

Open problems in the dynamical evolution of globular clusters

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Pisa, 18 September 2017; dedicated to Francesco Pegoraro's 70th birthday

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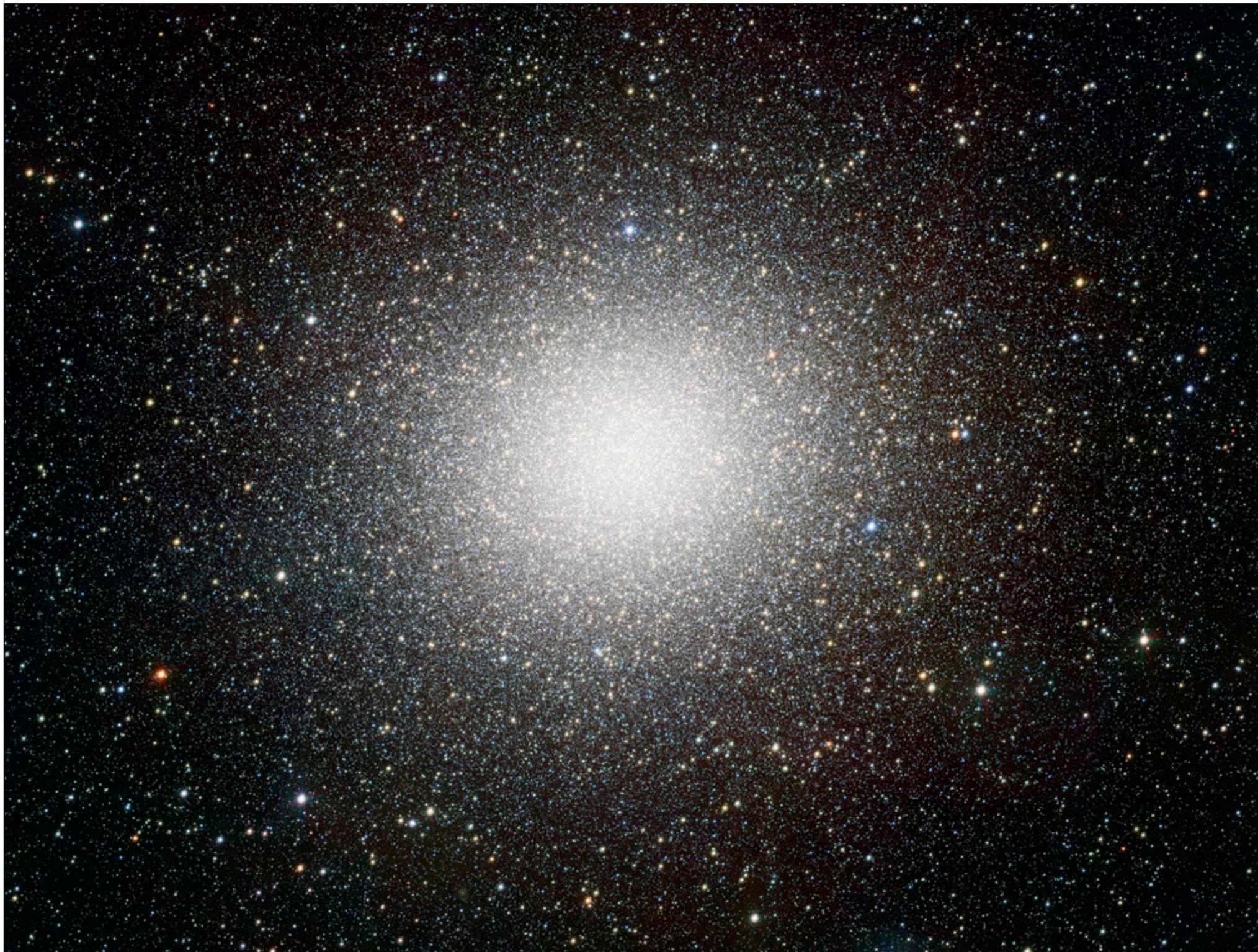
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M87

(I) THE VLASOV-POISSON SYSTEM OF EQUATIONS AND THE SUCCESS OF THE KING MODELS

For globular clusters, the dynamical time is very short, because a typical star velocity is a few km/s and a typical half-mass radius is a few pc.

[recall that $1 \text{ pc} / (1 \text{ km/s}) \sim 1.02 \text{ Myr}$]

Two-star relaxation times are larger.

In turn, globular clusters in our Galaxy are very old, with age $\sim 10 \text{ Gyr}$.

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla_x f - \nabla_x \Phi \cdot \nabla_v f = 0$$

