Quasideuteron distribution in nuclei

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- We compute the distribution of quasideuterons (**QD**) in doubly closed shell nuclei and infinite nuclear matter.

- The ground states of ^{16}O and ^{40}Ca are described in ls coupling using a realistic hamiltonian including the Argonne v_8' and the Urbana IX models of two– and three–nucleon potentials, respectively.

- The nuclear wave function contains central and tensor correlations, and correlated basis functions theory is used to evaluate the distribution of neutron-proton pairs, which have the deuteron quantum numbers, as a function of their total momentum.

- By computing the number of deuteron–like pairs we are able to extract the Levinger's factor and compare to both the available experimental data and the predictions of the local density approximation, based on nuclear matter estimates.

- The agreement with the experiments is **EXCELLENT**, whereas the local density approximation is shown to sizably overestimate the Levinger's factor in the region of the medium nuclei.

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Formalism

In a A–nucleon system the distribution of **QD** pairs, whose center of mass is in the orbital state specified by the quantum number X, can be written

$$P_D(X) = \frac{1}{2J_D+1} \langle A | (a_D^\alpha)^\dagger (X) a_D^\alpha (X) | A \rangle \ ,$$

where $|A\rangle$ denotes the A–body ground state and $J_D = 1$ is the spin of the deuteron. The operator $a(a^\dagger)_D^\alpha(X)$ annihililates (creates) a deuteron with the quantum number X in the $\alpha = 1,2,3$ Cartesian state.

In configuration space the above expression takes the form

$$P_D(X) = \frac{1}{2J_D+1} \frac{1}{2} \int d^3r_1 d^3r_2 d^3r_{1'} d^3r_{2'} \Psi_{D,cm}(X; \mathbf{R}_{12}) \times \rho_D^{(2)}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_{1'}, \mathbf{r}_{2'}) \, \Psi^*_{D,cm}(X; \mathbf{R}_{1'2'}) \ ,$$

where $\rho_D^{(2)}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_{1'}, \mathbf{r}_{2'})$ is a *generalized two–body density matrix* defined by

$$\rho_D^{(2)}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_{1'}, \mathbf{r}_{2'}) = A(A-1) \int d\widetilde{R} \, \Psi_A^*(\mathbf{r}_1, \mathbf{r}_2, \widetilde{R}) \psi_{D,rel}^\alpha(12) \times |00\rangle\langle 00| \left(\psi_{D,rel}^\alpha(1'2')\right)^* \Psi_A(\mathbf{r}_{1'}, \mathbf{r}_{2'}, \widetilde{R}) \ ,$$

where $|00\rangle$ is the spin–isospin singlet NN state and summation over the repeated indices is understood.

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A realistic A–body wave function, accounting for both short– and intermediate–range correlations induced by the strong nuclear interaction, is given in CBF theory by

$$\Psi_A(R) = \mathcal{S}\Big[\prod_{i<j} F(ij)\Big]\Phi_0(R)\,,$$

where $R \equiv (\mathbf{r}_1, \ldots, \mathbf{r}_A)$, $\mathcal{S}$ is a symmetrization operator and Φ_0 is the Slater determinant of single particle orbitals $\phi_\alpha(i)$, which are eigenfunctions of a suitable single particle hamiltonian. For nuclear matter, the orbitals $\phi_\alpha(i)$ are plane waves corresponding to a noninteracting Fermi gas of nucleons with momenta $|\mathbf{k}| \le k_F = (6\pi^2\rho_{NM}/\nu)^{1/3}$. $\nu = 4$ and ρ_{NM} are the NM spin–isospin degeneracy and density, respectively.

Pandharipande, Wiringa (1979)

The two–body correlation operator, $F(ij)$, is given by the sum of 6 central and non–central spin–isospin dependent components,

$$\begin{aligned} F(ij) = {} & f_c(r_{ij}) + f_\sigma(r_{ij})(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) + f_\tau(r_{ij})(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \\ & + f_{\sigma\tau}(r_{ij})(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) + f_t(r_{ij}) T_{\alpha\beta}(\widehat{\mathbf{r}}_{ij})\sigma_i^\alpha \sigma_j^\beta \\ & + f_{t\tau}(r_{ij}) T_{\alpha\beta}(\widehat{\mathbf{r}}_{ij})\sigma_i^\alpha \sigma_j^\beta (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)\ , \end{aligned}$$

where the $f_p(r)$ correlation functions are variationally fixed by minimizing the ground state energy. All the correlation functions heal to zero, except $f_c(r \to \infty) \to 1$, while the tensor operator reads

$$T^{\alpha\beta}(\widehat{\mathbf{r}}_{ij}) = 3\widehat{\mathbf{r}}_{ij}^\alpha \widehat{\mathbf{r}}_{ij}^\beta - \delta^{\alpha\beta}\ .$$

