Theoretical estimates of spectroscopic factors (and final state interactions) in $(e, e'p)$

Outline

- Preliminaries: overlaps, spectral functions and all that
- Spectroscopic factors in infinite nuclear matter
- Spectral functions of finite nuclei within the Local Density Approximation
- Final state interactions: nuclear transparency
- Conclusions

• Overlaps are well (and *uniquely*) defined quantities for interacting many-body systems

$$\chi_n(\mathbf{r}_1) = \int d^3r_2 \dots d^3r_{\rm A} \; \Psi_n^{\rm A-1}(\mathbf{r}_2 \dots \mathbf{r}_A)^\dagger \Psi_0^{\rm A}(\mathbf{r}_1 \dots \mathbf{r}_A)$$

• They are directly related to the spectral function, yielding the energy-momentum probability distribution

$$P(\mathbf{k}, E) = \sum_n \left| \langle \Psi_n^{\rm A-1} | a_{\mathbf{k}} | \Psi_0^{\rm A} \rangle \right|^2 \delta(E - E_n + E_0)$$

• If $|\Psi_n^{\rm A-1}\rangle$ is a bound state χ_n carries information on single nucleon dynamics

• Within the mean field picture $\chi_n \to \phi_n^{MF}$

- *In principle* the spectroscopic factors

$$Z_n = \int d^3r \ \ |\chi_n(\mathbf{r})|^2$$

can be extracted from the (non trivial !) analysis of the $(e, e'p)$ x-section, that in the Plane Wave Impulse Approximation picture reduces to

$$\sigma = K \sigma_{ep} \ P(\mathbf{p} - \mathbf{q}, \omega - T_p) \ ,$$

where σ_{ep} is the elctron scattering cross section off a *bound moving* nucleon

- Realistic theoretical spectral functions and a quantitative understanding of final state interactions (FSI) of the knocked out nucleon are required

Spectroscopic factors in infinite nuclear matter

- As momentum is a good quantum number, the spectral function at $|\mathbf{k}| < k_F$ exhibits only one peak

- The spectroscopic factor is defined as

$$Z_k = \left| \langle \Phi^{1h}_{\mathbf{k}} | a_{\mathbf{k}} | \Psi_0 \rangle \right|^2 \ ,$$

where $|\Phi^{1h}_{\mathbf{k}}\rangle$, is the *one-hole* (A-1)-nucleon state carrying momentum $\mathbf{k}$

- Z_k *does not* coincide with the occupation number of the state $|\Phi^{1h}_{\mathbf{k}}\rangle$, $n(\mathbf{k})$, given by

$$n(\mathbf{k}) = \langle \Psi_0 | a^\dagger_{\mathbf{k}} a_{\mathbf{k}} | \Psi_0 \rangle = \sum_n |\langle \Phi^n_{\mathbf{k}} | a_{\mathbf{k}} | \Psi_0 \rangle|^2$$

$\{|\Phi^n_{\mathbf{k}}\rangle\}$, being the complete set of (A-1)-nucleon states of momentum $\mathbf{k}$

• $P(\mathbf{k},E)$, Z_k and $n(\mathbf{k})$ of nuclear matter at equilibrium density ($\rho = 0.16$ fm^{-3}) have been calculated using realistic a realistic nuclear hamiltonian of the form

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} V_{ijk}$$

and the set of *correlated* states

$$|n) = \frac{F|n_{\rm FG}\rangle}{\langle n_{\rm FG}|F^\dagger F|n_{\rm FG}\rangle^{1/2}}$$

• The correlation operator F, whose structure reflects the structure of the interaction potential, is determined by minimization of the ground state expectation value of the nuclear hamiltonian

$$E_0^V = \langle \Psi_0 | H | \Psi_0 \rangle$$

• The correlated states are orthogonalized by a transformation that preserves diagonal matrix elements

$$|n) \rightarrow |n\rangle = \widehat{T}|n) \quad , \quad (n|H|n) = \langle n|H|n\rangle$$