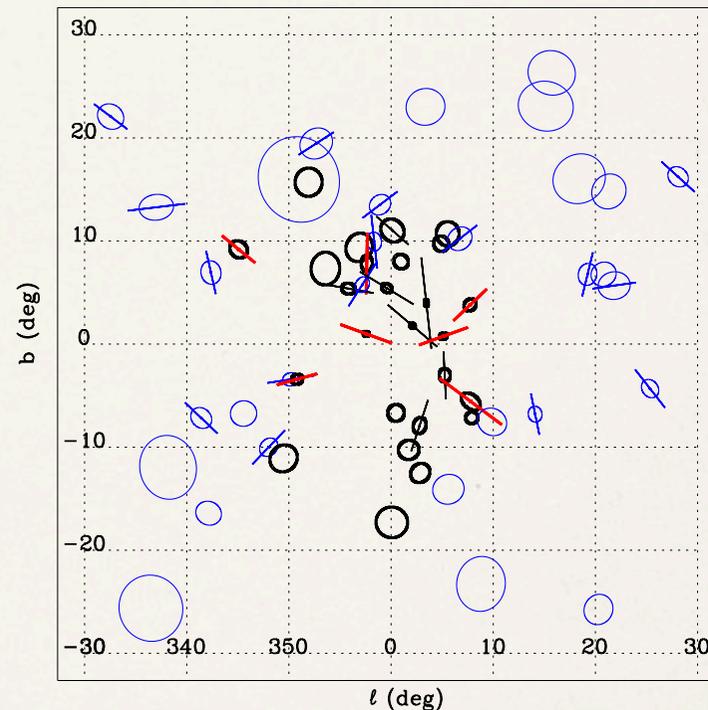
$$\Delta \Phi = 4\pi G \int f d^3v + 4\pi G \rho_{ext}$$

For stellar dynamics, the general quantitative description is by means of the Vlasov-Poisson system of equations, for which we consider the possible presence of external fields (e.g., dark matter or tidal forces).

TIDES AND SHAPES OF GLOBULAR CLUSTERS

Tidal interaction with the host galaxy is the physical basis of the truncation considered in the King models (see below). But tides would make the stellar system triaxial, with the long axis in the direction of the center of the host galaxy. Is it really true? What can we say about the shapes of globular clusters? Are the observed shapes primarily due to tides or rotation?

Very little has been done on this problem (White & Shawl 1987; Chen & Chen 2010). In contrast, note that this issue has led to the discovery of the important role of pressure anisotropy in elliptical galaxies.

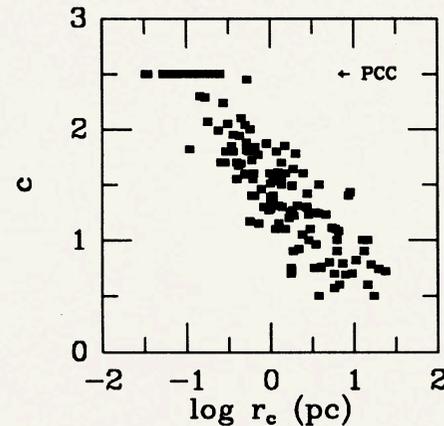


Mais, direz-vous, ces amas sont beaucoup plus petits que la Voie Lactée, dont ils font même probablement partie, et bien qu'ils soient plus denses, ils nous donneront plutôt quelque chose d'analogue à de la matière radiante ; or, les gaz n'atteignent leur équilibre adiabatique que par suite des chocs innombrables des molécules. Il y aurait peut-être moyen d'arranger cela. Supposons que les étoiles de l'amas aient justement assez d'énergie pour que leur vitesse s'annule quand elles arrivent à la surface ; alors, elles pourront traverser l'amas sans choc, mais arrivées à la surface, elles reviendront en arrière et le traverseront de nouveau ; après un grand nombre de traversées, elles finiront par être déviées par un choc ; dans ces conditions, nous aurions encore une matière que l'on pourrait regarder comme gazeuse ; si par hasard il y avait eu dans l'amas des étoiles dont la vitesse était plus grande, elles en sont sorties depuis longtemps, elles l'ont quitté pour n'y plus revenir. Pour toutes ces raisons, il serait curieux d'examiner les amas connus, de

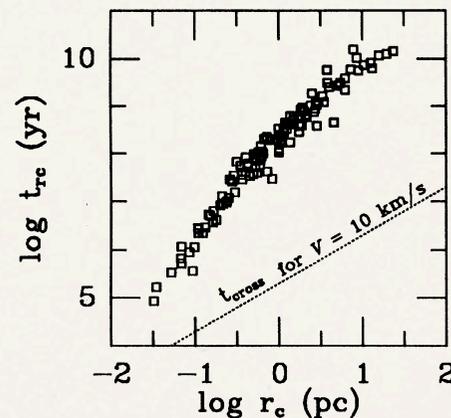
“Science et méthode”, Henri Poincaré (1908)

The globular clusters in our Galaxy are characterized by a wide range of relaxation times. For many of them relaxation effects are thought to be significant. The parameters illustrated below are obtained by fitting the observations with King models.

Concentration (dimensionless depth of the potential well)



Central relaxation time



Djorgovski & Meylan (1994)

EFFECTS OF RELAXATION

Star-star relaxation is the physical basis of the isotropic, quasi-Maxwellian King models (see below). But collisionality leads to other important phenomena, all responsible for the dynamical evolution of globular clusters:

- evaporation
- core collapse [gravothermal catastrophe and oscillations; see also Sormani & GB (2013)]
- equipartition and mass segregation [see also Spitzer's (1969) instability; Trenti & van der Marel (2013)]
- physical encounters, multiple encounters, anomalous effects involving or producing binaries

How much of these effects can be measured? So far great attention has been given to producing powerful and realistic N-body simulations [e.g., see Heggie & Hut (2003)]. Much remains to be done in terms of physical understanding and actual observations. Furthermore, good relaxation would lead to mass segregation: is this incompatible with the use of simple one-component King models?