A. Fabrocini, A. de Saavedra, and G. Co' (2000)

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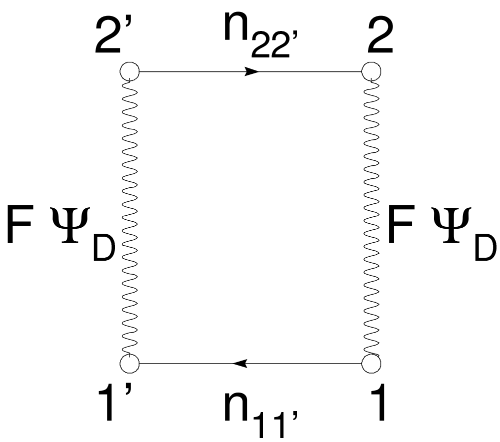
The dressed leading order approximation



$$\rho_D^{(2)}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_{1'}, \mathbf{r}_{2'}) \approx \frac{2J_D+1}{4\pi} \rho^{(1)}(\mathbf{r}_1, \mathbf{r}_{1'}) \Sigma(\mathbf{r}_{12}, \mathbf{r}_{1'2'}) \rho^{(1)}(\mathbf{r}_2, \mathbf{r}_{2'}),$$

where $\rho^{(1)}(\mathbf{r}_1, \mathbf{r}_{1'})$ is the one–body density matrix and

$$\Sigma(\mathbf{r}, \mathbf{r}') = \frac{1}{16} \left[U(r)U(r') + W(r)W(r')Q(\widehat{\mathbf{r}} \cdot \widehat{\mathbf{r}}') \right] ,$$

with $Q(x) = (3x^2 - 1)/2$ and

$$U(r) = u_D(r) - \Delta u(r) ;$$
$$W(r) = w_D(r) - \Delta w(r) .$$

The $\Delta u(r)$ and $\Delta w(r)$ functions account for the medium correlations effect on the bare components of the DWF:

$$\begin{aligned} \Delta u(r) &= u_D(r) \left[h_c(r) - f_\sigma(r) + 3f_\tau(r) + 3f_{\sigma\tau}(r) \right] \\ &- \ 2\sqrt{2} w_D(r) \left[f_t(r) - 3f_{t\tau}(r) \right] \ ; \\ \Delta w(r) &= w_D(r) \left[h_c(r) - f_\sigma(r) + 3f_\tau(r) + 3f_{\sigma\tau}(r) \right] \\ &- \ 2\sqrt{2} \left(u_D(r) - \frac{w_D(r)}{\sqrt{2}} \right) \left[f_t(r) - 3f_{t\tau}(r) \right] . \end{aligned}$$

In order to evaluate $\mathcal{P}_D(\mathbf{k}_D)$ in spherically symmetric nuclei, the one–body density matrix can be expressed in terms of the *natural orbits* (NO):

$$\rho^{(1)}(\mathbf{r}_1, \mathbf{r}_{1'}) = \nu \sum_l \frac{2l+1}{4\pi} P_l(\widehat{\mathbf{r}} \cdot \widehat{\mathbf{r}}') \sum_n n_{nl}\, \phi_{nl}^{NO}(r_1)\, \phi_{nl}^{NO}(r_{1'}).$$

