfer reaction into the continuum (resonant states) just above threshold

"Surface Approximation": phase shifts(or ANC) are measured

2. Electromagnetic excitation and dissociation

electromagnetic matrix elements are measured

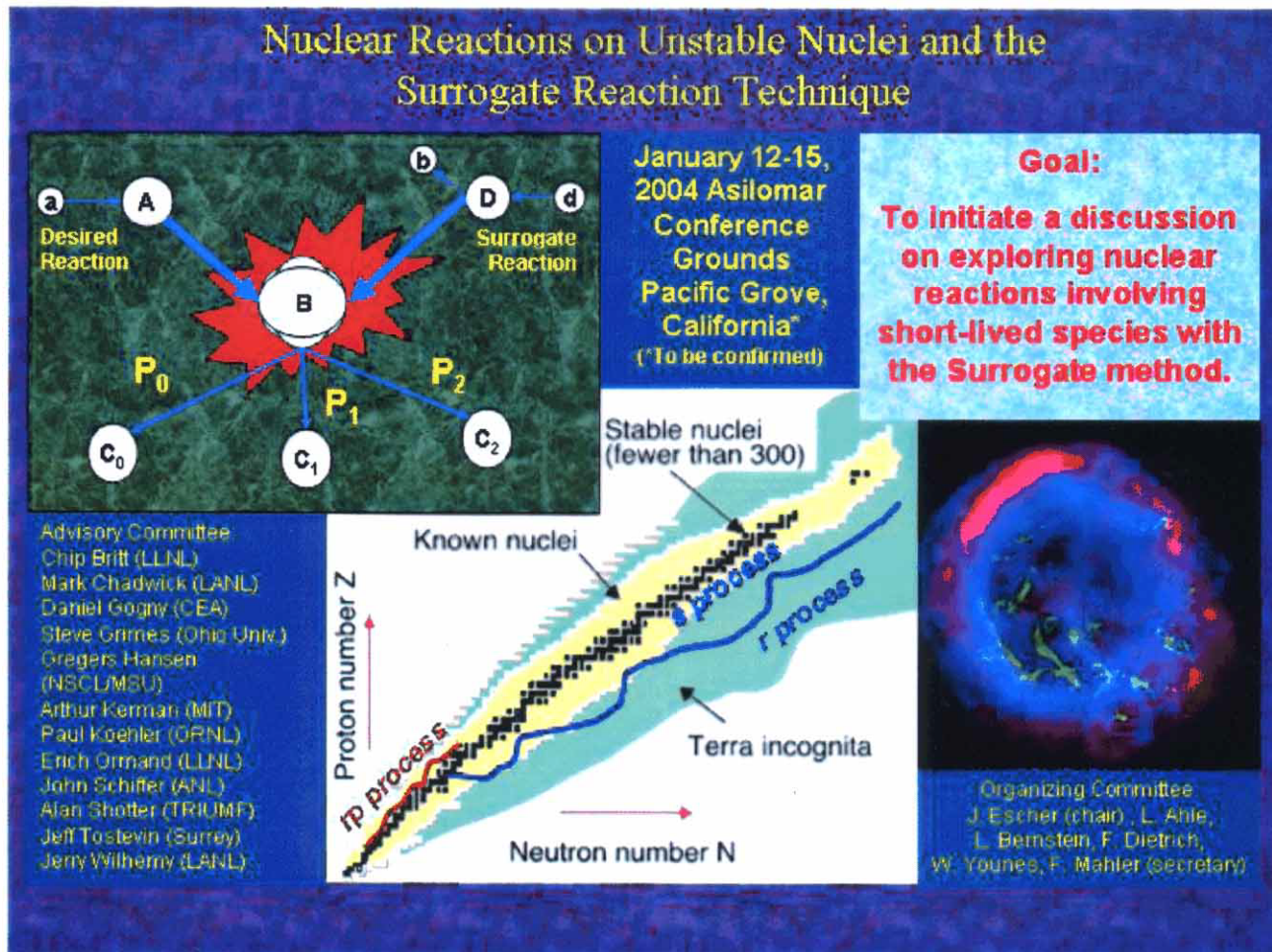
not spectroscopic factors

relation to spectroscopic factors in special situations

important parameters

A Toy Model for Coulomb Dissociation of Halo Nuclei

Effective range theory of Halo Nuclei



desired reaction

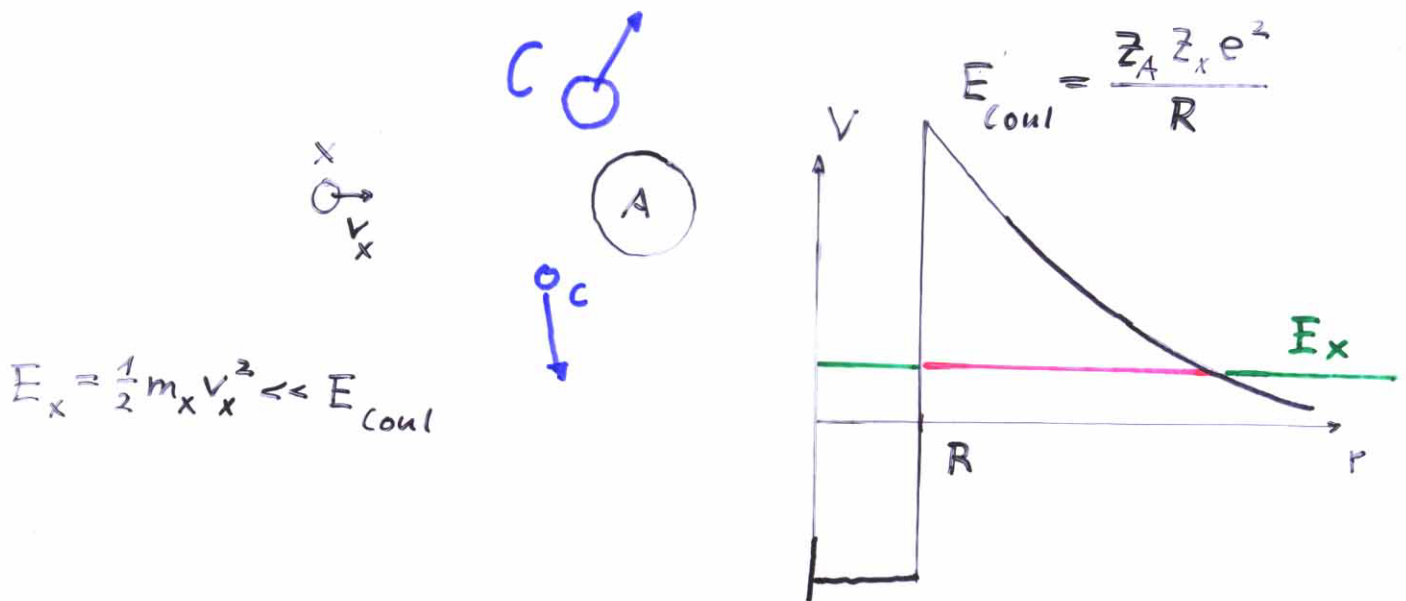


surrogate reaction

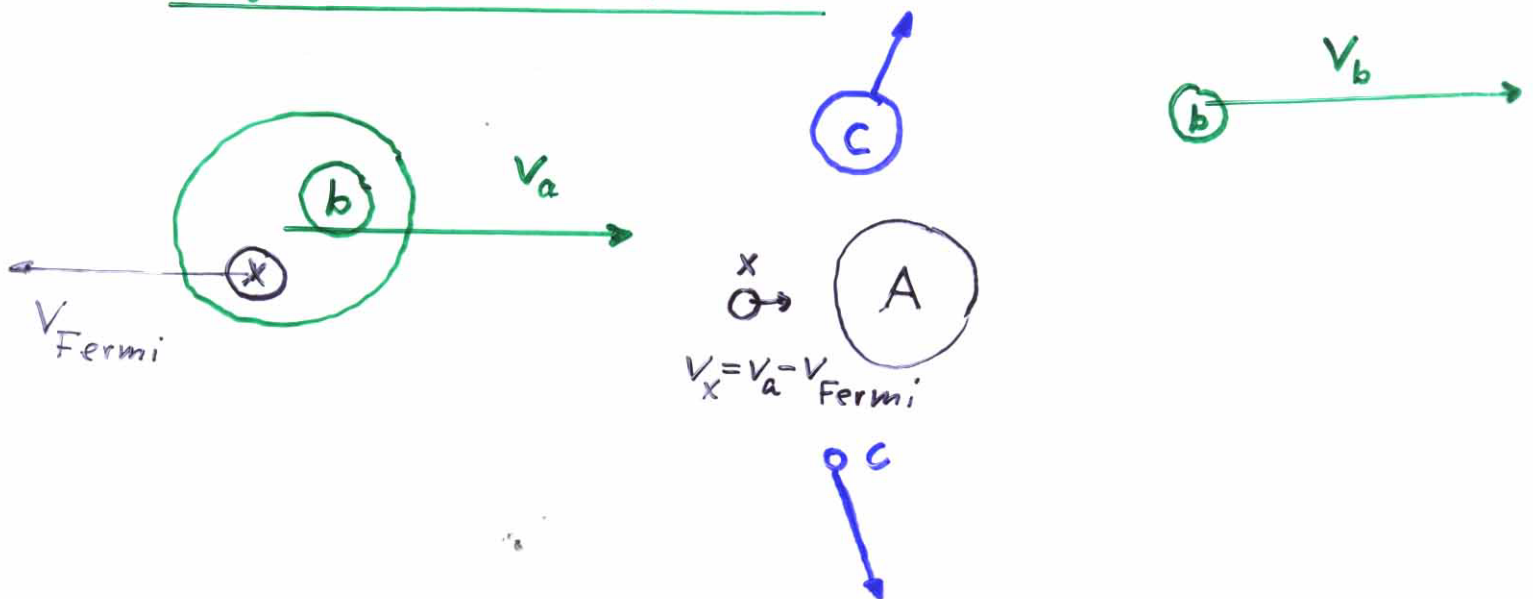


Trojan Horse Method

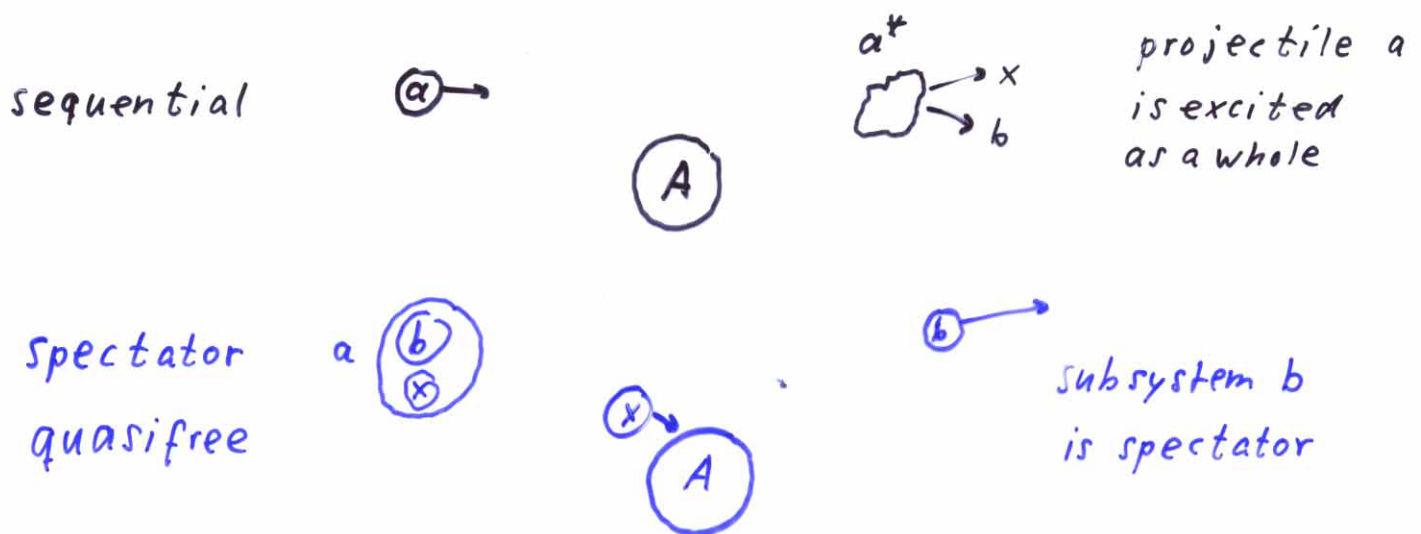
charged particle reaction $A + x \rightarrow c + C$



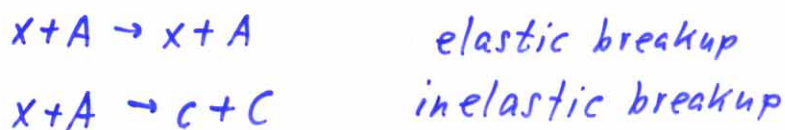
trojan horse $a = (b + x)$



Mechanisms for Breakup Reactions



reactions of $A+x$ subsystem:



for THM to work, the spectator (or quasifree) mechanism must be dominant
