Spin-orbit effects in knock-out reactions

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- **The toy model**

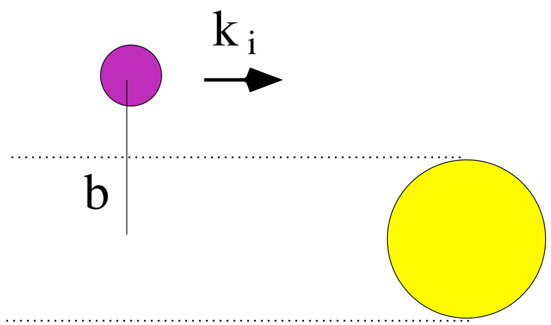
- **The model**

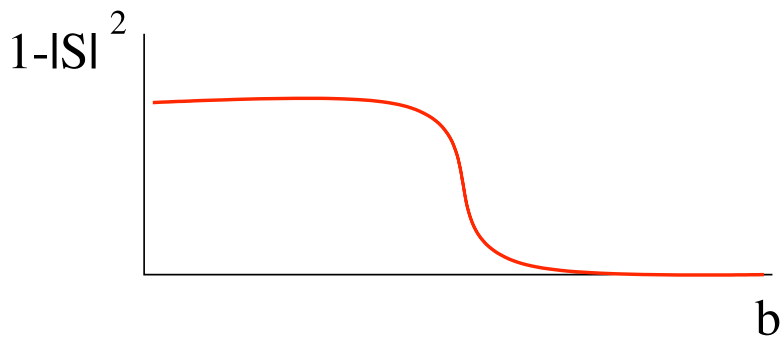
- **Review of approximations related to the sizes of the nuclei**

- **Analysing powers**

- **Sensitivity to the neutron optical potential**

Toy model for tails



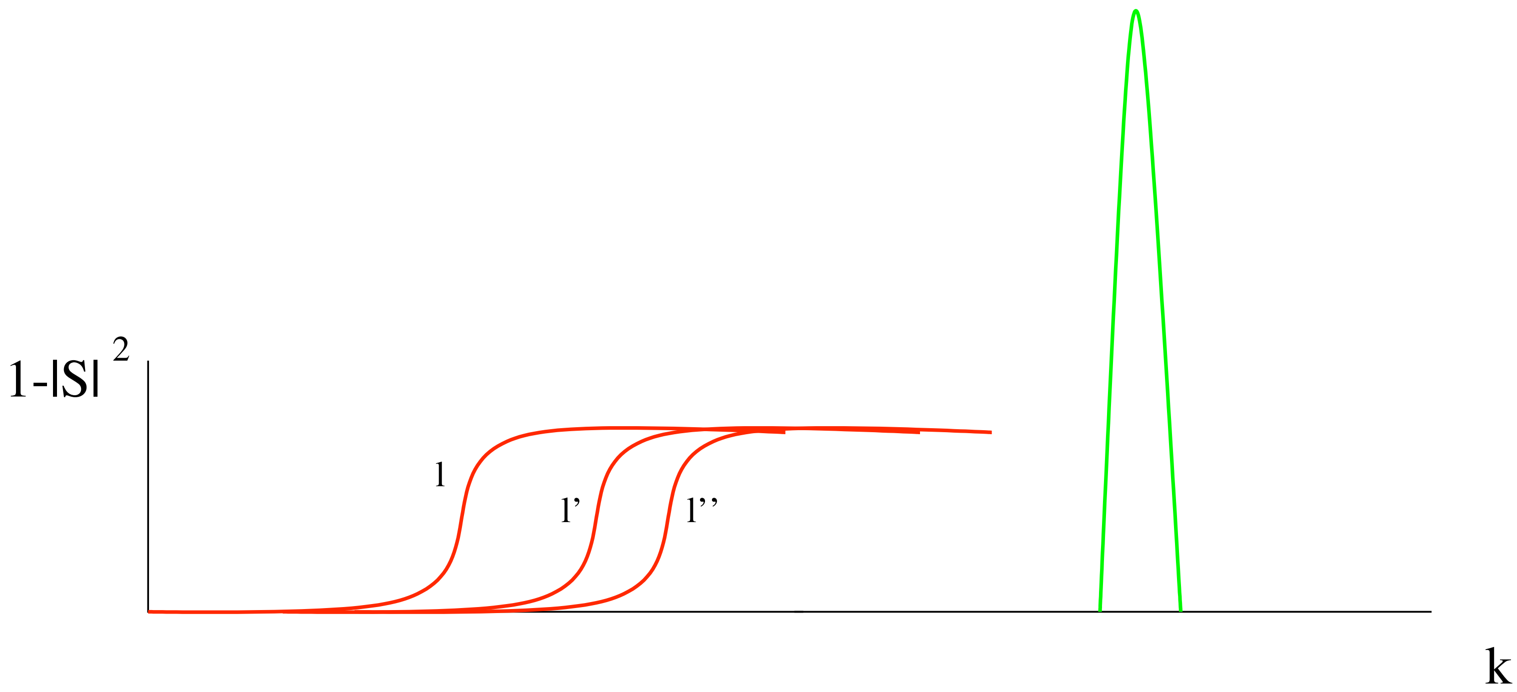


$$S(k,b) \simeq S(b) \quad k_i b = l$$

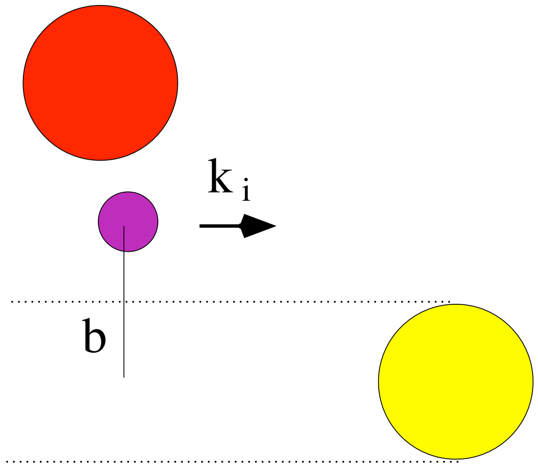
$$\sigma = \frac{\pi}{2}\sum_l (2l+1)\ (1-|S_l(l/k_i)|^2) = \int dk \frac{d\sigma}{dk} =$$