• The hamiltonian is split according to

$$H = H_0 + H_I$$

$$\langle m|H_0|n\rangle = \delta_{mn}\langle m|H|n\rangle \quad , \quad \langle m|H_I|n\rangle = (1-\delta_{mn})\langle m|H|n\rangle$$

• If correlated states have large overlaps with the eigenstates of the hamiltonian the matrix elements of H_I are small

- The spectral function, rewritten in the form

$$P(\mathbf{k},E) = \frac{1}{\pi}\ Im\ \langle\Psi_0|\frac{1}{H - E_0 - E - i\eta}|\Psi_0\rangle\ ,$$

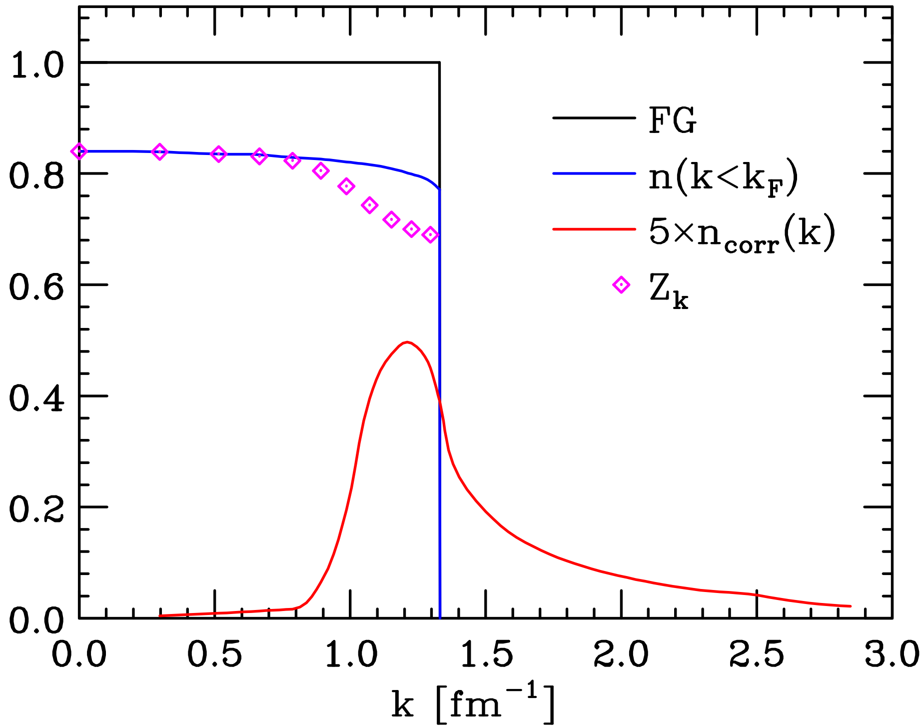
with $(\Delta E_0 = E_0 - E_0^V)$

$$\frac{1}{H - E_0 - E - i\eta} = \frac{1}{H_0 - E_0^V - E - i\eta}\ \sum_m (-)^m \left(\frac{H_I - \Delta E_0}{H_0 - E_0^V - E - i\eta}\right)^m$$