A. Fabrocini, G. Co' (2001)

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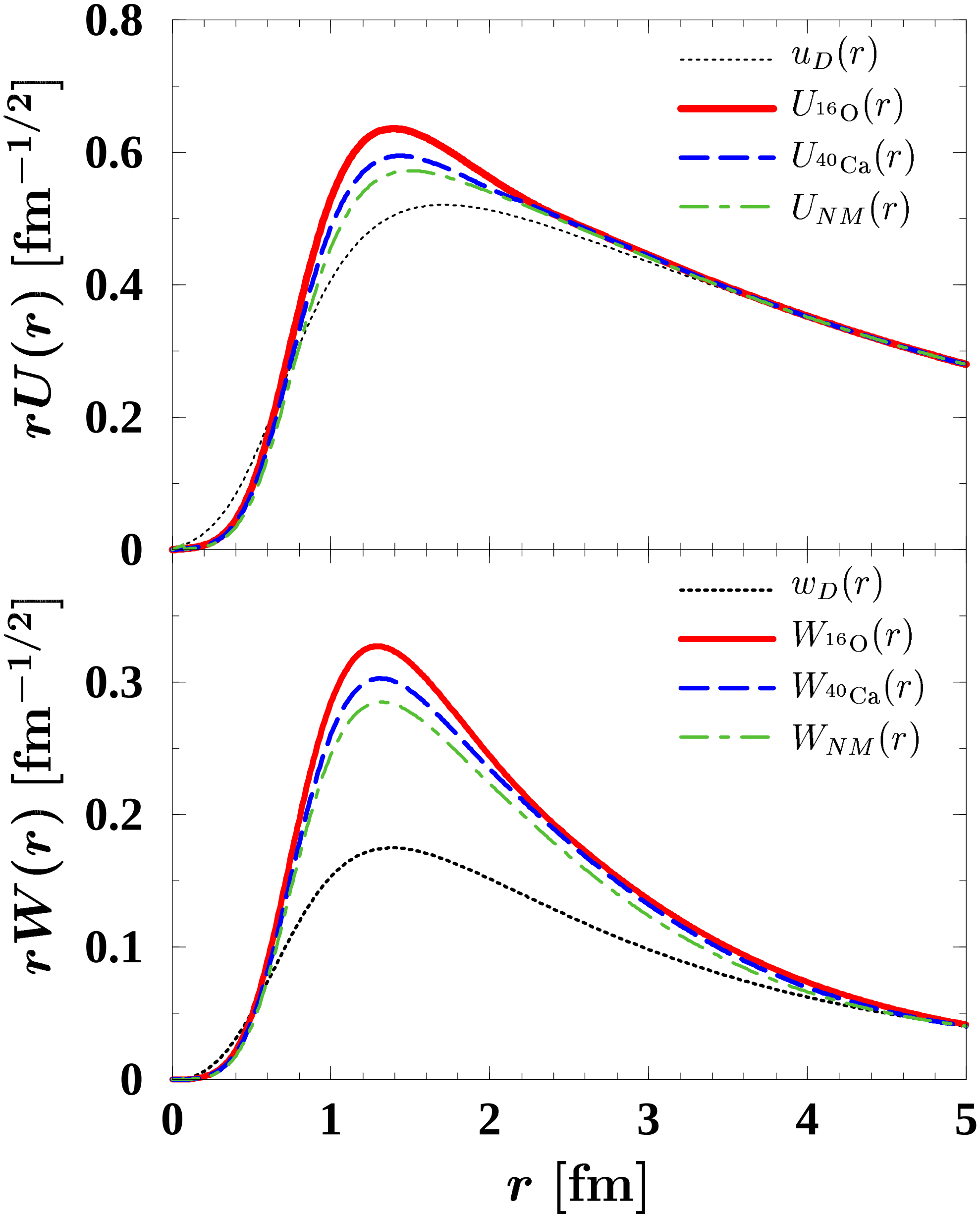
Figure 1: Radial components $U(r)$ and $W(r)$ of the AU8′ QD wave functions in ^{16}O, ^{40}Ca and nuclear matter. **Upper panel:** the solid and dashed lines show the radial dependence of $U_A(r)$ for ^{16}O and ^{40}Ca respectively. The dot-dashed and dotted lines correspond to the nuclear matter $U_{NM}(r)$ and the bare $u_D(r)$. **Lower panel:** as in the upper panel for the D-wave components of the QD and deuteron wave functions.



At small relative distances both $U(r)$ ($r \lesssim 1$ fm) and $W(r)$ ($r \lesssim 0.5$ fm) are slightly suppressed with respect to $u_D(r)$ and $w_D(r)$. On the other hand, they are appreciably enhanced at larger distances. These effects are more visible for the lightest nucleus.