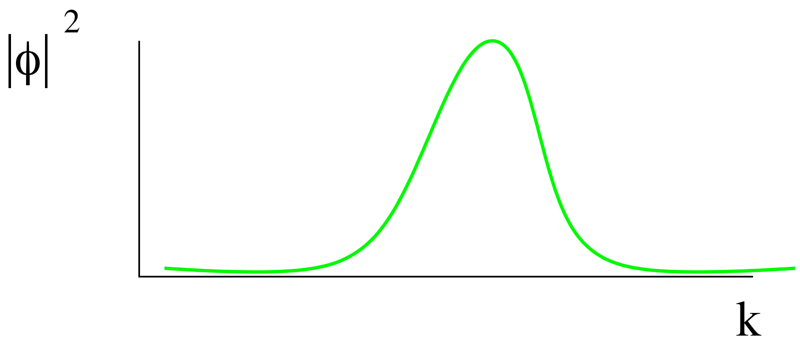
$$= \int dk \frac{\pi}{2}\sum_l (2l+1)\ (1-|S_l(l/k)|^2)\ \delta(k-k_i)$$

Toy model



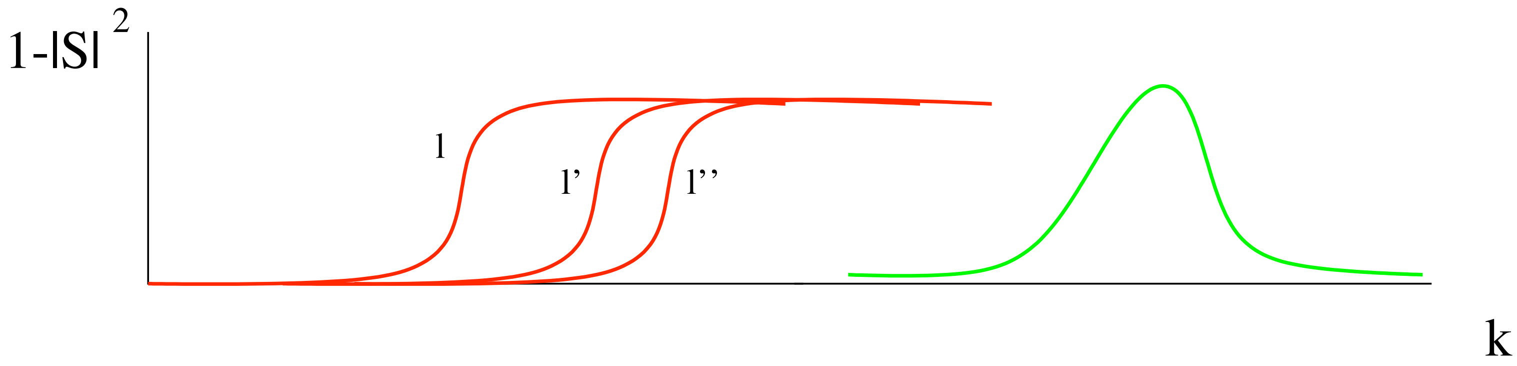
The core deliveries

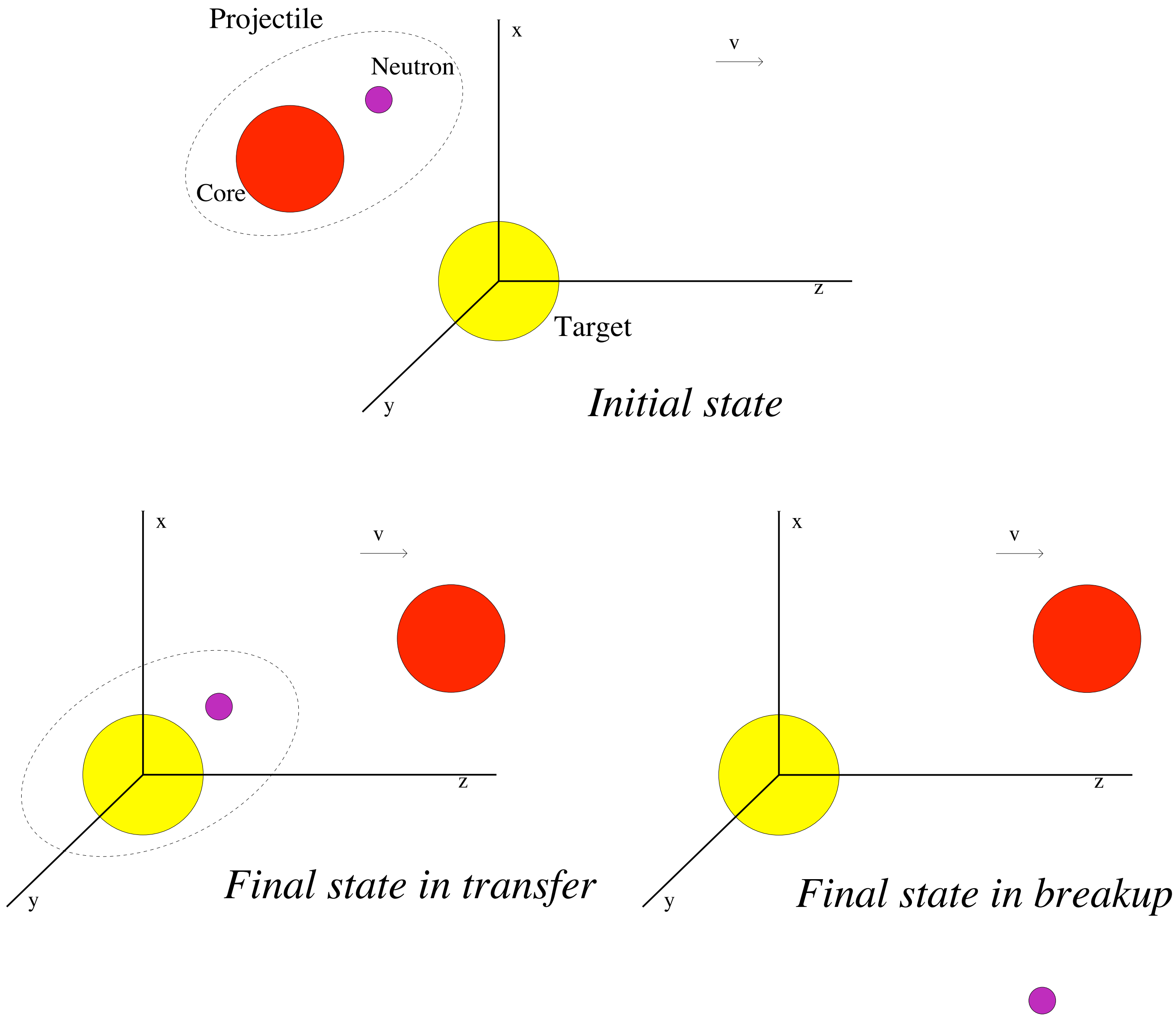




$$\frac{d\sigma}{dk} = \frac{\pi}{2}\sum_l (2l+1)\ (1-|S_l(l/k)|^2)\ |\phi(k)|^2$$