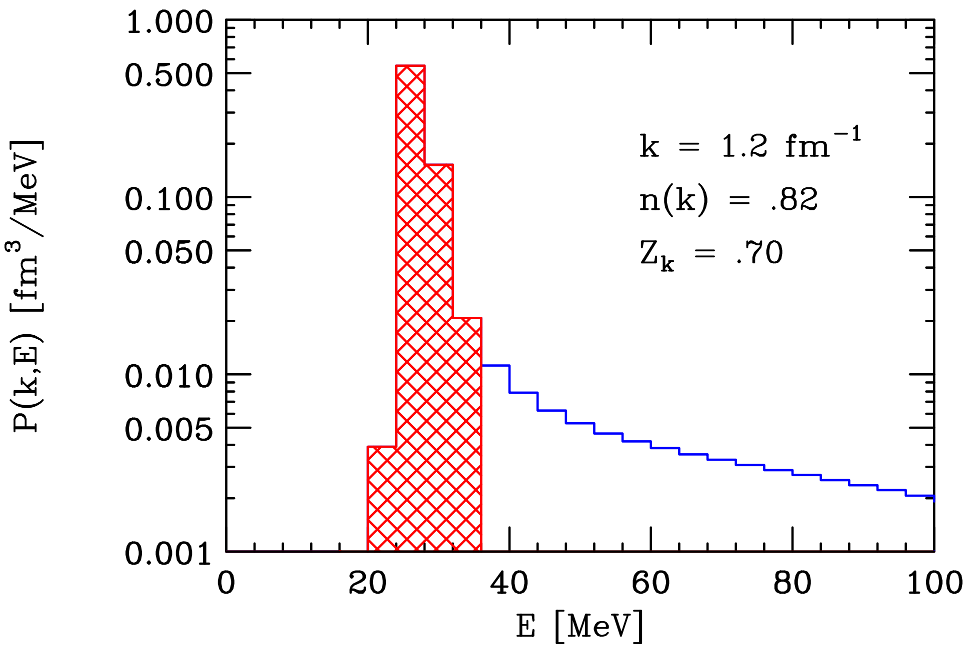
$$|\Psi_0\rangle = \sum_m (-)^m \left(\frac{H_I - \Delta E_0}{H_0 - E_0^V}\right)^m |0\rangle$$

has been calculated including the correlated one hole and two hole-one particle intermediate states

• while the calculated Z_k is discontinuous at $|\mathbf{k}| = k_F$ and vanishes at $|\mathbf{k}| > k_F$, the contribution to $n(\mathbf{k})$ from states $|\Phi^n_{\mathbf{k}}\rangle \neq |\Phi^{1h}_{\mathbf{k}}\rangle$ is continuous across the Fermi surface

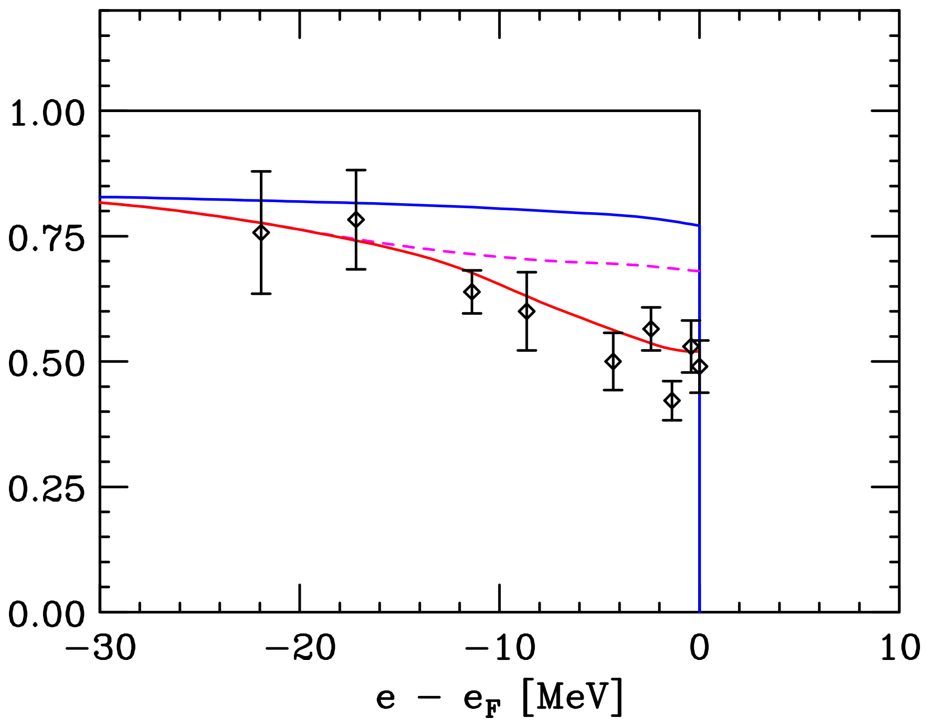


- the difference between Z_k and $n(k)$ naturally emerges from the analysis of the spectral function at fixed $|\mathbf{k}| < k_F$