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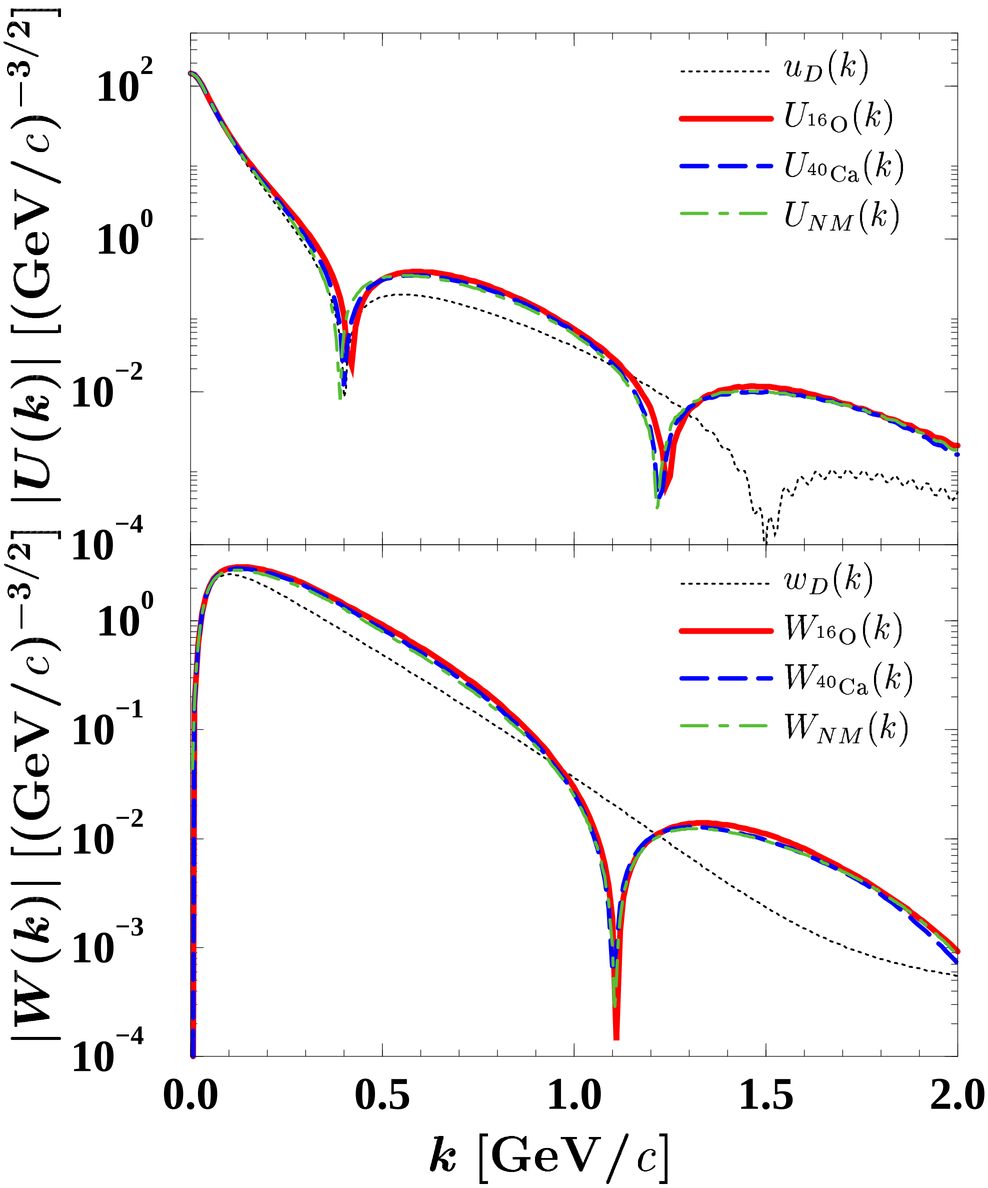
Figure 2: Radial components $U(k)$ and $W(k)$ in momentum space of the AU8′ QD wave functions in ^{16}O, ^{40}Ca and nuclear matter. **Upper panel:** the solid and dashed lines show the radial dependence of $U_A(r)$ for ^{16}O and ^{40}Ca respectively. The dot-dashed and dotted lines correspond to the nuclear matter $U_{NM}(r)$ and the bare $u_D(r)$. **Lower panel:** as in the upper panel for the D-wave components of the QD and deuteron wave functions.



The nuclear medium shifts the second minimum of $|u_D(k)|$ towards lower values of k. The Argonne v_8' $|w_D(k)|$ does not exhibit any diffraction minimum, which, however, appears in $|W(k)|$.