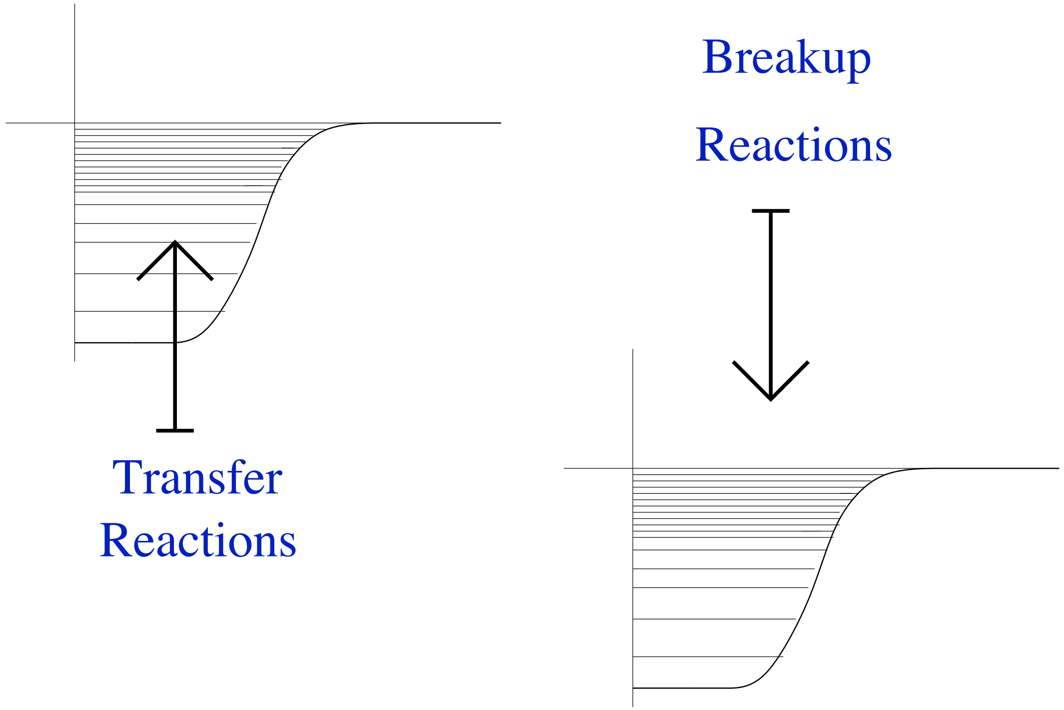
Toy model





The reaction

Transferring to the continuum



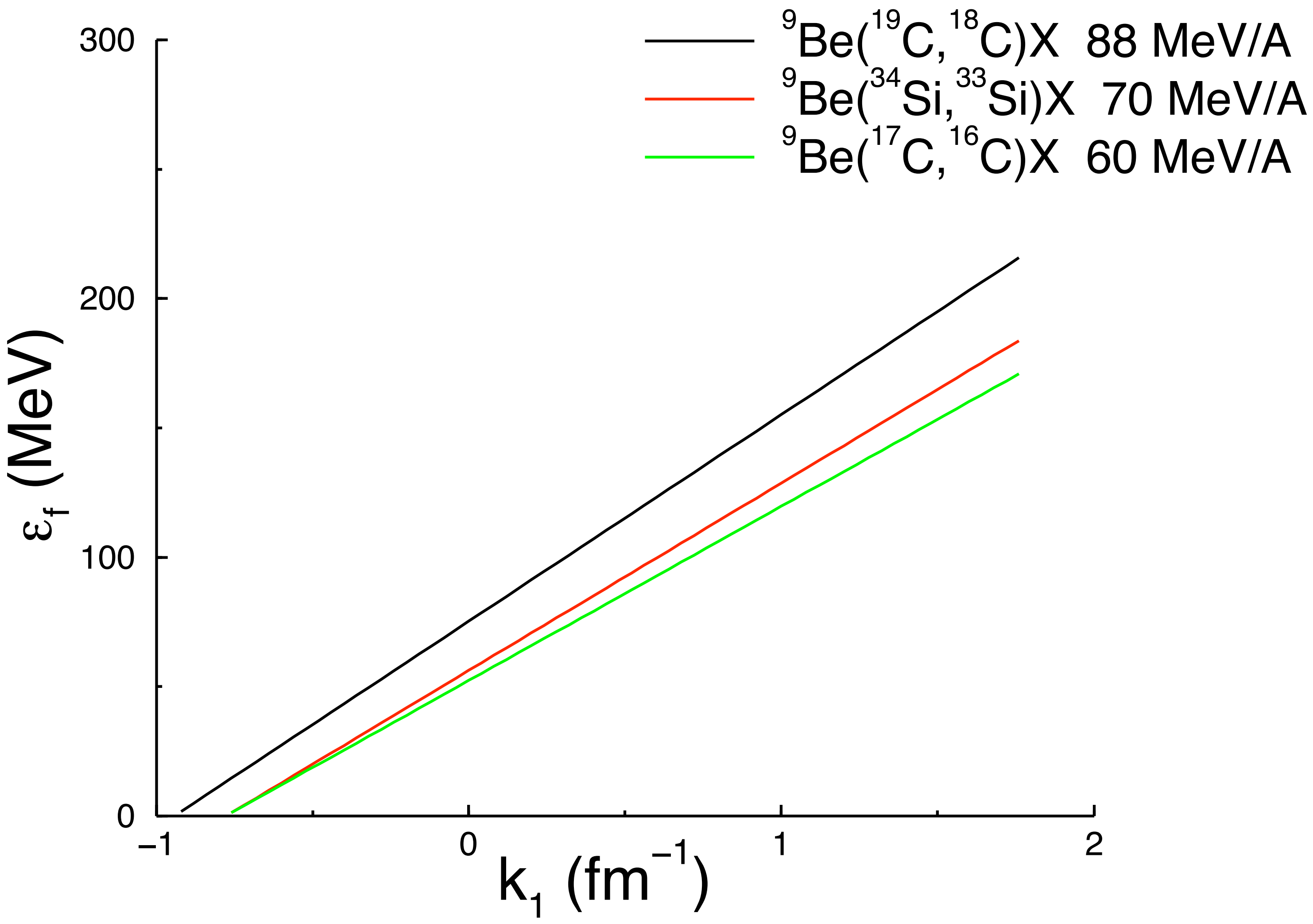
Cross-section

(A. Bonaccorso and D. M. Brink, Phys. Rev. **C38** (1988), 1776, L. Lo Monaco and D. M. Brink, J. of Phys. **G11** (1985), 935...)