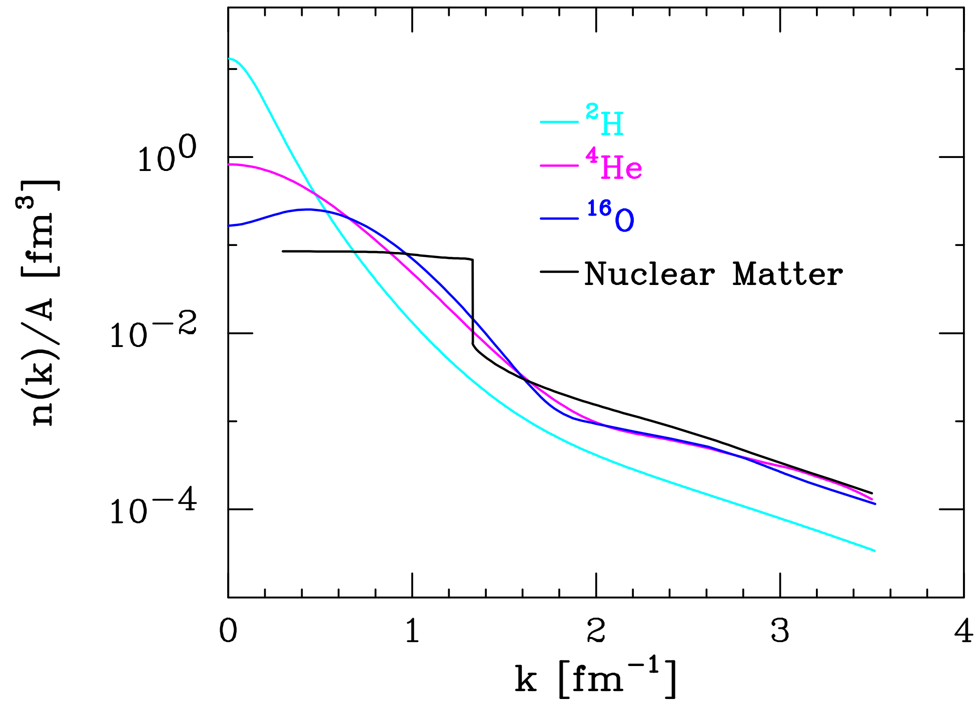
- integration over the peak region only yields Z_k
- integration over the whole energy range yields $n(k)$

- High resolution $(e, e'p)$ experiments measure Z_k
- Comparison to NIKHEF data. The solid red line includes *estimated* surface effects



Spectral function of finite nuclei within the local density approximation (LDA)

- **Bottom line**: scaling of the calculated $n(|\mathbf{k}| > k_F)$ with A (for A>2) suggests that the *correlation* (continuous) part of the momentum distribution at large $|\mathbf{k}|$ be nearly unaffected by surface effects



- The separation of one-hole and background contributions in the nuclear matter momentum distribution can be generalized to the spectral function

- Combine the correlation part extracted from nuclear matter calculations at different densities with a mean field spectral function yielding a reasonable fit of $(e, e'p)$ data