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QD distribution in a nuclei

$$\mathcal{P}_D(\mathbf{k}_D) = \sum_{\alpha,\alpha'} n_\alpha n_{\alpha'} \mathcal{P}_{\mathcal{D}}^{\alpha,\alpha'}(\mathbf{k}_D)\,,$$

with

$$\mathcal{P}_{\mathcal{D}}^{\alpha,\alpha'}(\mathbf{k}_D) = \frac{\nu^2}{16}\frac{(2\pi)^3}{4\pi}\left[|\Psi_S^{\alpha,\alpha'}(\mathbf{k}_D)|^2 + \sum_{s=-2}^{2}|\Psi_D^{\alpha,\alpha';\,s}(\mathbf{k}_D)|^2\right]\,,$$

where $\alpha = (nlm)$,

$$\Psi_S^{\alpha,\alpha'}(\mathbf{k}_D) = \int d^3k\,\phi_\alpha^{NO\dagger}\left(\frac{\mathbf{k}_D}{2}+\mathbf{k}\right)U(k)\,\phi_{\alpha'}^{NO}\left(\frac{\mathbf{k}_D}{2}-\mathbf{k}\right)\,,$$

$$\Psi_D^{\alpha,\alpha';\,s}(\mathbf{k}_D) = \sqrt{\frac{4\pi}{5}}\int d^3k\,\phi_\alpha^{NO\dagger}\left(\frac{\mathbf{k}_D}{2}+\mathbf{k}\right)W(k)\,\phi_{\alpha'}^{NO}\left(\frac{\mathbf{k}_D}{2}-\mathbf{k}\right)Y_{2s}(\widehat{\mathbf{k}})\,,$$

and

$$\phi_{nlm}^{NO}(\mathbf{q}) = \phi_{nl}^{NO}(q)\,Y_{lm}(\widehat{\mathbf{q}})\,,$$

$Y_{lm}(\widehat{\mathbf{q}})$ are the spherical harmonics.

In the independent particle model (IPM), $\Psi_A(R) \equiv \Phi_0(R)$, and $n_{nl}^{IPM} = 1$, $\phi_{nl}^{NO} \equiv \phi_{nl}$ for occupied states, whereas $n_{nl}^{IPM} = 0$ for unoccupied states. Deviations from IPM provide a measure of correlation effects, as they allow higher NO to become populated with $n_{nl} \neq 0$.

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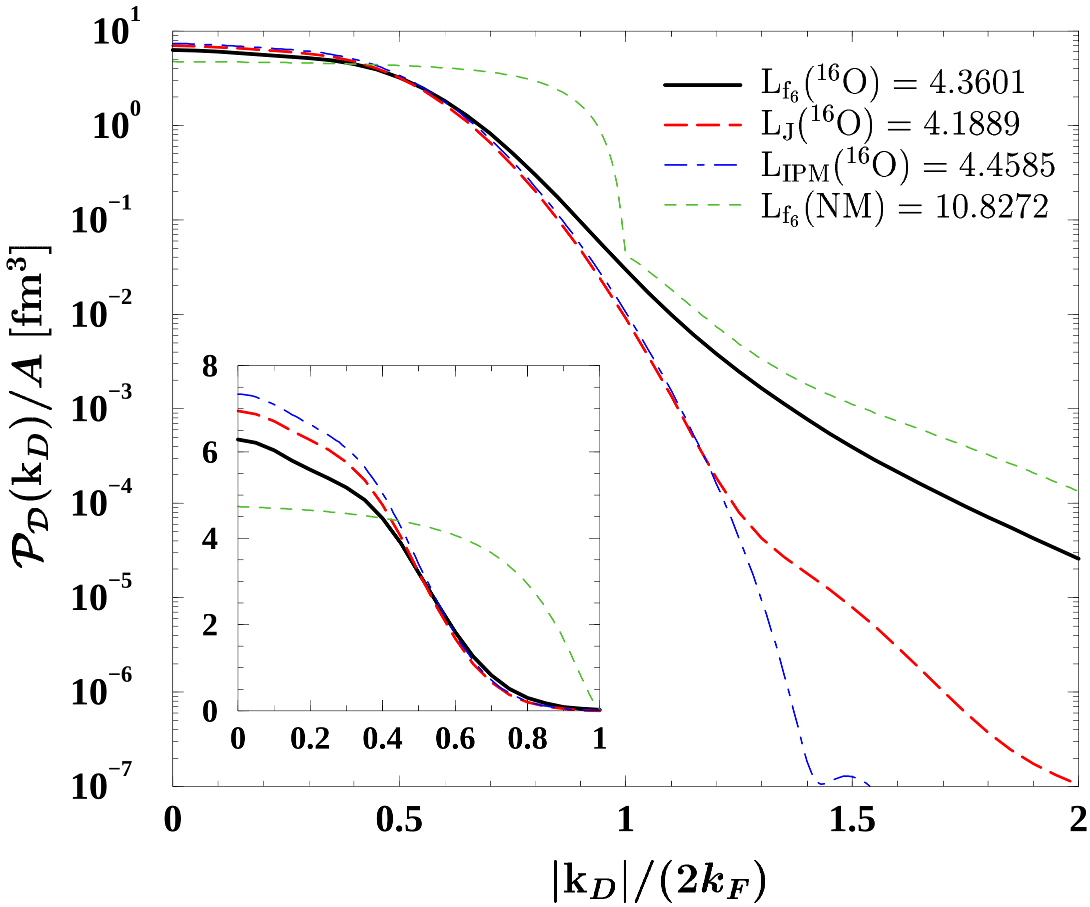
Figure 3: **Momentum distribution of QD pairs in ^{16}O** as a function of the total momentum $|\mathbf{k}_D|$. The solid, dashed and dash–dotted lines are the results obtained within the f_6 and Jastrow correlation models and IPM respectively. The short–dashed line displays the f_6 momentum distribution of the QD in nuclear matter at equilibrium density, $\rho_{NM} = 0.16$ fm^{-3}. The insert shows a blow up of the region $|\mathbf{k}_D|/(2k_F) < 1$, plotted in linear scale. The Levinger factors, L(A), for the various calculations are also reported.