$$\frac{d\sigma}{dk_1} = \frac{1}{\hat{l}_1^2}(16\pi)^2\frac{\hbar}{mvk_f}\sum_{l_2}\frac{1-|S_{l_2}|^2+|1-S_{l_2}|^2}{4}\sum_{m_1}|Y_{l_1m_1}(\beta_1,\pi)|^2$$
$$\times \sum_{m_2}|Y_{l_2m_2}(\beta_2,0)|^2\int P_{el}(b)|K_{m_1-m_2}(\eta b)|^2\ b\ db$$

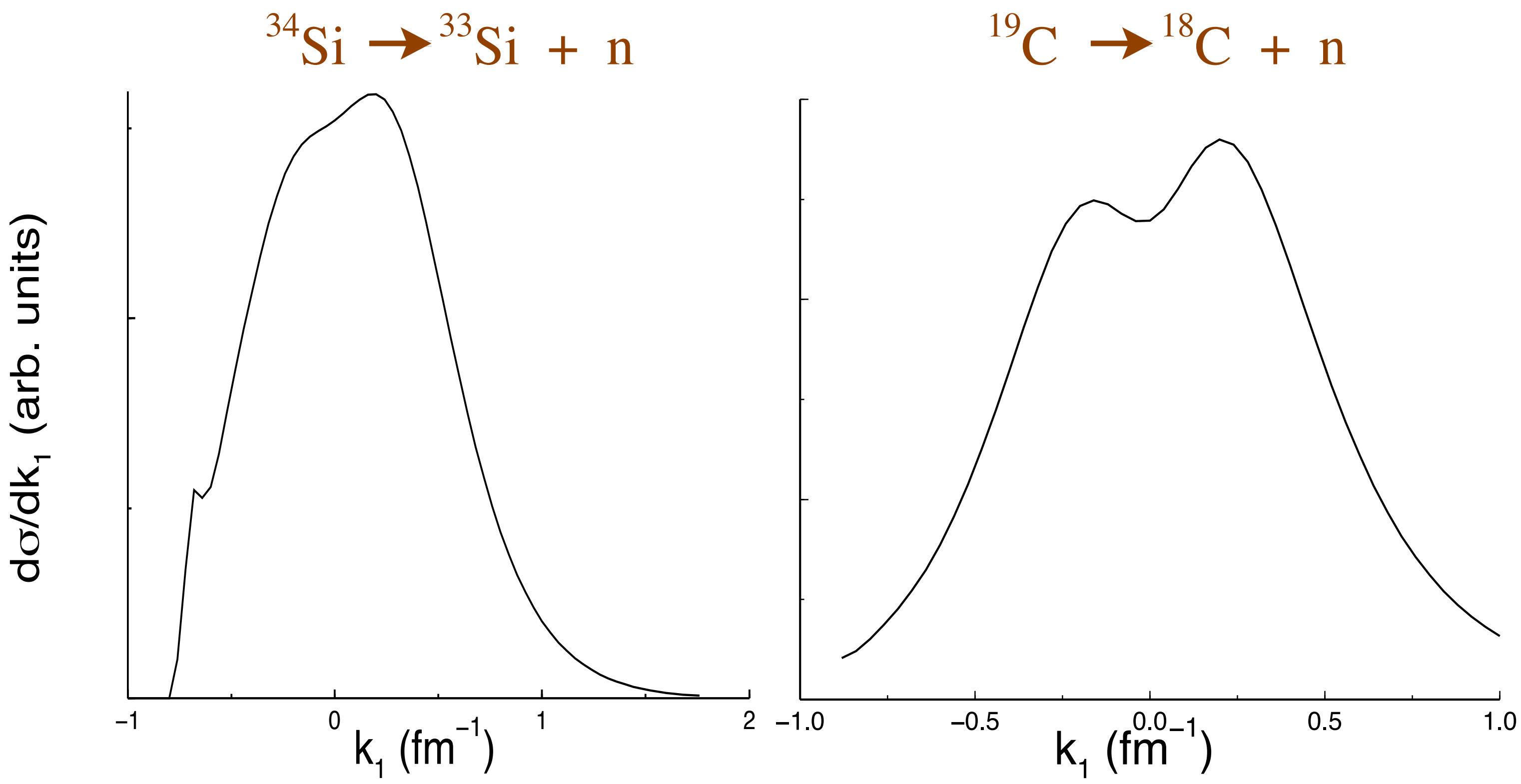
- $\beta_{1,2}$ depend on binding energies, beam velocity and masses,
- neutron-target spin-orbit force has been neglected,
- $\eta^2 = k_1^2 + \gamma_i^2 = k_2^2 - k_f^2$.

Typical energy ranges



Momentum distributions

(A. Bonaccorso, Phys. Rev. **C60** (1999), 054604,J. Enders et *al.*, Phys. Rev. **C65** (2002), 034318)



They are always asymmetric.

Usual approximations

The normal assumption is that ηb is big

$$K_{m_1-m_2}(\eta b) \simeq \sqrt{\frac{\pi}{2\eta b}} e^{-\eta b}(1 + \frac{\mu - 1}{8\eta b} + \frac{(\mu-1)(\mu-9)}{2(8\eta b)^2} + ...)$$