$$P^{LDA}(\mathbf{k}, E) = P_{MF}(\mathbf{k}, E) + P_{corr}(\mathbf{k}, E)$$

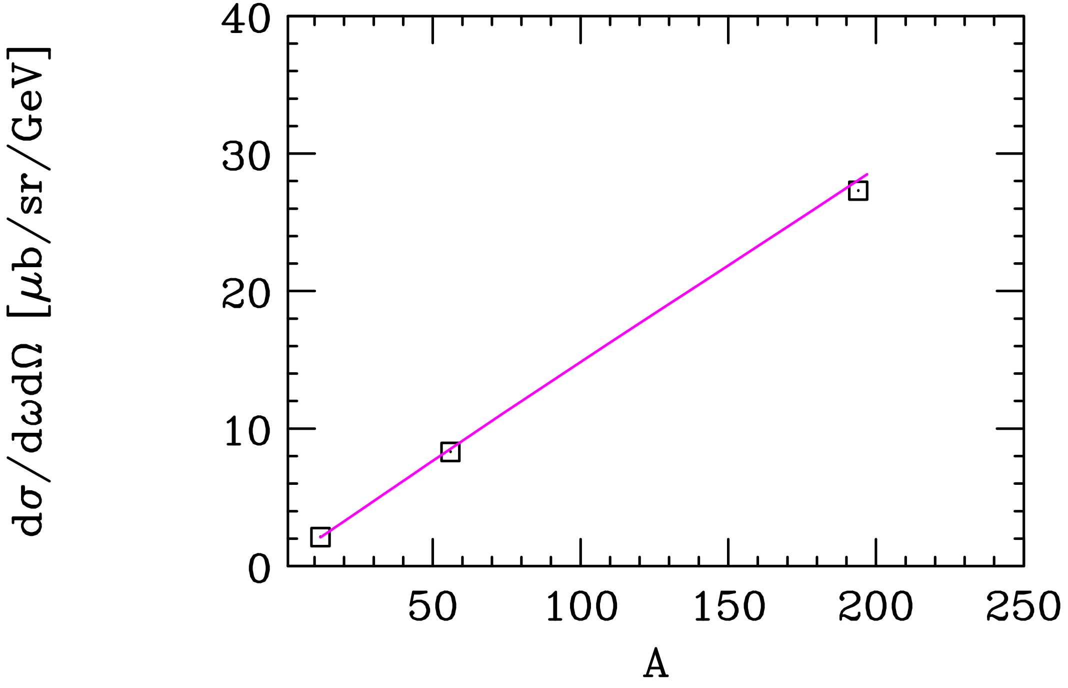
$$P_{corr}(\mathbf{k}, E) = \int d^3r \ \rho(\mathbf{r}) \ P^{NM}_{corr}(\mathbf{k}, E; \rho = \rho(\mathbf{r}))$$

$$P_{MF}(\mathbf{k}, E) = Z_n \sum_n |\phi^{MF}(\mathbf{r})|^2 F_n(E - E_n)$$

- The spectroscopic factors Z_n are constrained by the requirement

$$\int dE \ \frac{d^3k}{(2\pi)^3} \ P^{LDA}(\mathbf{k}, E) = Z$$

- PWIA cross sections obtained from LDA spectral functions provide a quantitative description of inclusive data
- Example: A-dependence of the JLab E89-008 data at $x = 1$ and $Q^2 \sim 1$



- Integrated strength form Carbon LDA spectral function

$$\int_{\Delta E} dE \int_{\Delta k} \frac{d^3k}{(2\pi)^3} P^{LDA}(\mathbf{k}, E)$$

- $\Delta k = 0 - 310$ MeV, $\Delta E = 15 - 22$ MeV (low Q^2, p-state) : $Z_p = .64$
- $\Delta k = 0 - 310$ MeV, $\Delta E = 30 - 50$ MeV (low Q^2, s-state) : $Z_s = .60$
- $\Delta k = 0 - 290$ MeV, $\Delta E = 30 - 80$ MeV (high Q^2, s-state) : $Z_s = .78$

- The integration region corresponding to the high Q^2 measurement is likely to include a sizeable amount of correlation strength

Final State Interactions within the high-energy approximation

A. Eikonal approximation : the outgoing proton moves along a straight trajectory in the direction of $\mathbf{p}$

B. Frozen approximation : the spectator nucleons are seen as a collection of fixed scattering centers

- Under assumptions A and B, one can construct the coordinate space distortion operator

$$\Omega_{\mathbf{p}}^{(-)}(\mathbf{r}) = \frac{1}{\rho_A(\mathbf{r})} \int d^3\mathbf{r}_1 \ldots d^3\mathbf{r}_A |\Psi_0(\mathbf{r}_1 \ldots \mathbf{r}_A)|^2$$

$$\times \frac{1}{A} \sum_{i=1}^{A} \prod_{j>i} \left[1 - \Gamma_{\mathbf{p}}(\mathbf{b}_i - \mathbf{b}_j)\theta(z_i - z_j)\right] \delta(\mathbf{r} - \mathbf{r}_i)$$