- NN correlations introduce high momentum components in the distribution. The full $\mathcal{P}_D(\mathbf{k}_D)$ is strongly enhanced with respect to $\mathcal{P}_D^{IPM}(\mathbf{k}_D)$ at large $|\mathbf{k}_D|$, and it is correspondingly depleted at small $|\mathbf{k}_D|$. The depletion is mostly due to the non–central tensor correlations.

- The effect of state–dependent correlations is large, as one can see by comparing the full $\mathcal{P}_D(\mathbf{k}_D)$ with the Jastrow model $\mathcal{P}_D^J(\mathbf{k}_D)$ (obtained by retaining only the scalar component in the two–body correlation operator).

- The tail of $\mathcal{P}_D(\mathbf{k}_D)$ is appreciably different from that of nuclear matter. At $|\mathbf{k}_D| = 4k_F$ the difference is still a factor ~ 10 for both ^{16}O and ^{40}Ca.

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Figure 4: **Momentum distribution of QD pairs in ^{40}Ca** as a function of the total momentum $|\mathbf{k}_D|$. The solid, dashed and dash–dotted lines are the results obtained within the f_6 and Jastrow correlation models and IPM respectively. The short–dashed line displays the f_6 momentum distribution of the QD in nuclear matter at equilibrium density, $\rho_{NM} = 0.16$ fm^{-3}. The insert shows a blow up of the region $|\mathbf{k}_D|/(2k_F) < 1$, plotted in linear scale. The Levinger factors, L(A), for the various calculations are also reported.