where $\mu = 4(m_1 - m_2)^2$ or

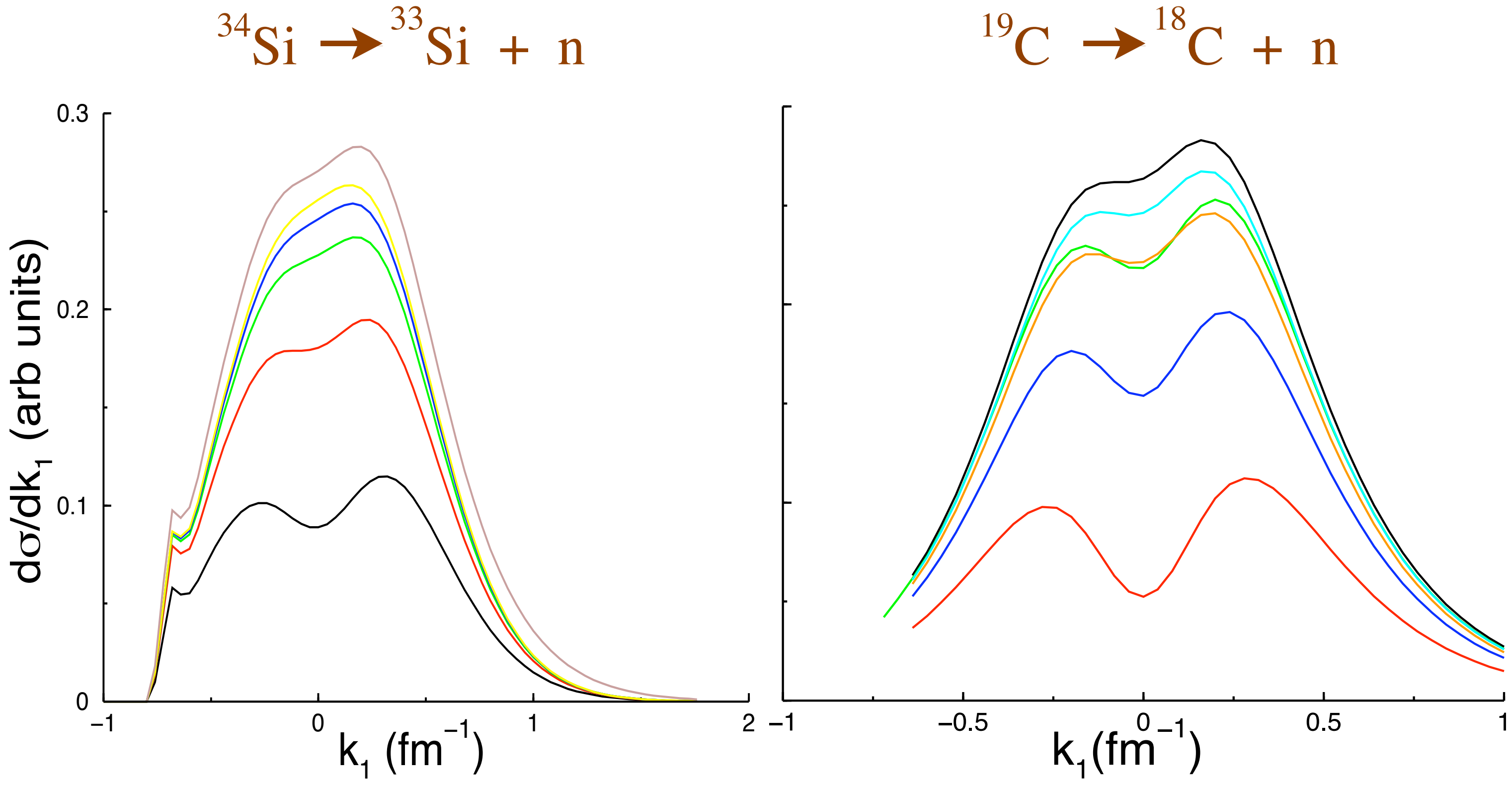
$$\frac{d\sigma}{dk_1} = \int d\mathbf{b} \sum_{l_2} \frac{\hbar}{mv} \frac{1}{k} |C_i|^2 \frac{|1 - S_{l_2}|^2 + 1 - |S_{l_2}|^2}{4} \frac{e^{\eta d}}{\eta d} \hat{l}_2^2 M_{l_1 l_2} P_{el}(b)$$

where

$$M_{l_1 l_2} = \frac{e^{-2\eta b}}{\eta b} 2\sqrt{\pi} \int dX e^{-X^2} P_{l_1}(A_i + B_i X^2) P_{l_2}(A_f + B_f X^2)$$

(F. Stancu and D. M. Brink, Phys. Rev. **C32** (1985), 1937, A. Bonaccorso, G. Piccolo and D. M. Brink, Nucl. Phys. **A441** (1985), 555)

Convergence



Suitability of the approximations depends on radii and binding energy, not on beam energy.

Polarisation

- Some angular momentum states are favored in fragmentation reactions, therefore beams coming from these reactions will be polarised (H. Okuno *et al*, Phys. Lett. **B335** (1994), 29, K. Matsuta *et al*, Phys. Lett. **B281** (1992), 214, K. Asahi *et al*, Phys. Lett. **B251** (1990), 488).

- Such a situation is handled by using *mixed states*, where there is a probability p_i to find the initial nuclei in a state $|\Psi_i>$, and therefore $<O>=\sum p_i <O>_i$.

- The initial state is thus described by a density matrix $\rho = \sum p_i \rho_i$.

Analysing powers

The analysing power is a tensor whose components provide a relationship between the cross-sections for polarized and unpolarized beams.

$$\frac{\sigma_m}{\sigma} = (1 + \sum_{kq} t^m_{kq} T_{kq})$$

They can be related to the probability amplitude

$$R_{n_1 n'_1} = \int \sum A_{n_1} A^*_{n'_1}$$

and since

$$N_{kq} = \sum_{n_1, n'_1} \widehat{k}(j_1 \ n_1 \ k \ q | j_1 \ n'_1) R_{n_1 n'_1}$$

an expression for $T_{kq} = N_{kq}/N_{00}$ has been obtained.
For a spin-1 particles beam

$$\frac{\sigma_{\pm 1}}{\sigma} = (1 + \frac{T_{20}}{\sqrt{2}}) \quad ; \quad \frac{\sigma_0}{\sigma} = (1 - \sqrt{2}\ T_{20})$$

How to get T_{20}

We need a general probability amplitude

$$\widetilde{A}_{\sigma}^{j_1,n_1}(\mathbf{k},\mathbf{K}) = \sum_{j_2 n_2}\sum_{m_1 m_2}\sum_{\sigma'\lambda}\sum_{L l_2}(l_2\ \lambda\ s\ \sigma|j_2\ n_2)(l_1\ m_1\ s\ \sigma'|j_1\ n_1)$$

$$\times\quad (l_2\ m_2\ s\ \sigma'|j_2\ n_2) i^{m_1-m_2}\widehat{L}e^{2i\delta_L}Y_{Lm_1-m_2}(\widehat{\mathbf{K}})K_{m_1-m_2}(\eta b)(1-S_{l_2})$$

$$\times\quad \frac{Y^*_{l_2\lambda}(\widehat{\mathbf{k}})}{k}(-1)^{m_1}Y_{l_1 m_1}(\beta_1,0)Y^*_{l_2 m_2}(\beta_2,0)$$

and then

$$R_{n_1 n_1'} = k^2\int d\mathbf{k}\ d\mathbf{K}\sum_{\sigma}\widetilde{A}_{\sigma}^{j_1,n_1}(\mathbf{k},\mathbf{K})(\widetilde{A}_{\sigma}^{j_1,n_1'}(\mathbf{k},\mathbf{K}))^*$$