- The profile function $\Gamma_{\mathbf{p}}$ is the Forier transform of the NN scattering amplitude at incident momentum $\mathbf{p}$ and momentum transfer $\mathbf{k}$, generally parametrized in the form

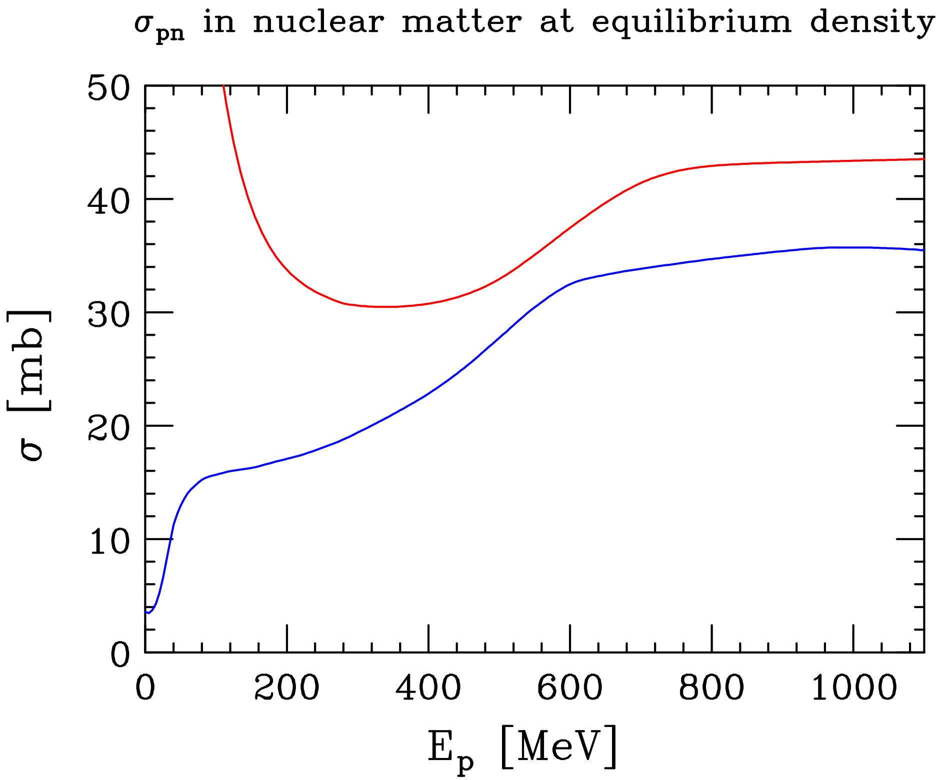
$$f_{\mathbf{p}}(\mathbf{k}) = \frac{|\mathbf{p}|}{4\pi}(i + \alpha_{\mathbf{p}})\sigma_{\mathbf{p}} \mathrm{e}^{-\beta_{\mathbf{p}}^2 \mathbf{k}^2}$$

where $\sigma_{\mathbf{p}}$ is the total NN cross section

- WARNING :

NN scattering in the nuclear medium may be appreciably modified by Pauli blocking and dispersive corrections

- Medium modified NN cross section, evaluated in nuclear matter at equilibrium density (Pandharipande & Pieper)



- At $T_p = 970$ MeV the free space cross section is reduced by $\sim 20\%$

- Local Density Approximation (LDA): experimental density + nuclear matter radial distribution functions at different densities

$$\Omega_{\mathbf{p}}^{(-)}(\mathbf{r}) = \frac{1}{\rho(\mathbf{r})} \int d^3\mathbf{r}_1 \ldots d^3\mathbf{r}_A |\Psi_0(\mathbf{r}_1 \ldots \mathbf{r}_A)|^2$$

$$\times \frac{1}{A} \sum_{i=1}^{A} \left[1 - \sum_{j>i} \Gamma_{\mathbf{p}}(\mathbf{b}_i - \mathbf{b}_i) \theta(z_i - z_j) + \ldots \right] \delta(\mathbf{r} - \mathbf{r}_i)$$