DYNAMICAL EVOLUTION

So the dynamical evolution of globular clusters is expected to be driven by a number of factors, among which

- instabilities
- external forcing
- collisionality

THE TRADITIONAL KING MODELS

(“lowered isothermal spheres”)

- 1) isotropic, because constructed under the hypothesis of significant relaxation;
- 2) generally applied to fit the photometric profiles (constant M/L);
- 3) tidally-truncated (truncation non-unique; see McLaughlin, van der Marel 2005), but spherical;
- 4) nonrotating;
- 5) generally applied as one-component models;
- 6) a one-parameter family of models, exhibiting different degrees of concentration.

Of course, as noted above, globular clusters are very complex systems, in which physical encounters, multiple encounters, anomalous effects involving or producing binaries, stellar evolution and other phenomena go well beyond the simple view of classical stellar dynamics and for which the use of more and more realistic simulations is clearly needed and welcome. Yet, the success of simple models would greatly help to understand physical issues and physical mechanisms.

Great progress in the observations is making it possible, even now, to probe the six-dimensional phase-space structure of some clusters, so that more advanced models are demanded.

I.R. King (1966)

$$f_K = A \left[\exp(-aE) - \exp(-a\Phi(r_{tr})) \right] \quad \text{for } E < \Phi(r_{tr})$$

$$f_K = 0 \quad \text{for } E > \Phi(r_{tr})$$

$$E = \frac{1}{2}v^2 + \Phi(r)$$

$$\psi \equiv a \left[\Phi(r_{tr}) - \Phi(r) \right]$$

$$\hat{\Delta}\psi = -\hat{\rho}_K[\psi]$$

$$\hat{\rho}_K[\psi] = \exp(\psi) \gamma\left(\frac{5}{2}, \psi\right)$$

ψ is the dimensionless
escape energy at radius r

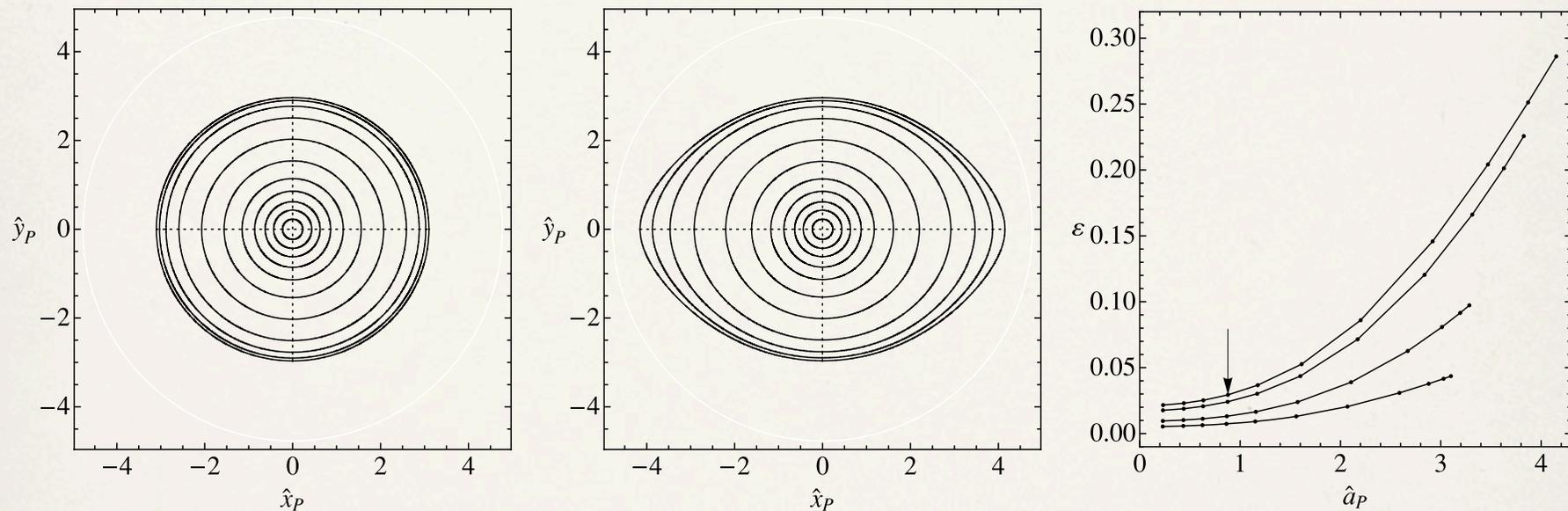
for $\psi > 0$

A, a, r_{tr} are positive constants: 2 scales (e.g., mass M and core-size r_0), 1 dimensionless parameter [e.g., “concentration” $c = \log(r_{tr}/r_0)$ or $\Psi = \psi(0)$]

(II) BEYOND THE KING MODELS

- a) Construction of the collisionless analogues of the Roche ellipsoids, produced by the action of an external tidal field on an otherwise spherical, self-gravitating system;
- b) Construction of models for rotating globular clusters, characterized by axisymmetry, differential rotation, and pressure anisotropy;
- c) Construction of quasi-relaxed two-component stellar systems by considering separate parameters for the distribution functions of heavy and light stars.

a) By replacing the single-star energy by the Jacobi integral in the King distribution function and inserting the tidal field in the Poisson equation (under the assumption that the globular cluster is moving on a circular orbit around the galaxy center):



GB, A. L. Varri (2008) *Astrophys. J.*; A.L. Varri, GB (2009) *Astrophys. J.*

b) Simple models for rotating globular clusters, characterized by axisymmetry, differential rotation, and pressure anisotropy, based on a relatively simple choice of distribution function [Varri & GB (2012); Bianchini, Varri, GB, Zocchi (2013)]:

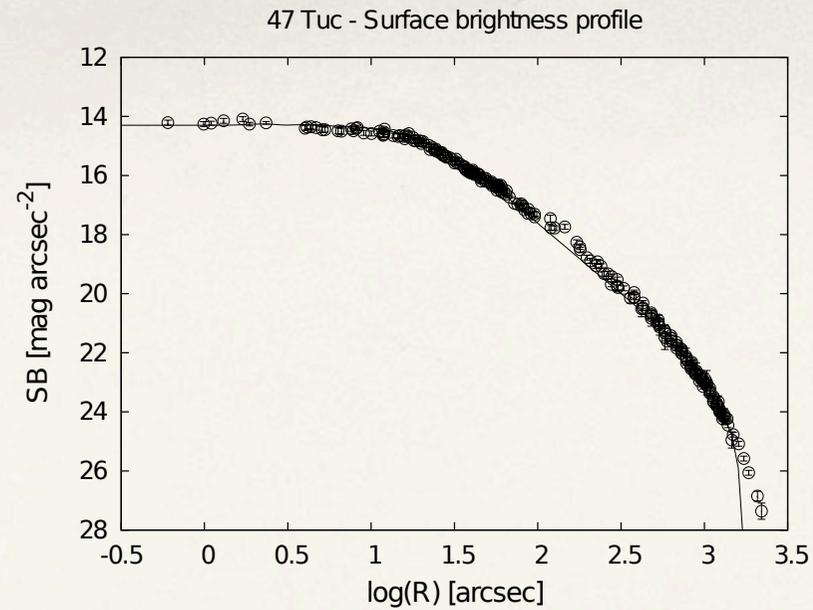
$$I(E, J_z) \equiv E - \frac{\omega J_z}{1 + bJ_z^{2c}}$$

$$f_{WT}^d(I) = Ae^{-aE_0} \left[e^{-a(I-E_0)} - 1 + a(I - E_0) \right]$$

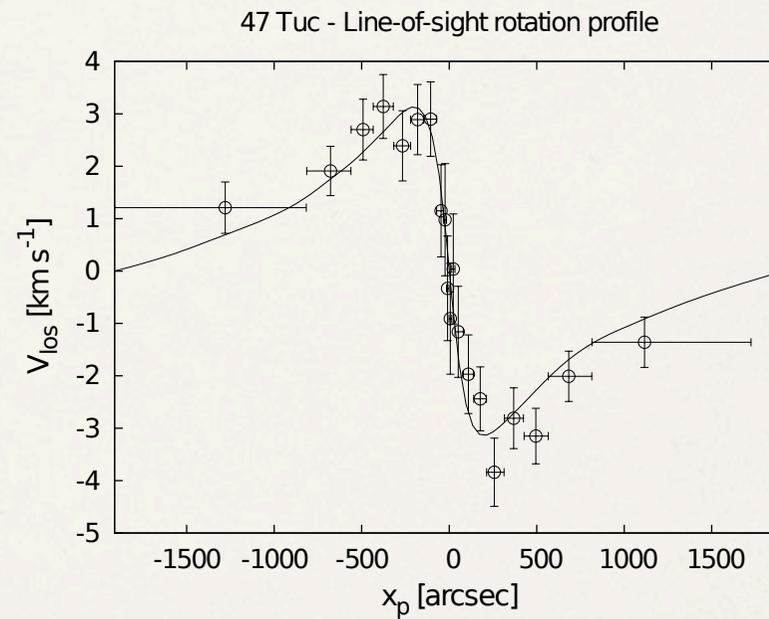
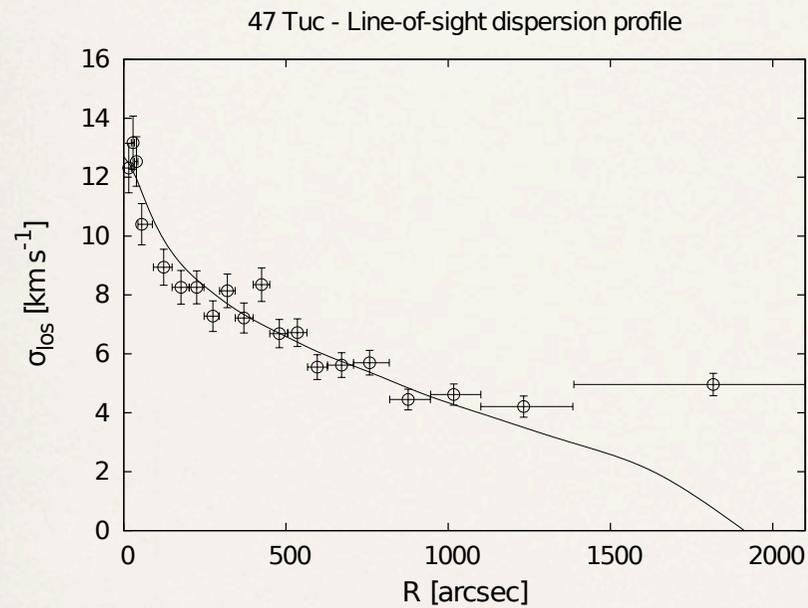
$$\text{if } E \leq E_0$$

$$f_{WT}^d(I) = 0 \quad \text{otherwise}$$

Photometric data:
Trager et al (1995) +
Noyola & Gebhardt
(2006)