Our results for T_{20}

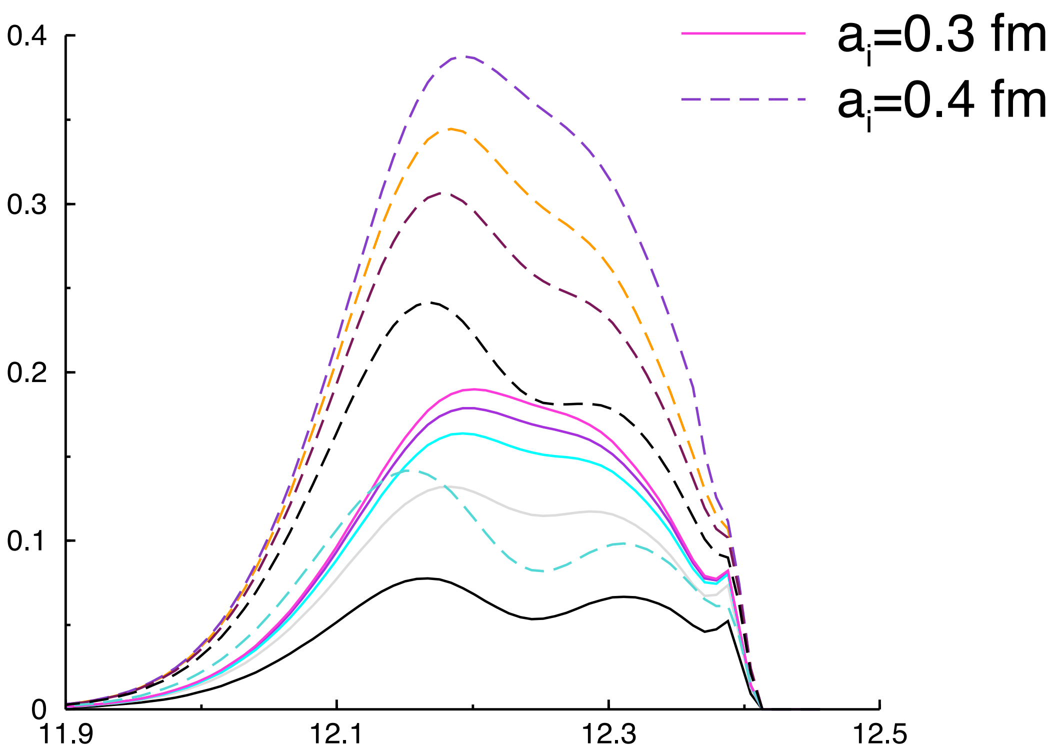
We have calculated some numbers for stripping of ^{17}C at 60 MeV/A, where an eikonal model calculation gives $T_{20} = 0.23$

(R.C. Johnson and J.A. Tostevin, *Analysing power of neutron removal reactions with beams of neutron-rich nuclei*, in: 'Spins in Nuclear and Hadronic Reactions', Proceedings of the RCNP-TMU Symposium (Tokyo, Japan 26 - 28 October 1999), (ed H Yabu, T Suzuki and H Toki, World Scientific (Singapore), October 2000), 155-164)

Approx.	T_{20}
0th order	-0.24
1st order	0.10
2nd order	0.24
3rd order	0.28
M-function	0.18
Bessel function	0.32

Examining the optical potential

(J. H. Dave and C. R. Gould, Phys. Rev. **C28** (1983), 2212)



- We find a strong sensitivity to the diffuseness of the imaginary part.

- The energies are too big for the range of validity of the potential

A new optical potential: JLM

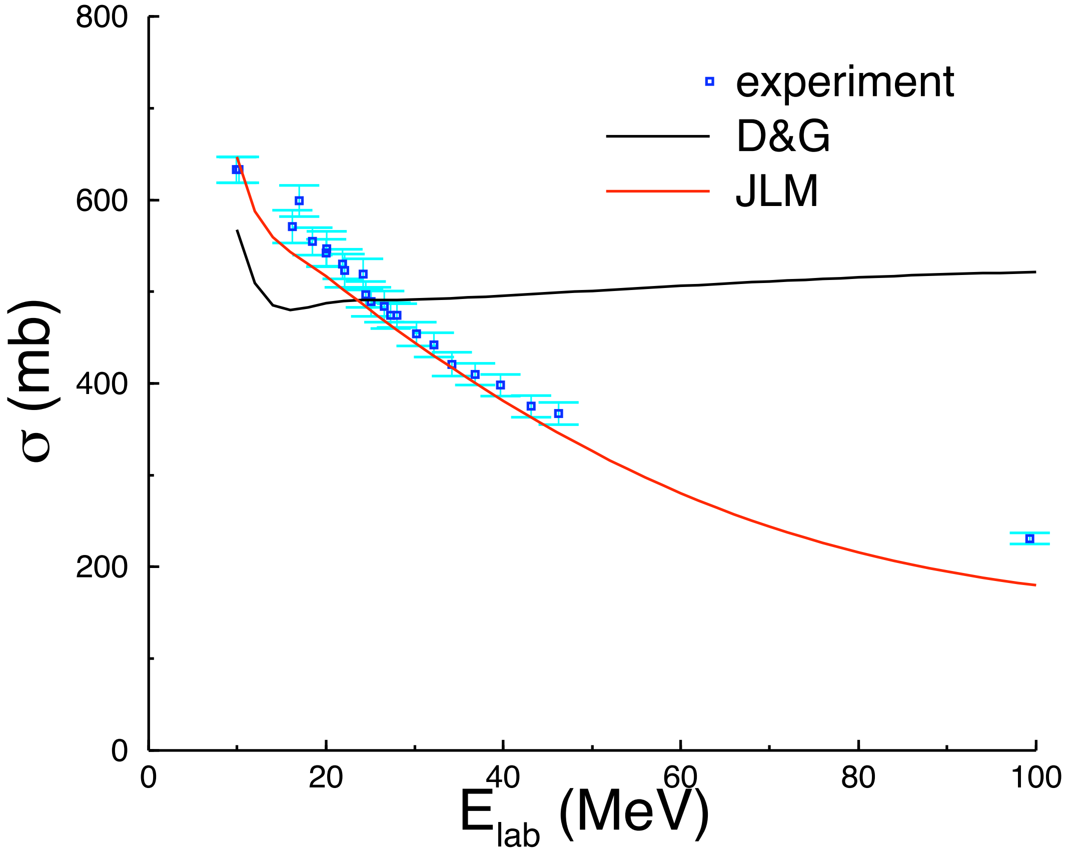
- It uses Reid's hard core nucleon-nucleon interaction

- The interaction is folded with the nuclear matter density

- Its range of validity includes our region of interest
  (Jeukenne et *al.*, Phys. Rev. **C16** (1977), 80, Bauge et *al.* Phys. Rev. **C58** (1998), 1118.)