ensure by choosing kinematical conditions

various models (semiclassical, DWBA, eikonal, ...)

Bonaccorso, Brink:

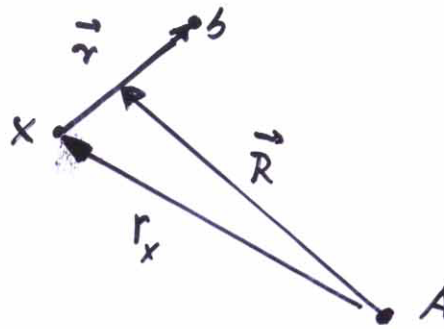
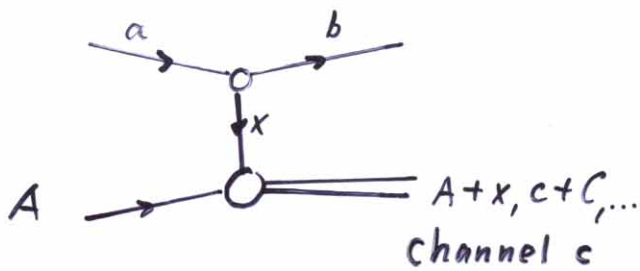
breakup probability $\frac{dP}{d\Omega_n} \sim \sum_j [1 - |S_j|^2 + (1 - |S_j|^2) B_j]$

↑
S-matrix for x - A interaction

see also C.A. Bertulani, G. Baur, M.S. Hussein NPA 526 (1994) 751

M.S. Hussein, K. McVoy NPA 445 (1985) 124

Post-form DWBA



elastic breakup

$$T \sim \int d^3R d^3r \chi_b^{(-)*} \chi_x^{(-)*} V_{bx}(r) \psi_{bx}(r) \chi_a^{(+)}(\vec{R})$$

for numerical calculations $V_{bx} \sim \delta(\vec{r})$ $D_0 \equiv \int d^3r V_{bx}(r) \psi_{bx}(r)$
"zero range"

inelastic breakup

$$T \sim D_0 \int d\xi_A \Phi_{B,c}^{(-)*}(\xi_A, r_x) \Phi_A(\xi_A) \dots$$

↑
internal coordinates of A

"wave function of transferred particle"