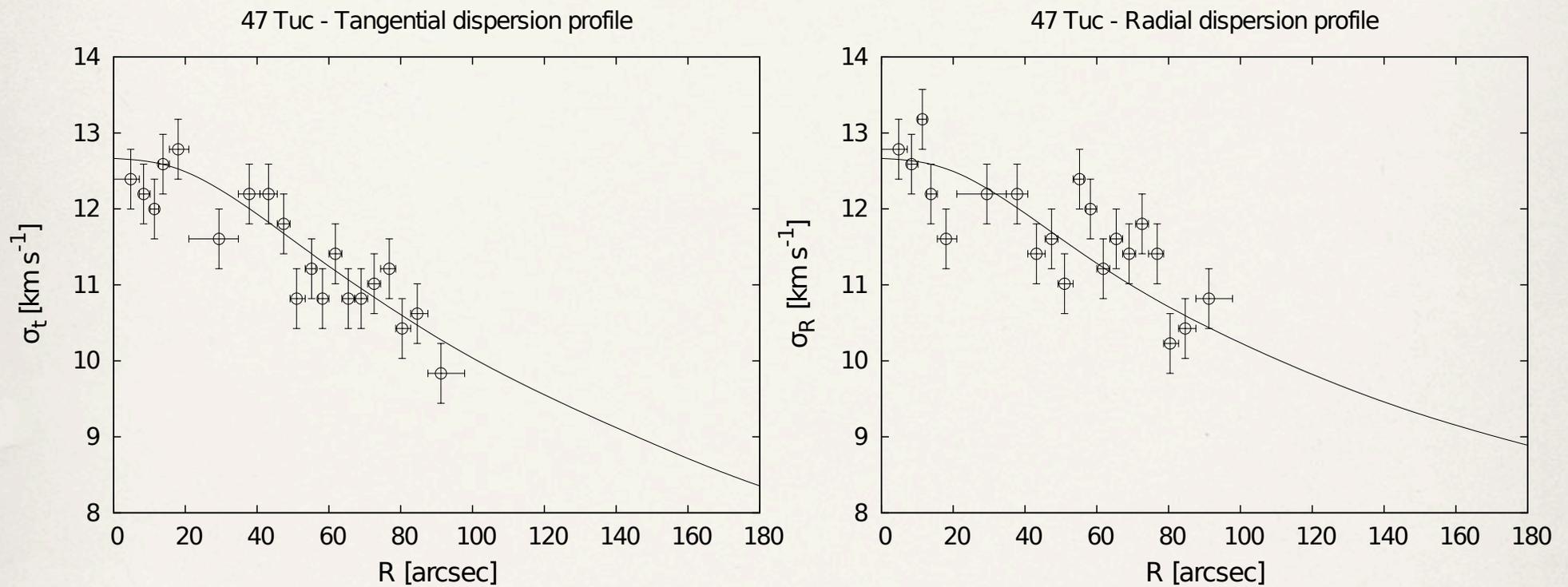


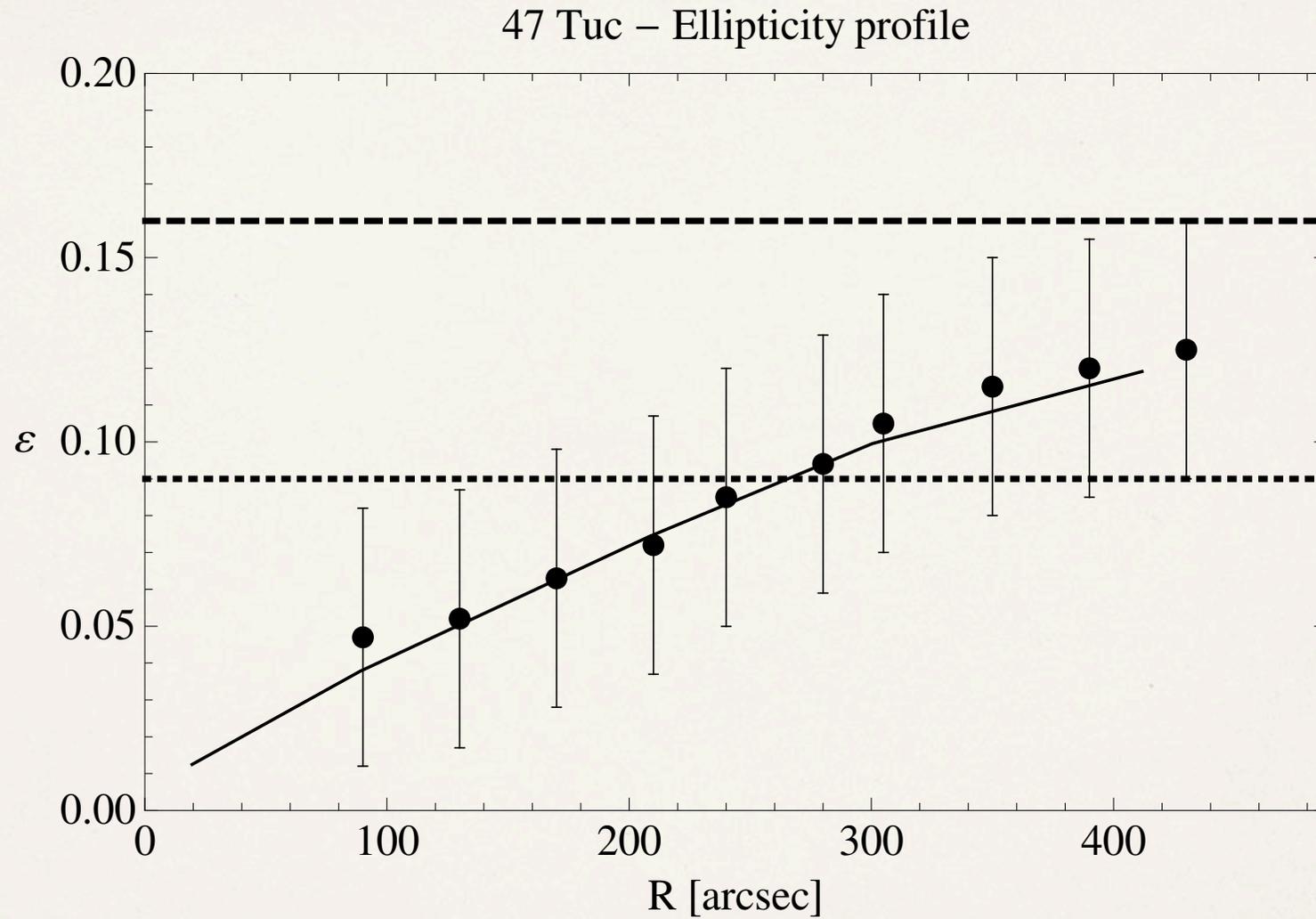
2,476 l.o.s. velocity data: Gebhardt
et al. (1995) + Lane et al. (2011)