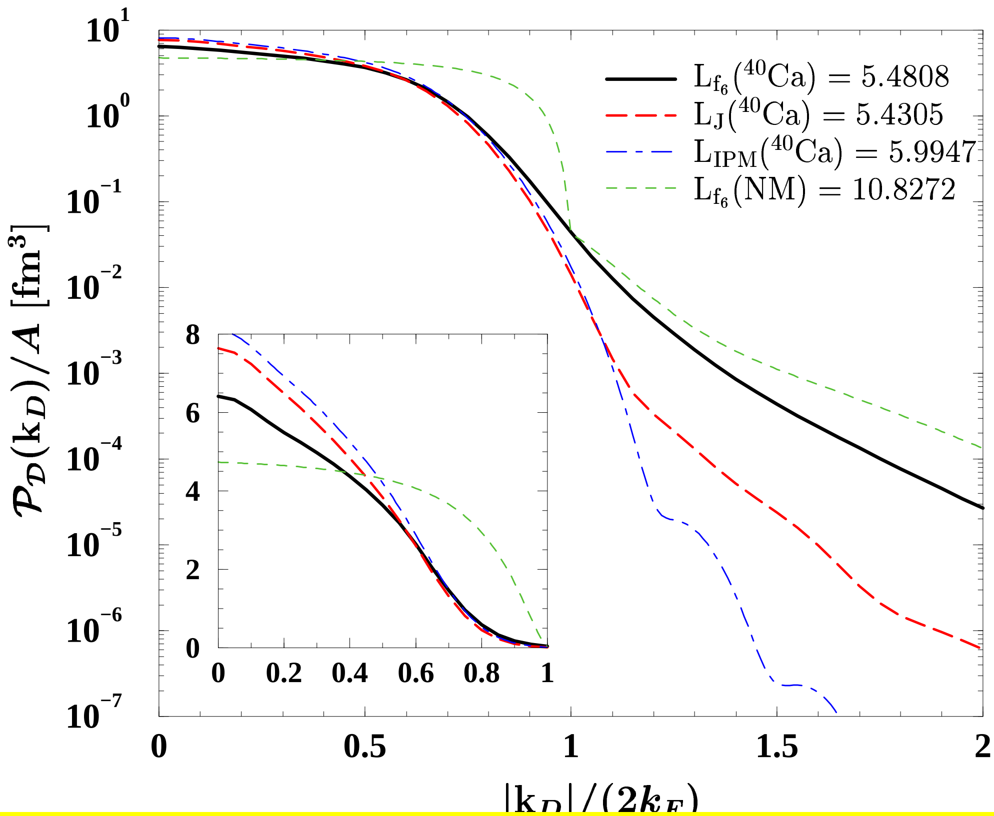
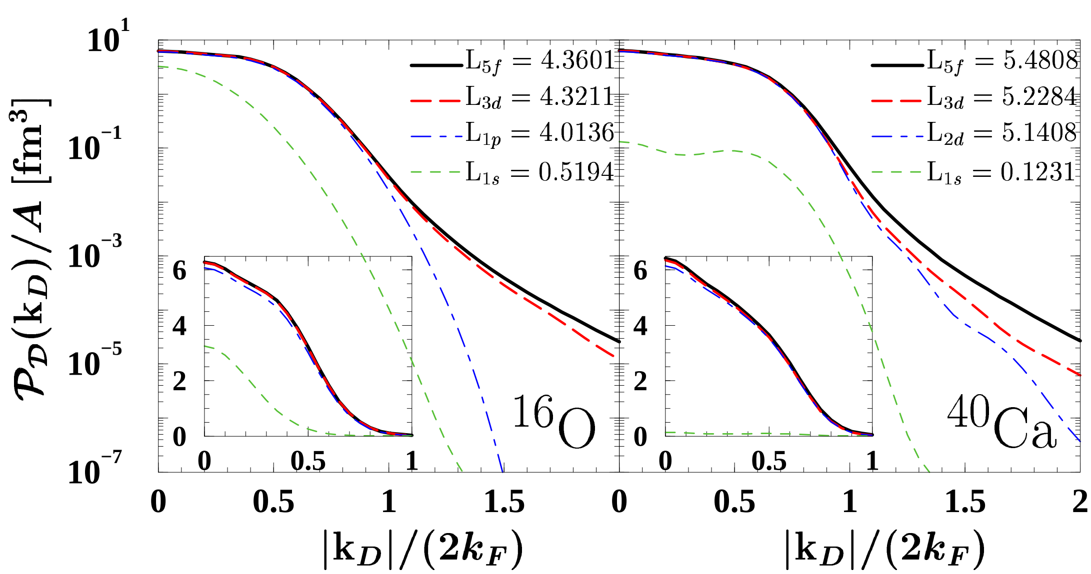


Figure 5: **Convergence** of $\mathcal{P}_D(\mathbf{k}_D)/A$ for ^{16}O and ^{40}Ca in the number of natural orbits. The results have been obtained within the f_6 correlation model.



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Levinger's factor

Within the Levinger's **QD** model (1951) the nuclear photoabsorption cross section $\sigma_A(E_\gamma)$, above the giant dipole resonance and below the pion threshold, is

$$\sigma_A(E_\gamma) = \mathcal{P}_D \; \sigma_{QD}(E_\gamma) \; ,$$

where E_γ is the photon energy and $\mathcal{P}_D$ is interpreted as the effective number of the NN pairs of the QD type

$$\mathcal{P}_D = \mathrm{L}(A) \left[\frac{Z(A-Z)}{A} \right] \; ,$$

where $\mathrm{L}(A)$ is the so called Levinger's factor.

$$\mathrm{L}(A) = 4\frac{\mathcal{P}_D}{A} = 4(2J_D+1)\int \frac{d^3k_D}{(2\pi)^3}\,\frac{\mathcal{P}_D(\mathbf{k}_D)}{A}$$