"surface approximation" $\sim \delta_{\ell_x c} j_{\ell_x} + (S_{\ell_x c} - \delta_{\ell_x c}) h_{\ell_x}^+$

Inclusive cross section $A + a \rightarrow b + X$

use $\sum_{c \neq \ell_x} |S_{\ell_x c}|^2 = 1 - |S_{\ell_x \ell_x}|^2$

M. Ichimura PRC **41** (1990) 834 relation among

theories of inclusive breakup reactions

Trojan Horse Method

Volume 37B, number 3

PHYSICS

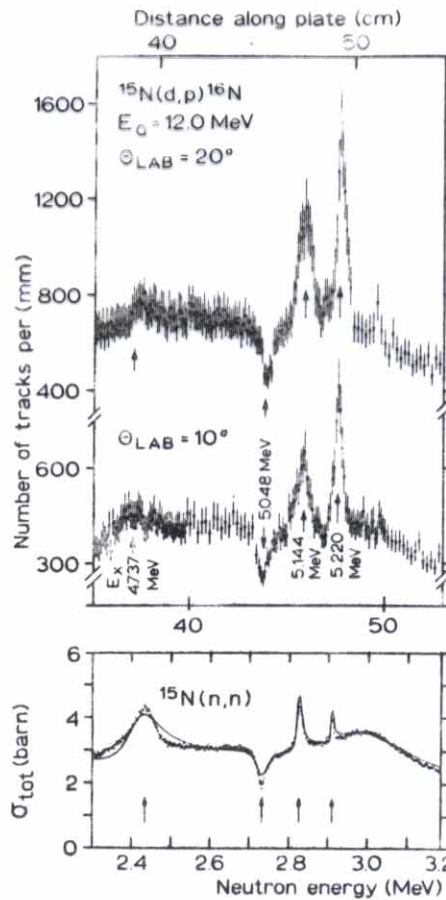
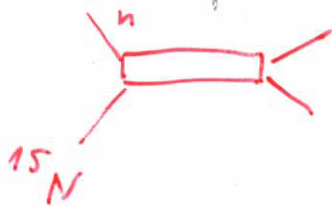
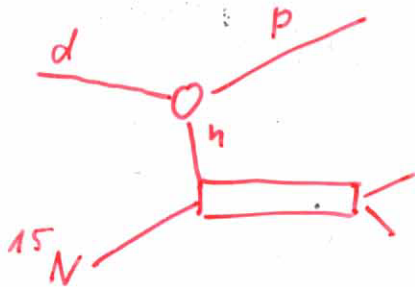


Fig.1 Details of proton spectra from $^{15}\text{N}(d,p)^{16}\text{N}$ (upper part) in comparison with the $^{15}\text{N}(n,n)$ total cross section of Zeitnitz et al. [4] (lower part). Resonance energies E_x in ^{16}N are from the neutron data. Their positions expected in the proton spectra are indicated by arrows. (solid line: R -matrix fit).

Compare also the method to extract n - n scattering

from $n + d \rightarrow n + n + p$

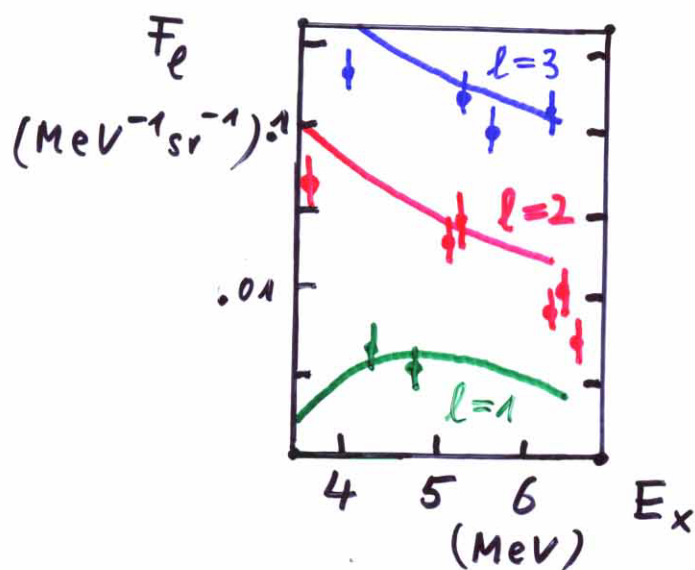
Parallelism

G. Baur, D. Trautmann Phys. Rep. 25C (1976) 293

$$\frac{d^2\sigma}{d\Omega_p dE_p}(d,p) = F_\ell(q_n, q_d, q_p) \cdot \sigma_\ell(n,n)$$

"stripping enhancement factor"
depends on a cut-off radius R

results for $^{15}\text{N}(d,p)^{16}\text{N}^* / ^{15}\text{N}(n,n)^{15}\text{N}$



similar results for $^{24}\text{Mg}(d,p)^{25}\text{Mg}^* / ^{24}\text{Mg}(n,n)^{24}\text{Mg}$

investigate unbound nuclei:



P. Santi et al. at NSU

G. Blanckin, A. Bonaccorso and N. Vinh Mau nucl-th /

0402050

$E_x \rightarrow 0$ Threshold behaviour

$x = \text{neutral particle}$

$$\frac{d^2\sigma(\text{inelastic})}{d\Omega dE} \sim q_n \left(\frac{1}{q_n^{l_n+1}} \right)^2 \underbrace{(1 - |S_{l_n l_n}|^2)}_{\sim q_n^{2l_n+1}} \sim \text{finite} \quad (\neq 0)$$

$\uparrow$
from DWBA
integral

$(l_n=0: \text{Fermi's } \frac{1}{v} \text{-law in n-A scattering})$

$$\frac{d^2\sigma(\text{elastic})}{d\Omega dE} \xrightarrow{q_n \rightarrow 0} 0$$

$x = \text{charged particle}$

$$\sigma(A+x \rightarrow c+C) \sim \frac{e^{-2\pi\eta}}{q_x^2}$$

$$\frac{d^3\sigma(\text{inelastic})}{d\Omega_c d\Omega_b dE_b} \sim q_x |S_{l_x c} T_{l_x m_x}^{(+)}|^2 \rightarrow \text{finite} (\neq 0)$$