12,974 HST proper motions: McLaughlin et al. (2006)

Inclination: Anderson & King (2003) ~ 45 degrees





c) Setting up a simple two-component model of globular clusters (each component is spherical and described by an anisotropic $f^\wedge(\nu)$ distribution function):

A&A 590, A16 (2016)

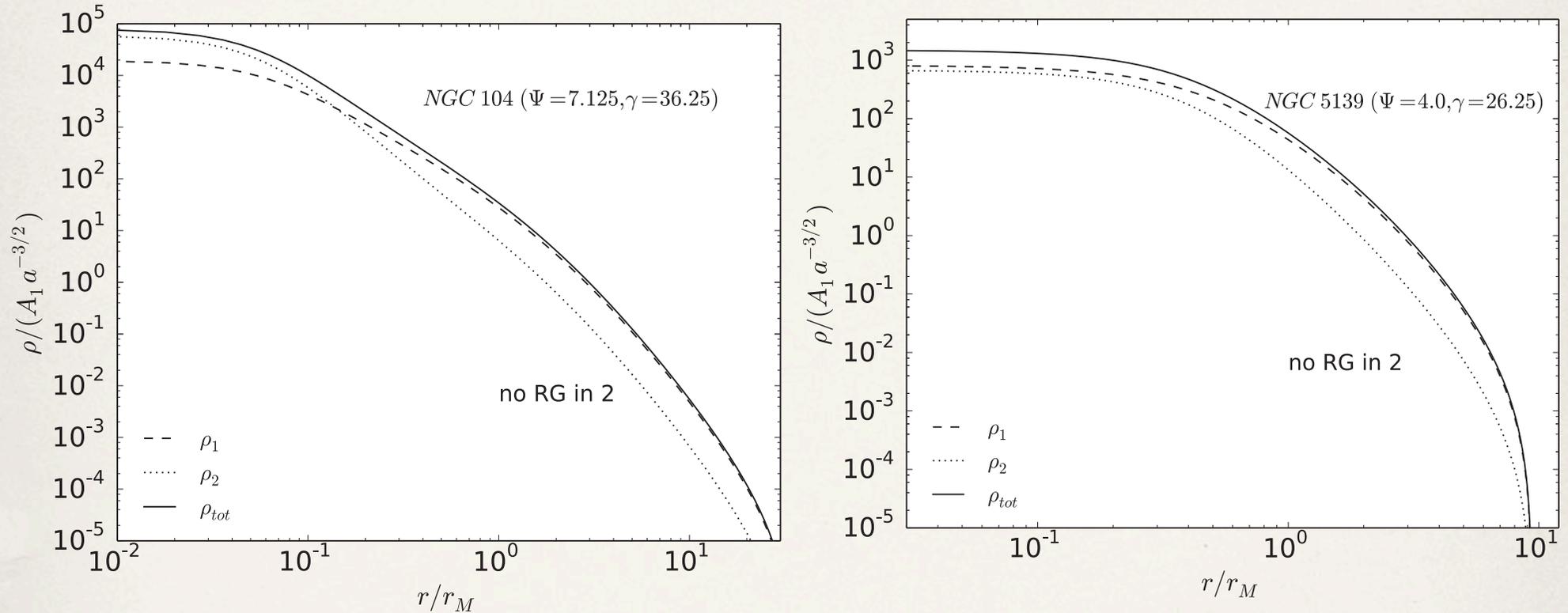


Fig. 10. *Left panel:* density profiles of each component and the total density profile for the best-fit model of 47 Tuc (NGC 104), obtained by the procedure in which RG stars are not included in the heavy component (see text); *right panel:* corresponding density profiles for the best-fit model of ω Cen (NGC 5139).