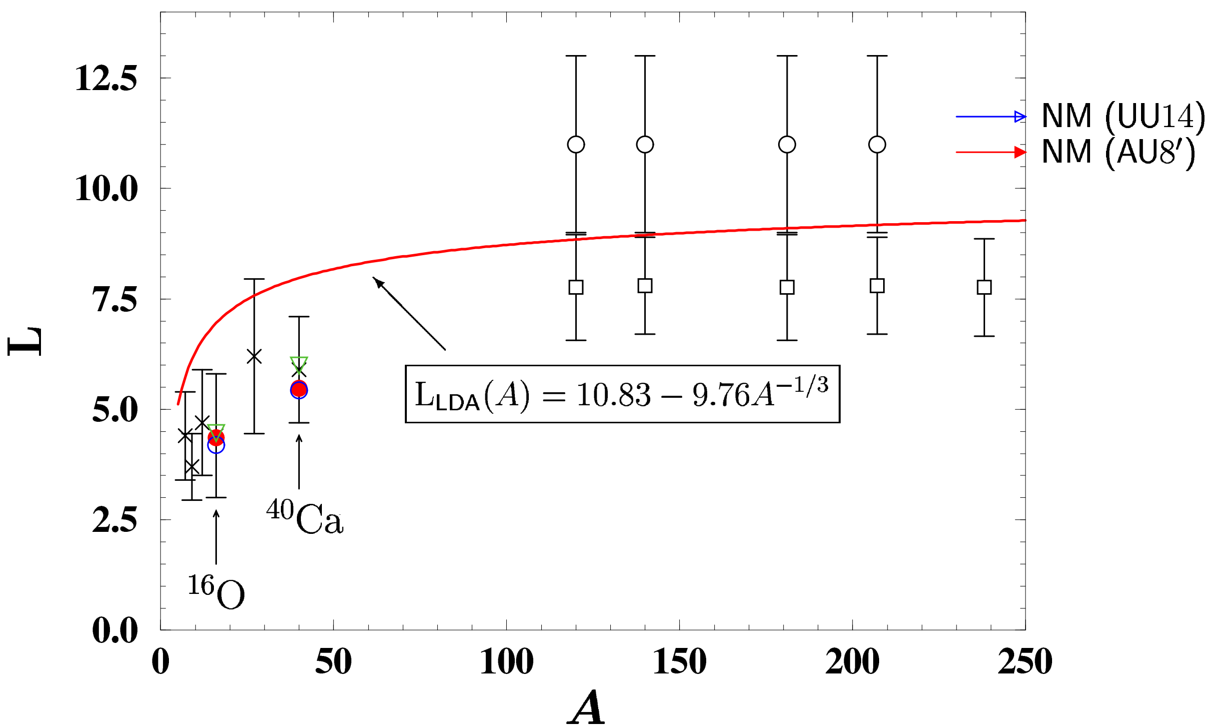

Figure 6: **Levinger's factor** $\mathrm{L}(A)$ for ^{16}O, ^{40}Ca and nuclear matter (shown by the arrows for the UU14 and AU8′ forces). The filled circles, the empty circles and the triangles show the Levinger's factors obtained within the f_6 and Jastrow correlation models and the IPM, respectively. The LDA is also reported (solid line).

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Conclusion

☆ **CBF** theory has been applied to microscopically compute the distribution of QD pairs, $\mathcal{P}_D(\mathbf{k}_D)$, in doubly closed shell nuclei ^{16}O and ^{40}Ca and nuclear matter, starting from the realistic Argonne v_8' plus Urbana IX potential.

☆ NN correlations produce a high momentum tail in $\mathcal{P}_D(\mathbf{k}_D)$ and, correspondingly a depletion at small $\mathbf{k}_D$ for both nuclei and nuclear matter. These effects are mainly due to the presence of the state–dependent correlations associated with the tensor component of the one pion exchange interaction. Contrary to what happens for the one–nucleon momentum distribution, this tail sizably differs from that of nuclear matter.

☆ Summation of $\mathcal{P}_D(\mathbf{k}_D)$ over $\mathbf{k}_D$ provides the total number $\mathcal{P}_D$ of QD pairs and consequently allows for an *ab initio* calculation of the Levinger's factor $\mathrm{L}(A)$. The Levinger factors for ^{16}O and ^{40}Ca are much smaller than the nuclear matter value and in very good agreement with the available photoreaction data. In addition, our results show that **LDA** overestimates $\mathrm{L}(A)$ in the region of the light–medium nuclei.

☆ The deuteron wave function is appreciably modified by the surrounding medium. While in the case of the S-wave component the difference is mostly visible at small relative distance ($r < 1$ fm), the D-wave component of the **QD** appears to be significantly enhanced, with respect to the deuteron $w_D(r)$, over the range $r < 2$ fm.

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