$\uparrow$
 $\sim e^{-\pi\eta}$

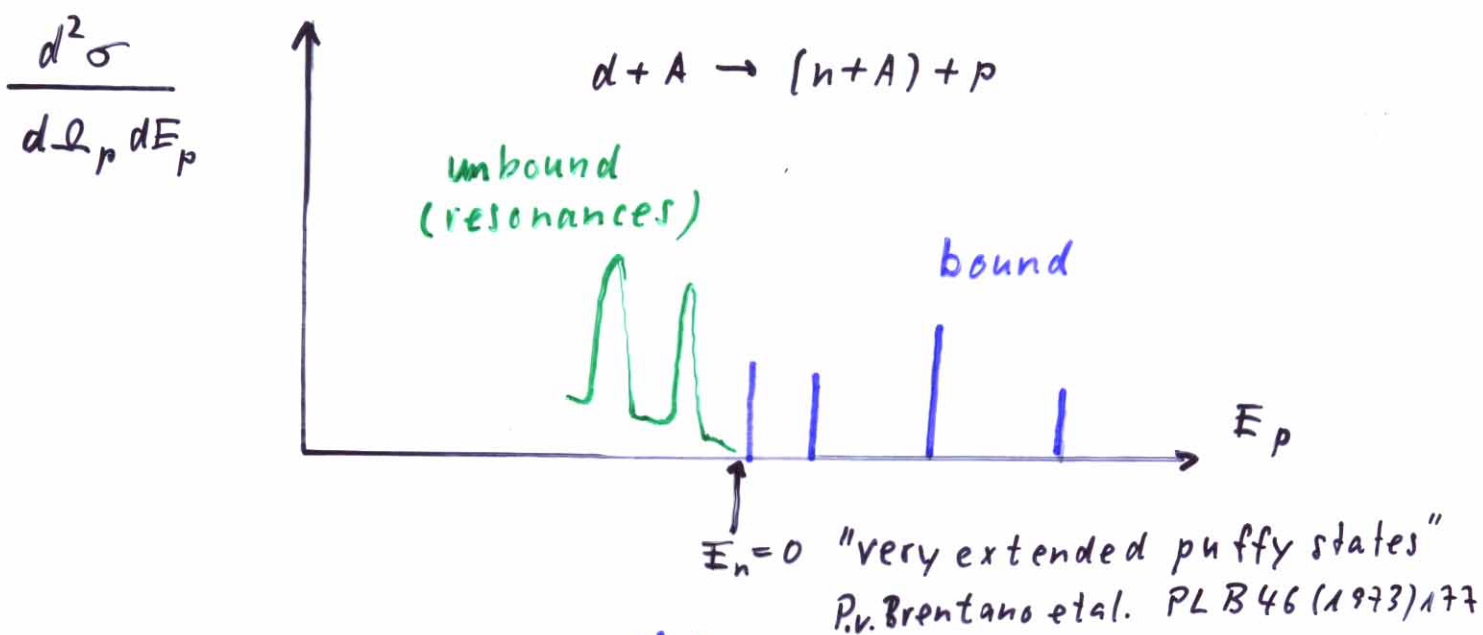
$\uparrow$
 $\frac{e^{+\pi\eta}}{\sqrt{q_x}}$

• DWBA-integral $T_{l_x m_x}^{(+)}$ must be known theoretically

• extraction of inelastic S -matrix element

$S_{l_x c}$ relies on surface approximation

Stripping to bound and unbound states: continuous transition



bound states $E_b = \frac{\hbar^2 \alpha^2}{2m}$

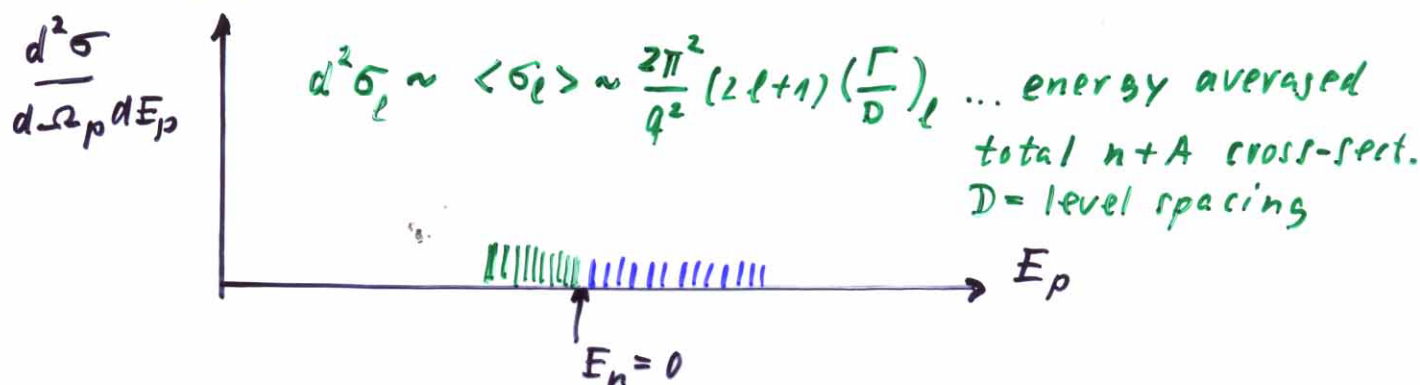
$$\psi_n(r) = \underbrace{\sqrt{S} \cdot N}_{\text{ANC}} \cdot h_l(i\alpha r) \quad r > R$$

unbound states (continuum) $E_n = \frac{\hbar^2 q^2}{2m}$

natural definition of spectroscopic factor: $\Gamma = \Gamma_{s.p.} \cdot S$

with $\Gamma_{s.p.} = \frac{\hbar^2}{mq} \cdot N^2$

for densely spaced levels:



see G. Baur, F. Rösler, D. Trautmann, R. Shyam Phys. Rep. 111 (1984) 333

Pampus, Bisplinghoff, Ernst, Hoyer-Kuckuk, Ramakao, Baur,
Rösel, Trautmann

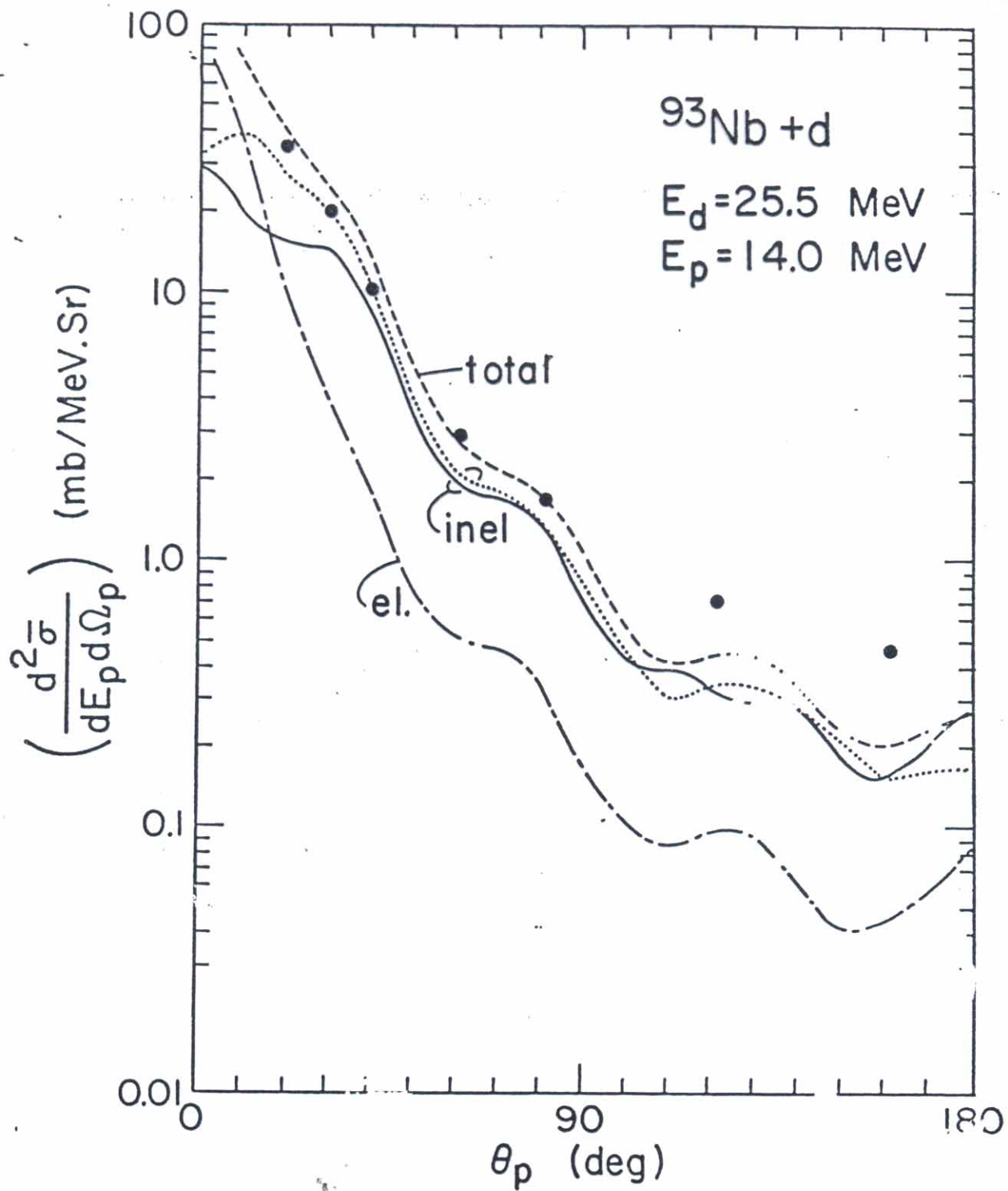


FIG. 1

Formalism for breakup reactions without use
of surface approximation:

A. Karano, M. Ichimura Phys. Lett. B 115 (1982) 81

⇒ a numerical check of the accuracy
of the surface approximation

Inelastic breakup mechanism clarified
in a series of papers ("IAV")

see M. Ichimura, N. Austern and C. M. Vincent

Phys. Rev. C37 (1988) 2264 · to trace back this story

Conclusion

- Trojan-horse method is indirect, needs theoretical analysis
S. Typel, G. Baur Ann. Phys. 305(2003) 228
- assumes simple reaction mechanism
- DWBA, or modified PWBA - methods
at higher energies: eikonal approach
- electron screening can be neglected

Experiments

- Catania group C. Spitaleri et al.
- $^{12}\text{C}({}^7\text{Li}, t\gamma){}^{16}\text{O}$ Heidelberg Heurler, Reiter et al.
not (yet) successful
- populate resonant states
- $^{22}\text{Na}({}^3\text{He}, d){}^{23}\text{Mg} \rightarrow {}^{22}\text{Na}(p, \gamma){}^{23}\text{Mg}$ C. Rolfs et al. 1995
- $^{17}\text{O}({}^3\text{He}, d){}^{18}\text{F} \rightarrow {}^{17}\text{O}(p, \gamma){}^{18}\text{F}$ V. Landre et al. 1989
- $^{15}\text{N}({}^7\text{Li}, t){}^{19}\text{F} \rightarrow {}^{15}\text{N}(\alpha, \gamma){}^{19}\text{F}$ F. de Oliveira et al. 1996
- application to radioactive beams EXL
a wide choice of targets and projectiles
- cf. K.E. Rehm et al $d({}^{56}\text{Ni}, p){}^{57}\text{Ni}$ 1998
study of transitions to bound states
extend to unbound states

Coulomb Dissociation Method

a "nuclear physics Primakoff effect"

- nuclear structure

GDR in unstable nuclei

low lying EA strength

- nuclear astrophysics

indirect method to study radiative capture



$$\Rightarrow S_{17}(E) \quad \text{for } {}^7\text{Be}(p, \gamma) {}^8\text{B}$$

↑
solar neutrinos

complementary to direct studies:

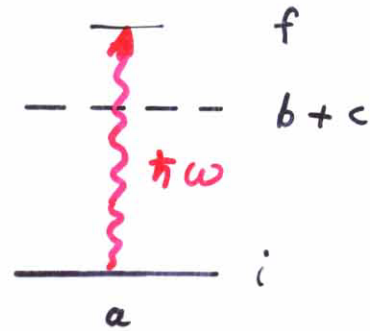
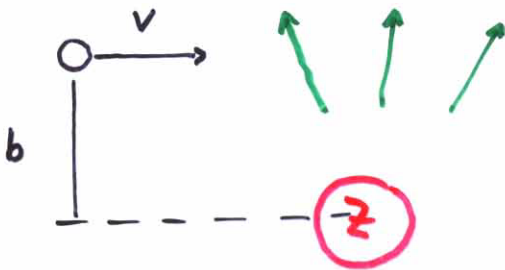
a few selected examples

unstable nuclei become accessible

e.g. r- and rp-process nuclei

2p-capture

Basic Parameters



collision time $\tau_{\text{coll}} = \frac{b}{\gamma v}$

$\gamma \equiv (1 - v^2)^{-\frac{1}{2}}$ ~~(≈ 1)~~

excitation time $\tau_{\text{exc}} = \frac{1}{\omega}$

adiabaticity parameter $f \equiv \frac{\tau_{\text{coll}}}{\tau_{\text{exc}}} = \frac{\omega b}{\gamma v}$

$f \ll 1$ excitation happens

$f \gg 1$ excitation is only virtual

strength parameter χ

$V_{\text{int}} \sim \frac{\hbar}{b^{\lambda+1}} \langle f | M(\pi\lambda) | i \rangle \cdot Ze$
 $\nwarrow$ electromagnetic matrix-el.
 $\pi\lambda = \text{multipolarity}$

$\chi = \frac{V_{\text{int}} \cdot \tau_{\text{coll}}}{\hbar} = Z \frac{\langle f | M(\pi\lambda) | i \rangle e}{b^{\lambda} \hbar v}$

$\chi \sim \text{number of exchanged photons}$

$\eta_{\text{Sommerfeld}} = \frac{Z Z_a e^2}{\hbar v}$ "monopole-monopole interaction"

lowest order: factorization

$$\sigma_c = \sum_{\pi\lambda} \frac{d\omega}{\omega} n_{\pi\lambda}(\omega) \sigma_{\pi\lambda}^{\pi\lambda}(\omega)$$

↑
multipolarity

equivalent photon spectra $n_{\pi\lambda}$

- $n_{E1} \sim \frac{1}{v^2} Z^2 \log \frac{\gamma v}{\omega R_{\min}}$
- $n_{M1} \sim \left(\frac{v}{c}\right)^2 n_{E1}$

ω_{max} ≈ $\frac{\gamma v}{R_{\min}}$

b-dependent equivalent photon spectra

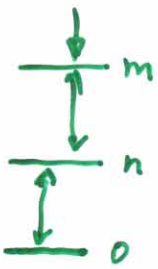
$$N_{E1}(\omega, b) \sim \frac{1}{b^2} \quad \text{for } b_{\min} < b \leq \frac{\gamma v}{\omega}$$

$$N_{E2}(\omega, b) \sim \frac{1}{b^4} \sim \frac{N_{E1}}{(k_y b)^2} \quad (k_y = \frac{\omega}{c})$$

↑
"adiabatic radius"

i.e. $N_{E2} \gg N_{E1}$ in many cases

Higher order effects



$$\Psi_n(t) = \sum_n a_n(t) e^{-i\omega_n t} \phi_n$$

with nuclear eigenstates $H_0 \phi_n = \hbar \omega_n \phi_n$

$\Rightarrow$ coupled equations

$$i\hbar \dot{a}_n(t) = \sum_m \langle n | V(t) | m \rangle e^{i(\omega_n - \omega_m)t} a_m(t)$$

solve numerically

or S-matrix-formalism

$$\tilde{V}(t) = e^{iH_0 t} V(t) e^{-iH_0 t}$$

$$a_n(\infty) = \langle n | T \left(\exp \left\{ -i \int_{-\infty}^{\infty} \tilde{V}(t) dt \right\} \right) | 0 \rangle$$

time ordering operator $T(A(t_1) \dots A(t_n)) = A(t_{p_1}) A(t_{p_2}) \dots$
 $t_{p_1} \geq t_{p_2} \geq \dots$

limits:

• weak coupling : 1st order, see above

• $\xi \equiv \frac{T_{\text{coll}}}{T_{\text{nuc}}} \ll 1$ sudden limit

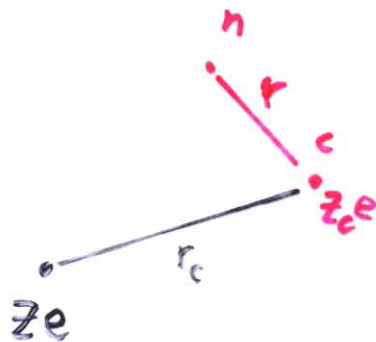
neglect T-operator all orders are included

Coulomb Dissociation of Neutron Halo Nuclei

A simple model

G. Baur, K. Hencken, D. Trautmann

Prog. Part. Nucl. Phys. 51 (2003) 487



$$H = T + \frac{ZZ_c e^2}{r_c} + V_{nc}(r)$$

short range

$$\phi_0 = \sqrt{\frac{\alpha}{2\pi}} \frac{e^{-\alpha r}}{r}$$

ground-state halo wave fctn.

$$\phi_{\vec{q}} = e^{i\vec{q} \cdot \vec{r}} - \frac{i}{\alpha + iq} \frac{e^{iqr}}{r}$$

continuum states

a theoretical laboratory to study

- higher order effects
"post-acceleration"
- quantal and semiclassical methods

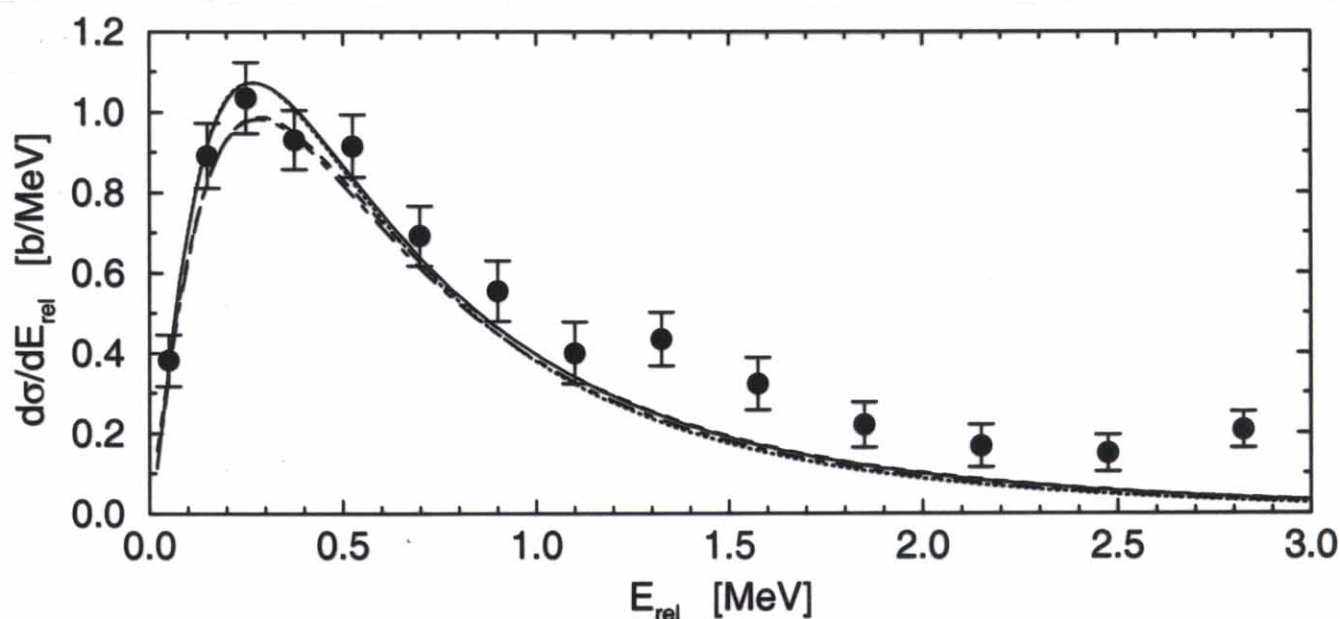
Comparison with experiment

halo nucleus $^{19}\text{C} \rightarrow ^{18}\text{C} + n$

{ ————— LO (with finite range correction)
 { - - - - - 1st order semiclassical calculation (with finite range w. f.)

{ — — — — — LO + NLO
 { - . - . - . - time-dependent Schrödinger eq.

S. Typel, G. Baur (2001)
PRC



T. Nakamura et al. ^{19}C on ^{208}Pb $E/A = 67 \text{ MeV/A}$

 $0 \leq \theta < 3^\circ$ angle integrated

Scaling laws

S. Typel, G. Baur PRC 64 (2001)

$$\underbrace{(c+n)}_{\equiv a} + Z \rightarrow c + n + Z$$

$$\vec{q}_a - \vec{q}_c - \vec{q}_n \equiv \vec{q}_{\text{out}}$$

$$x \equiv \frac{q_{\text{rel}}}{\omega} \quad \dots \text{shape parameter} \quad E_{\text{rel}} = \frac{\hbar^2 q_{\text{rel}}^2}{2\mu}$$

$$y = \frac{2 \tilde{Z} \tilde{Z}_c e^2 m_n}{\hbar v (m_n + m_c) b \omega} \quad \text{strength parameter}$$

excitation amplitude

$$a(q_{\text{rel}}) = a^{(1)} + a^{(2)} + a^{(3)} + \dots$$

breakup probability (integrated over $\Omega_{q_{\text{rel}}}$)

$$\sim |a^{(1)}|^2 : \quad \frac{dP_{\text{LO}}}{dq_{\text{rel}}} = \text{const. } y^2 \frac{x^4}{(1+x^2)^4}$$

$$\sim |a^{(2)}|^2 + 2\text{Re}(a^{(1)} a^{(3)*}) :$$

$$\frac{dP_{\text{NLO}}}{dq_{\text{rel}}} = \text{const. } y^4 \frac{x^2 (5 - 55x^2 + 28x^4)}{15 (1+x^2)^6}$$

$$\text{const.} \equiv \frac{16}{3\pi \omega}$$

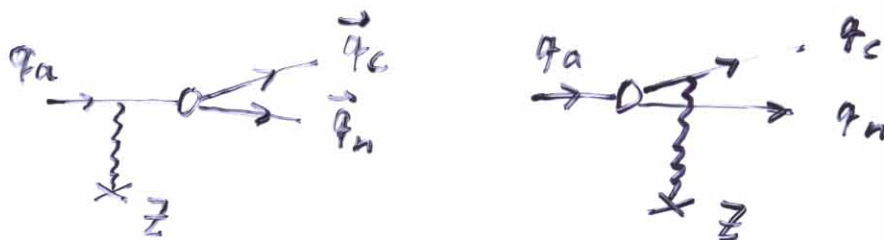
CWBA (Coulomb Wave Born Approximation)

$$T_{\vec{q}_a \rightarrow \vec{q}_c \vec{q}_n} = D_0 \int d^3R \chi_{\vec{q}_c}^{(-)}(\vec{R}) e^{-i\vec{q}_n \cdot \vec{R}} \chi_{\vec{q}_a}^{(+)}(\vec{R})$$

$$D_0 = \frac{\hbar^2}{2\mu} \sqrt{8\pi\alpha} \quad \mu = \frac{m_n m_c}{(m_n + m_c)}$$