Checking

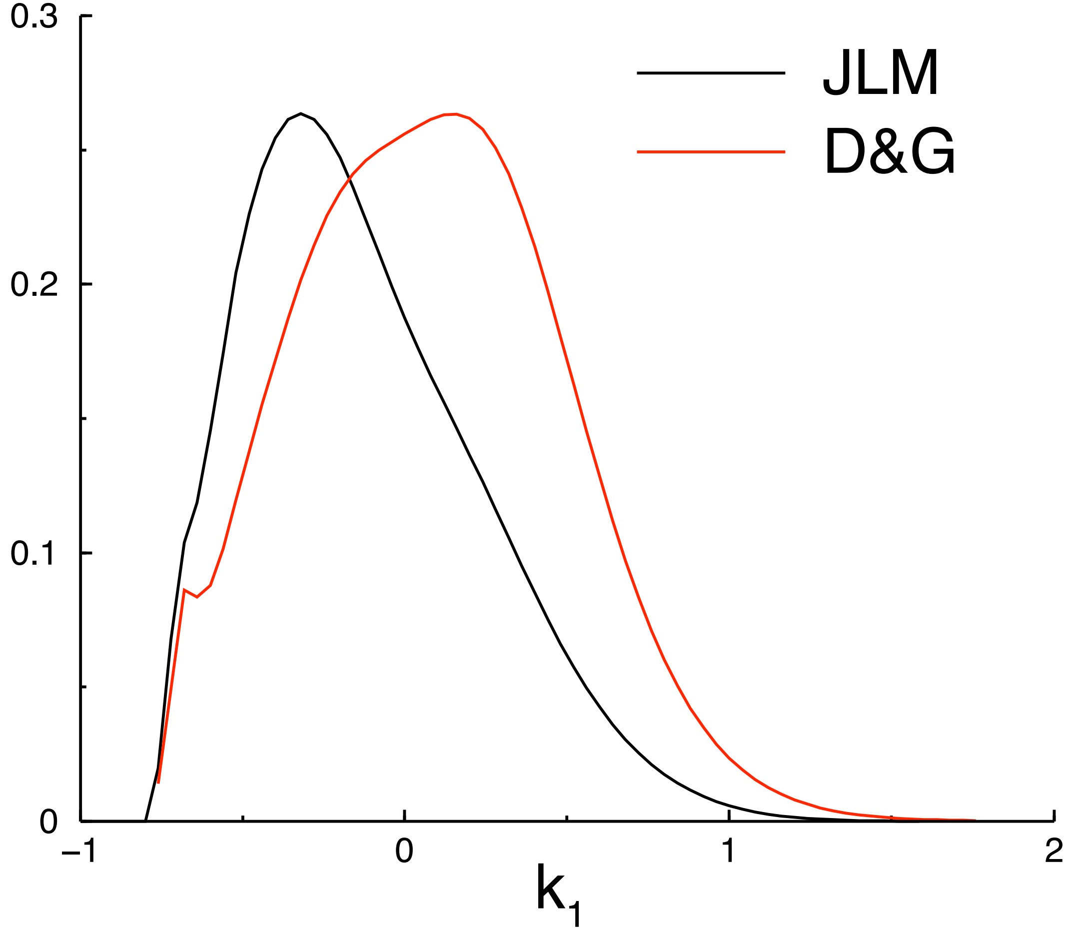
Total reaction cross-section in ^{9}Be(p,p)^{9}Be



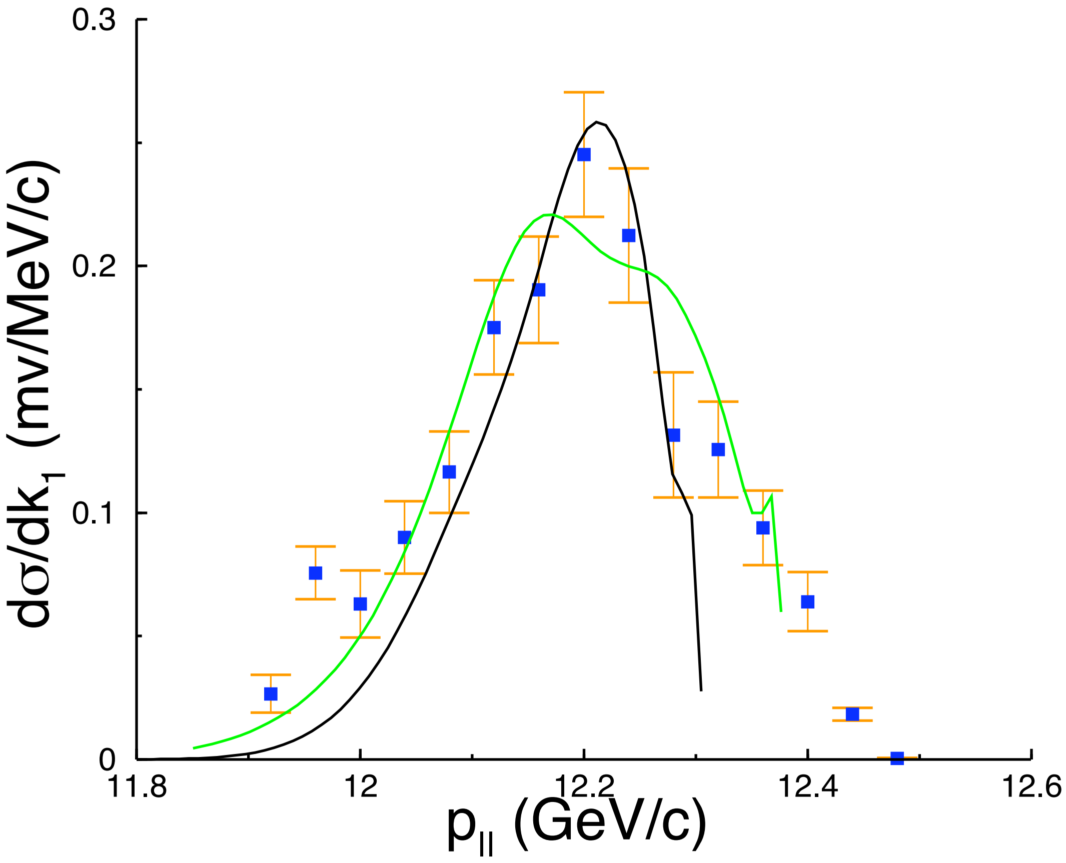
(W. Bauhoff et *al.* At. Dat. and Nuc. Dat. Tab. **35**, 429 (1986).)