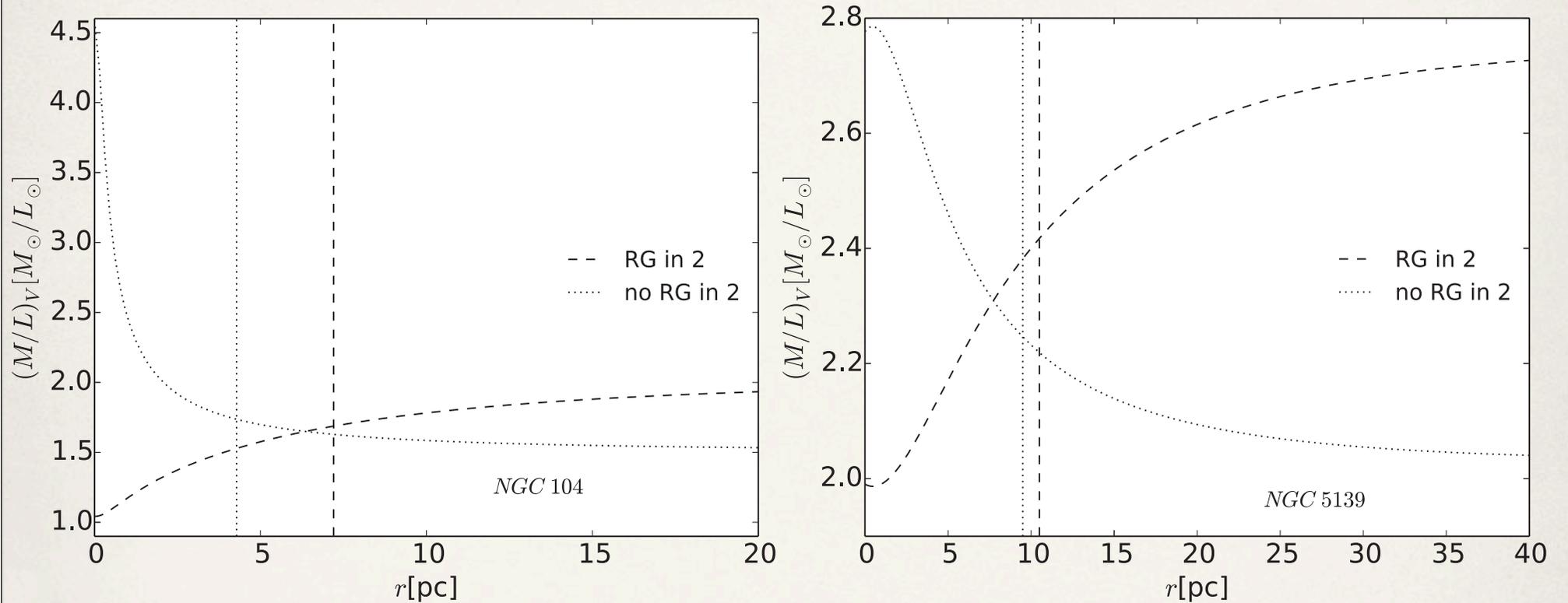


Fig. 11. Cumulative mass-to-light ratio as a function of the intrinsic radius r for the best-fit models of two globular clusters. The best-fit models are found by means of two different procedures, that is, by taking the heavier component as made of only dark remnants or by including in the heavier component the presence of red giants. The vertical lines indicate the position of the total half-mass radius.

(III) SOME OPEN PROBLEMS

a) instabilities (collisionless systems)

b) external forcing

c) weakly collisional systems

a) instabilities (collisionless systems)

Spherical, isotropic models with different types of truncation (see Part I).

1) What are the effects of the sharpness of the assumed truncation on the linear modes?

Spherical, anisotropic models, with excess of kinetic energy in the radial direction (see also Part IIc) are subject to the radial-orbit instability (Polyachenko & Shukhman 1981; REF 5).

Linear theory:

2) What is the behavior of the unstable $l=2$ modes in the vicinity of marginal stability?

3) What about the structure of damped modes? (may be excited by tidal interactions)

4) What about modes with different l values, in particular $l=1$ and $l=4$?

Nonlinear evolution:

5) Is there a general rule that would explain whether radial-orbit unstable equilibria would tend to evolve into prolate or oblate or triaxial shapes or is it just a matter of initial conditions? (See also the McLaurin-Jacobi-Dedekind transition.)

Spherical, anisotropic models, with excess of kinetic energy in the tangential directions.

6) Is there any evidence for instability? (largely unknown, because we do not have reasonable equilibrium states to work on; see REF 4)

Nonspherical, rotating, anisotropic models, e.g. of the kind introduced in Part IIb.

7) Can we develop a linear modal analysis (see REF 6)?

(little has been done on this topic; some simulations show that even some rotating toroidal configurations appear to be stable, but when rotation is too strong instability occurs)

b) external forcing (see Part IIa)

8) What can we say about external forcing for globular clusters not on a circular orbit?

c) weakly collisional systems

Spherical, isotropic, truncated stellar systems, with a small amount of collisionality.

9) Can we prove that King, or King-like models, undergo the so-called gravothermal catastrophe?

[Lynden-Bell & Wood (1968); GB & Trenti (2003). Note that a linear modal analysis of the kind reported in REF 5 should be adapted to the different outer boundary conditions imposed by the truncation on the models. Gravothermal catastrophe is expected also for systems made of stars with equal mass.]

10) Could we check the time-scale and the nature of the gravothermal catastrophe, which in a fluid model develops on the dynamical time-scale (see the study of the Ebert-Bonnor problem by Sormani & GB 2013)?