- Approximate

$$g(\mathbf{r}_1, \mathbf{r}_2) = \frac{\rho(\mathbf{r}_1, \mathbf{r}_2)}{\rho(\mathbf{r}_1)\rho(\mathbf{r}_2)} \approx g_{NM} \left[|\mathbf{r}_1 - \mathbf{r}_2|, \rho_A \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) \right]$$

- Effects of FSI

  - Z_n reduced by a transparency factor $T_{n\mathbf{p}}$ (from the imaginary part of the NN scattering amplitude)

$$\chi_n(\mathbf{r}) \to \psi_{n\mathbf{p}}(\mathbf{r}) = \Omega^{(-)}_{\mathbf{p}}(\mathbf{r})\chi_n(\mathbf{r})$$

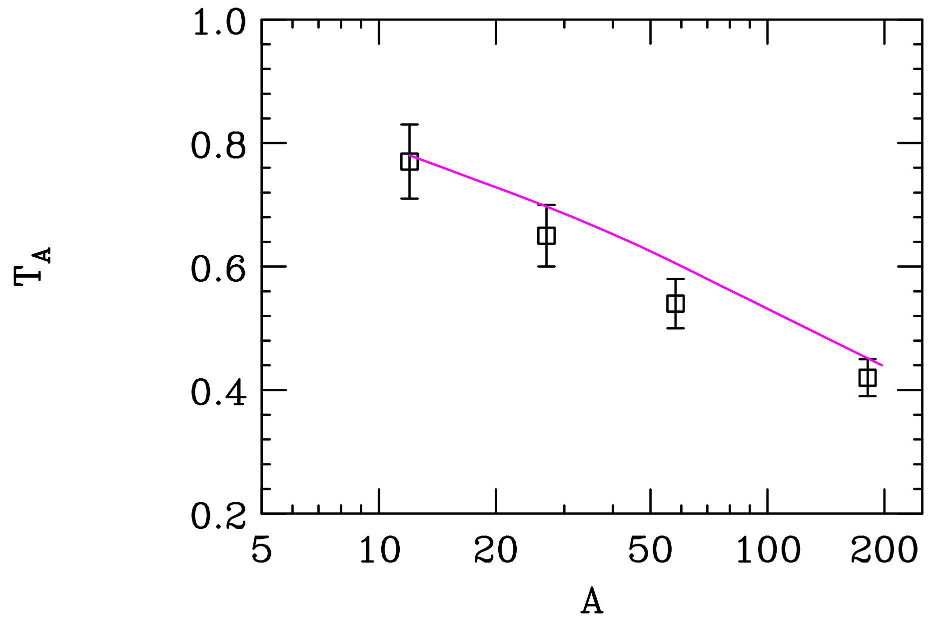
$$T_{n\mathbf{p}} = \frac{1}{Z_n} \int d^3r \, |\psi_{n\mathbf{p}}(\mathbf{r})|^2$$

$$Z_n \to \widetilde{Z}_n = T_{n\mathbf{p}} Z_n$$

  - Momentum distributions $|\psi_{n\mathbf{p}}(\mathbf{k})|^2$ shifted with respect to $|\chi_n(\mathbf{k})|^2$ (from the real part of the NN scattering amplitude)

How low can the proton energy be ?

- Compare theory to the A-dependence of nuclear transparency to a 200 MeV proton, measured at MIT



- WARNING: the calculated transparencies are significantly affected by a complicated pattern of correlation effects

Conclusions

- Spectroscopic factors (SF) are well and uniquely defined properties of interacting many-body systens
- Extraction of SF from $(e, e'p)$ data requires a quantitative understanding of reaction mechanisms beyond the PWIA picture (FSI, two-body currents ...)
- Even within PWIA, the presence of correlation strength extending down to low momentum must be carefully taken into account
- The results of *accurate* calculations based on microscopic many-body approaches provide a satistactory description of the data (Monte Carlo for ^7Li, Green's Function for ^{16}O, CBF for nuclear matter ...)