A new cross-section

$^{9}Be(^{34}Si,^{33}Si)X$



How much are we leaving out?



It depends on binding energy, beam energy and width of the state

T_{20} with the new potential

Approx.	previous T_{20}	T_{20} with JLM
0th order	-0.24	-0.23
1st order	0.10	0.12
2nd order	0.24	0.25
3rd order	0.28	0.31
M-function	0.18	0.19
Bessel function	0.32	0.38

Including the spin-orbit interaction

After some Racah algebra

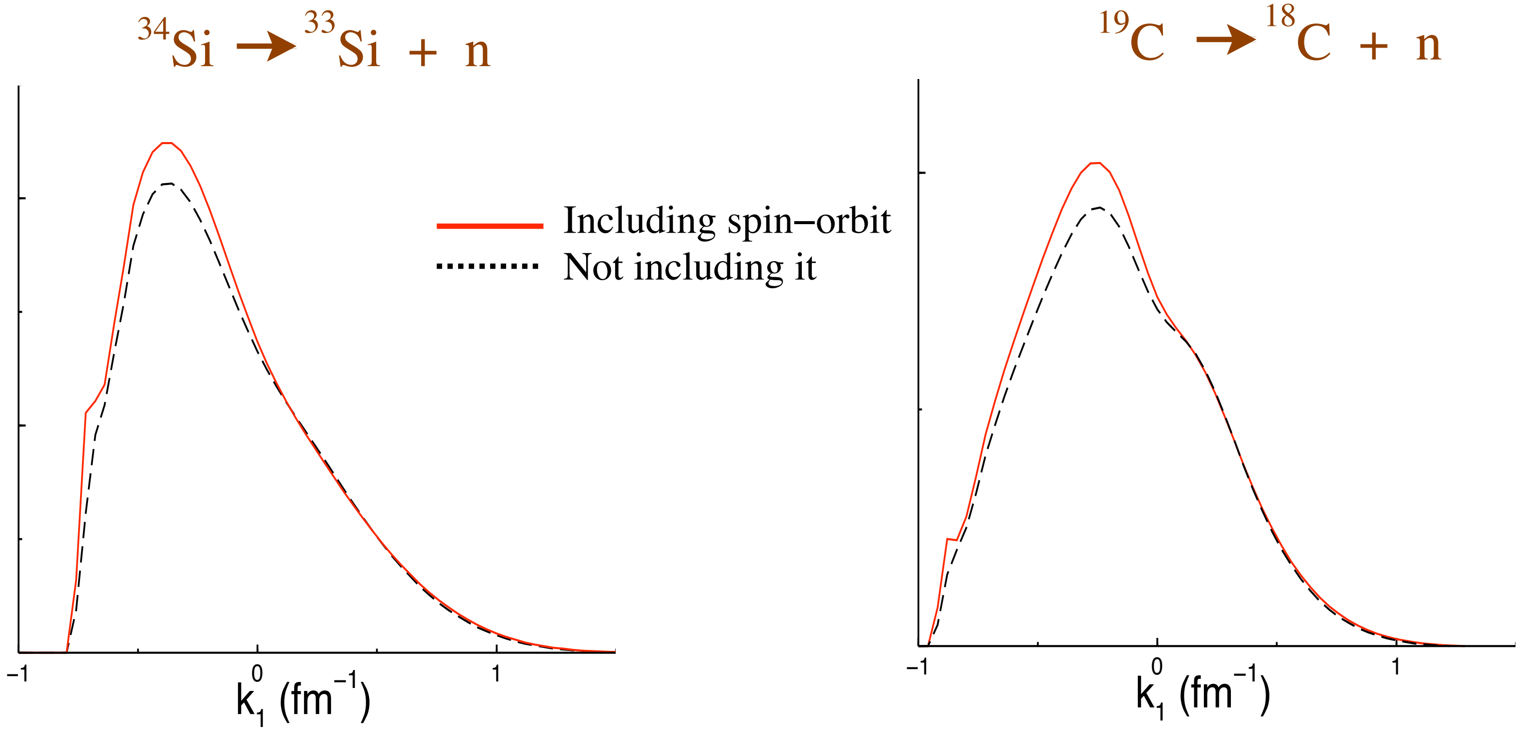
$$\frac{d\sigma}{dk_1} \propto \int db\ bP_{el}(b) \sum_{n_1} \sum_{j_2 n_2} \sum_{m_1 m_2} \sum_{\sigma'} \sum_{l_2} (l_1\ m_1\ s\ \sigma'|j_1\ n_1)(l_2\ m_2\ s\ \sigma'|j_2\ n_2)$$

$$\times \quad |K_{m_1-m_2}(\eta b)|^2 |1 - S_{l_2 j_2}|^2 (-1)^{m_1} Y_{l_1 m_1}(\beta_1, 0) Y^*_{l_2 m_2}(\beta_2, 0) \sum_{m'_2} \sum_{\sigma''} \sum_{m'_1}$$

$$\times \quad (l_1\ m'_1\ s\ \sigma''|j_1\ n_1)(l_2\ m'_2\ s\ \sigma''|j_2\ n_2)(-1)^{m'_1} Y_{l_1 m'_1}(\beta_1, 0) Y^*_{l_2 m'_2}(\beta_2, 0)$$

(H. Hashim and D. M. Brink, J. Phys. **G11** (1988), 107)

Spin-orbit effects



In the T_{20} calculation we get now 0.37 rather than 0.38

Conclusions

- The validity of assumptions over radii has been examined, and they have been found to work better for cross-sections than for analysing powers,

- a more realistic optical potential has been used, whose effect seems to push the cross-section towards the region of momentum that is forbidden by the model,

- the effect of the spin-orbit force appears to be negligible in the calculation of both total cross-section and analysing powers.

Acknowledgments