For $\eta_a \gg 1, \eta_c \gg 1$ $\xi \equiv \eta_c - \eta_a \ll 1$ $\eta_{a,c} = \frac{Z z_c e^2}{\hbar v_{a,c}}$

$$T \approx T_{\text{Born}} \sim \frac{1}{q_{\text{rel}}^2 + \alpha^2} - \frac{1}{(\vec{q}_{\text{rel}} - \Delta\vec{p})^2 + \alpha^2}$$



$$\Delta\vec{p} = \frac{m_n}{m_a} \vec{q}_{\text{coul}}$$

$$\vec{q}_{\text{coul}} = \vec{q}_a - \vec{q}_c - \vec{q}_n$$

$$b = \frac{2\eta_a}{q_{\text{coul}}}$$

↑
impact parameter

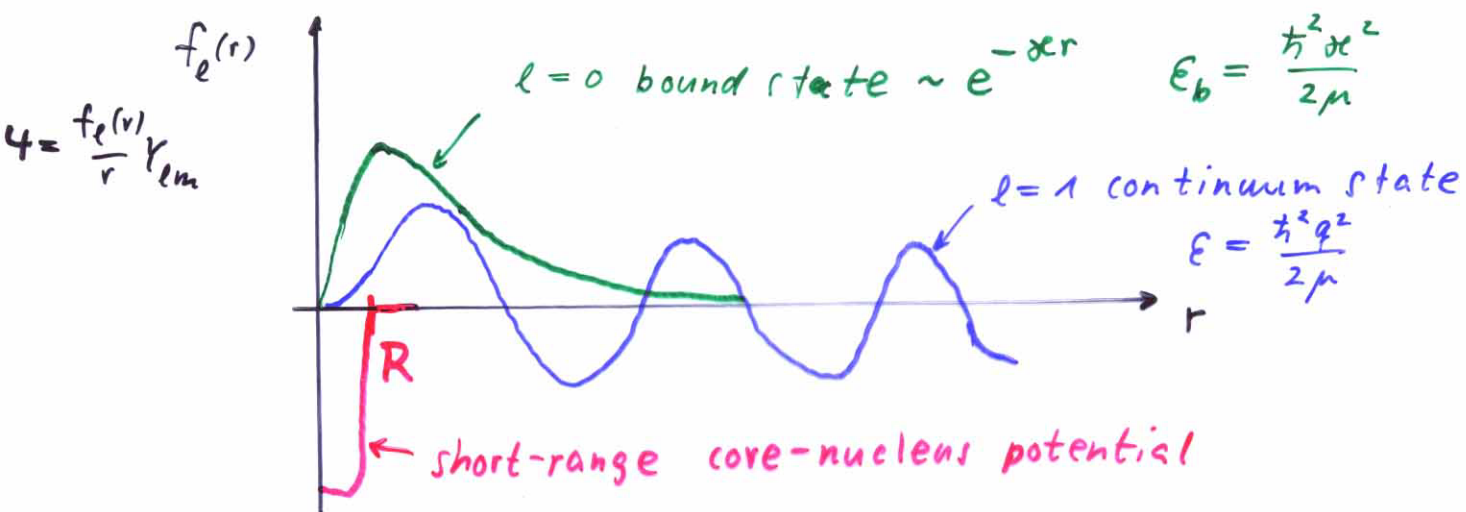
For fast projectiles: $\eta_a \gg 1, \eta_c \gg 1, \xi \ll 1$

$\Rightarrow$ no post-acceleration

(cf. Bush et al. PRL 1988: $^{11}\text{Be} \rightarrow ^{10}\text{Be} + n$:

...no evidence for previously suggested
velocity difference

Effective range theory of Halo Nuclei



$$\frac{1}{\alpha} \gg R \quad \text{halo nucleus}$$

$$q \sim \alpha \rightarrow q \cdot R \ll 1$$

radial dipole matrix-element

$$R_{l_i l_f}^1 \equiv \int_0^\infty dr e^{-\alpha r} \cdot r^2 j_1(qr) = \frac{2q}{(\alpha^2 + q^2)^2}$$

shape of $B(E1)$ -distribution determined
by a few low energy constants

$$R \rightarrow 0 : \quad \underline{\alpha}$$

$$x = \frac{q}{\alpha}$$

$$B(E1) \sim \frac{x^2}{(1+x^2)^4}$$

examples: $d, {}^{11}\text{Be}, {}^{19}\text{C}, {}^{15}\text{C} = ({}^{14}\text{C} + n)$

spectroscopic
factor

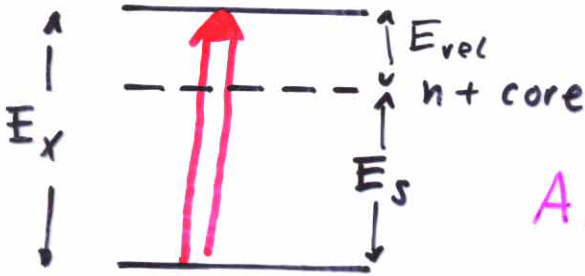
$$S = 0.75$$

(T. Aumann et al.)

calculation of low lying E1-strength in

simple model : bound state (s-wave) in short range pot.

→ plane wave continuum state



• determine from experiment:

Asymptotic Normalisation Constant

$$\frac{dB(E1)}{dE_x} = S \left| \left\langle q \left| e \frac{Z}{A} r Y_m^1 \right| N_0 \sqrt{\frac{\kappa}{2\pi}} \exp(-\kappa r) / r \right\rangle \right|^2$$

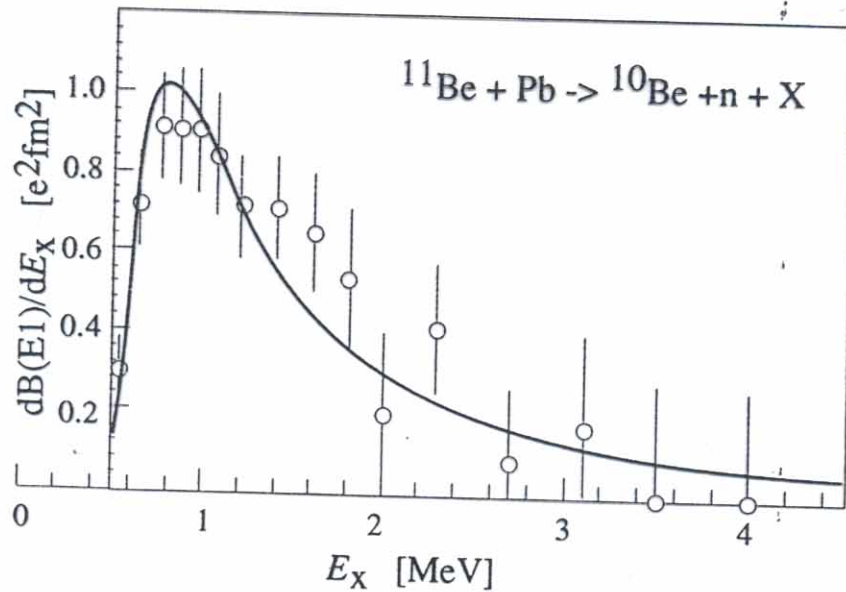
$$= S \frac{\exp(2\kappa r_0)}{1 + \kappa r_0} \frac{3\hbar^2}{\pi^2 \mu} e^2 \left(\frac{Z}{A} \right)^2 \frac{\sqrt{E_s} (E_x - E_s)^{3/2}}{E_x^4}$$

Baur, Berikiani 1986
or Blatt-Weisskopf 1952
deuteron photo-
disintegration
(5.1)

Fig. 5.4 E1 strength distribution deduced from the EMD cross section of ^{11}Be .

T. Nakamura et al.

(from Tanikata '95)



$$E_x = E_s + E_{rel}$$

• determine E_s (separation energy) from shape:

$$\sim \frac{E_{rel}^{3/2}}{(E_s + E_{rel})^4}$$

Chapter 2a: Applications to nuclear structure, transparency 21

Prototype of Halo Nucleus: Deuteron

$B = 2.23 \text{ MeV}$

see Blatt and Weisskopf (1952)

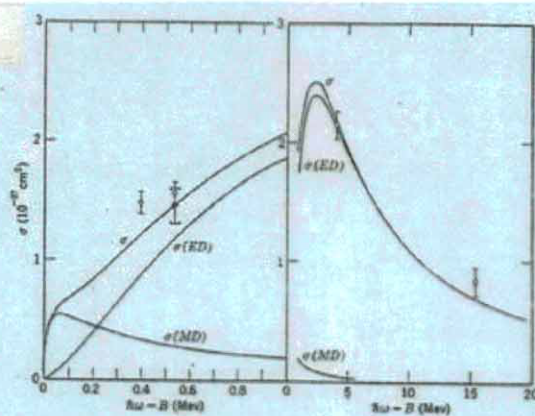


FIG. 4.1. Photo-disintegration cross section of the deuteron. ED means electric dipole absorption, MD means magnetic dipole absorption.

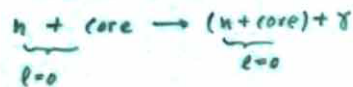
low lying MA strength



threshold behaviour: $\sigma_{\gamma}^{MA} \sim (\mu_n - \mu_p)^2 \frac{q_{rel}}{(q_{rel}^2 + q_{bind}^2)}$

($\sigma_{capt.} \sim \frac{1}{q_{rel}}$)

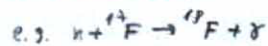
other cases?



no angular momentum
or (coulomb barrier)

→ odd-odd halo nuclei?

anyway, probably too
difficult to see in
(coulomb dissociation)



(not a halo nucleus, $E_n = 9.45 \text{ MeV}$)

systematic corrections for finite range R

normalization of (single particle) bound state wave function

$$\sqrt{2\kappa} \frac{e^{-\kappa r}}{r} \rightarrow \sqrt{\frac{2\kappa}{1-\kappa r_0}} \frac{e^{-\kappa r}}{r}$$

$$q^{2\ell+1} \cot \delta_\ell = -\frac{1}{a_\ell} + \frac{r_\ell}{2} q^2 + \dots \quad \text{effective range expansion}$$

$$d = (p+n) : \quad \frac{1}{\kappa} = 4.3 \text{ fm} \quad r_0 = 1.6 \text{ fm}$$

continuum wave function

$$\text{free wave} \quad j_\ell(qr) \xrightarrow{r>R} \cos \delta_\ell j_\ell + \sin \delta_\ell n_\ell$$

neglect contribution to radial integral from $r < R$:

$s \rightarrow p$ excitation

$$B(E1) \sim \left(\cos \delta_1 \frac{x}{(1+x^2)^2} + \sin \delta_1 \frac{1+3x^2}{2x^2(1+x^2)^3} \right)^2$$

$$\cos \delta_1 \approx 1 \quad \sin \delta_1 \approx -a_1 q^3 = -x^3 (\underbrace{x^3 a_1}_{\text{small}})$$

Kalassa, Baur (1954, 1956), Mengoni et al. (1955), Type I

$p \rightarrow s$ excitation:

$$\delta_0 = -a_0 \cdot q$$

halo nature
less pronounced
due to centrifugal barrier

Proton Halo Nuclei

e.g. ${}^8\text{B} = ({}^7\text{Be} + p)$

$$\begin{aligned} R_{\ell_i, \ell_f}^1 &\cong \int_R^\infty dr f_{\ell_i}(r) \cdot r \cdot (F_{\ell_f}(\eta, qr) + \tan \delta_{\ell_f} G_{\ell_f}) \\ &\cong R_{\ell_i, \ell_f}^F + \tan \delta_{\ell_f} R_{\ell_i, \ell_f}^G \end{aligned}$$

ANC-method $\delta_{\ell_f} = 0$

$$C_0^2 k \cot \delta_0 = -\frac{1}{a} - \frac{1}{a_{\text{Bohr}}} L(\eta) \quad C_0^2 = \frac{2\pi\eta}{e^{2\pi\eta} - 1} \quad \delta_0 \sim e^{-2\pi\eta}$$

behaviour of Coulomb functions

$$r \sim 0 \quad F_0 \sim C_0 \quad G_0 \sim \frac{1}{C_0}$$

$$r \rightarrow \infty \quad F \rightarrow \sin(\dots) \quad G \rightarrow \cos(\dots)$$

$$R_{\ell_i, \ell_f}^F \sim \text{independent of } R$$

$\tan \delta_{\ell_f} R_{\ell_i, \ell_f}^G \dots$ strongly dependent on R , small for halo nuclei (S. Type)

2-neutron-halo nuclei

example: Migdal (1973)

two interacting neutrons in a potential well

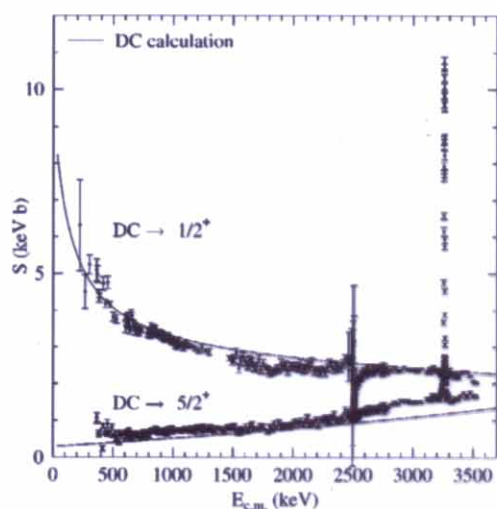


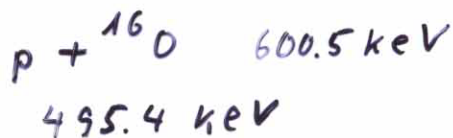
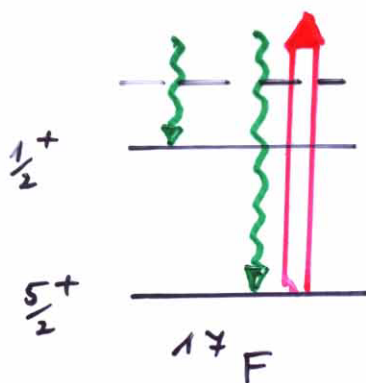
FIG. 3. Experimental capture cross section of the reactions $^{16}\text{O}(p, \gamma_0)^{17}\text{F}$ and $^{16}\text{O}(p, \gamma_1)^{17}\text{F}$ compared to a DC calculation. Note the strikingly different branching ratio between $\text{DC} \rightarrow 5/2^+$ and $\text{DC} \rightarrow 1/2^+$.

Radiative capture



$$\sigma = \frac{1}{E} e^{-2\pi\eta} \cdot S$$

↑
astrophysikalischer
S-Faktor



Coulomb dissociation



- only ground state branch
- would be a test of the method

Conclusion Outlook

Coulomb Excitation and Dissociation:

a powerful tool for nuclear structure

- $E1, E2, M1, \dots$ strength in unstable (light and heavy) nuclei
- applications to nuclear astrophysics
- model investigation of halo nucleus Coulomb breakup: 1st order theories are OK for high projectile energies
- effective range methods for halo nuclei
- applications to UPC (ultra-peripheral collisions) at RHIC and LHC