11) Could we quantify the establishment of equipartition and mass segregation? In particular, could the simple two-component models developed in Part IIc be better justified and more tractable for the purpose (see also REF 1 and REF 2)?

12) Could we describe the development of evaporation induced by collisionality, also as a function of the truncation imposed?

CONCLUSIONS

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Raise important general questions; formulate the physical questions raised in a rigorous mathematical form; address the problems with an open mind and an interdisciplinary approach.

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A comment on the Gravo-thermal Catastrophe

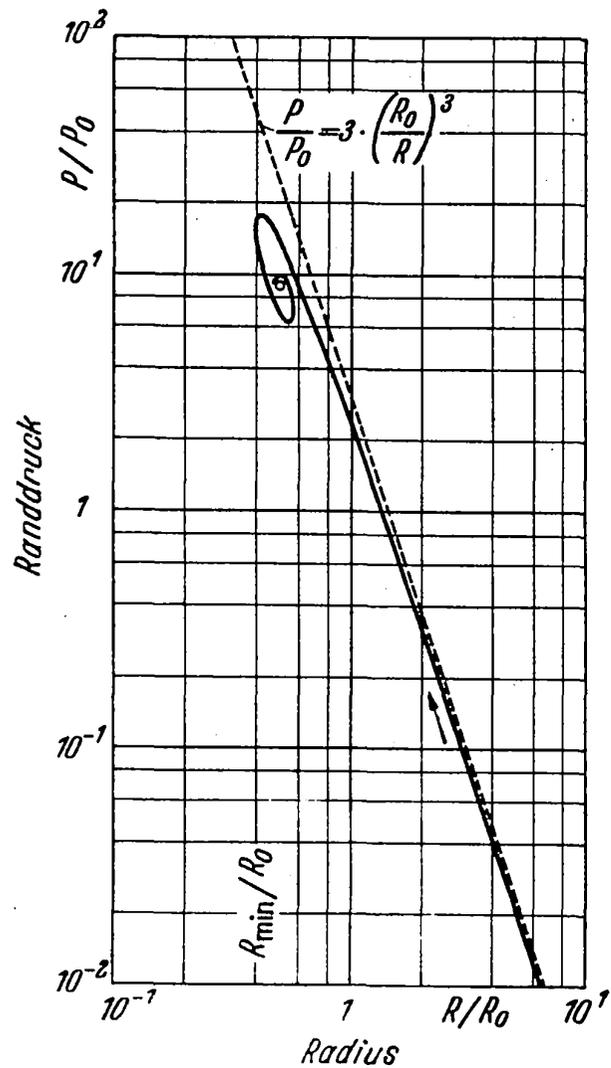
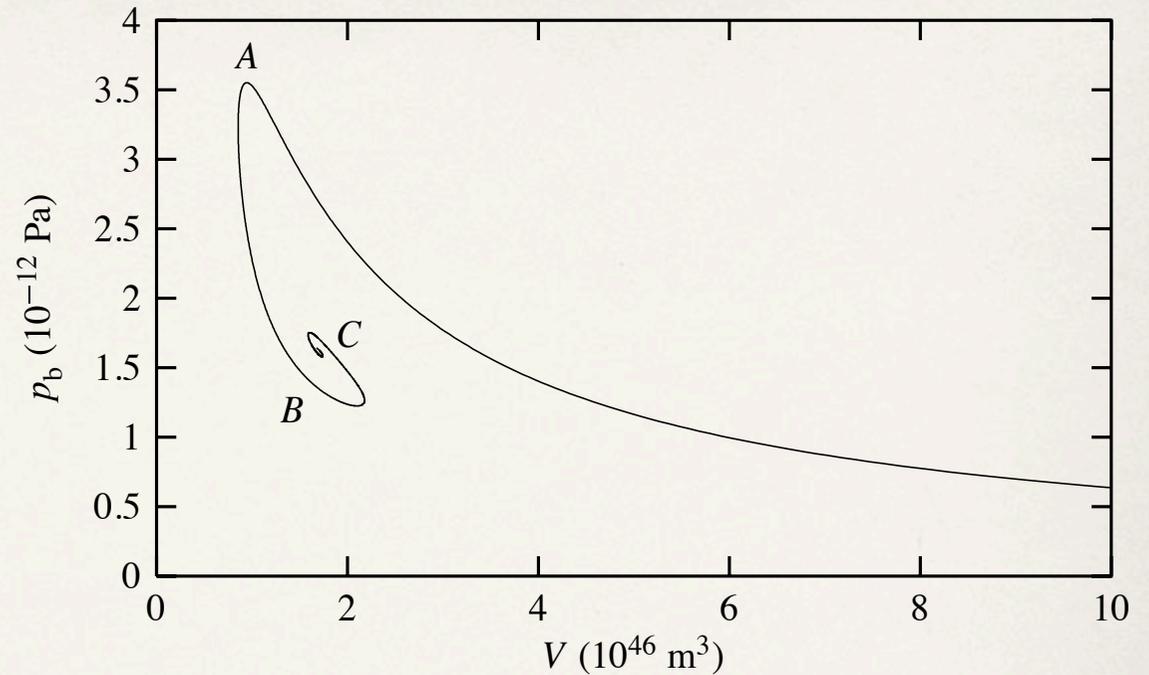


Abb. 2. Randdruck der Gaskugel.

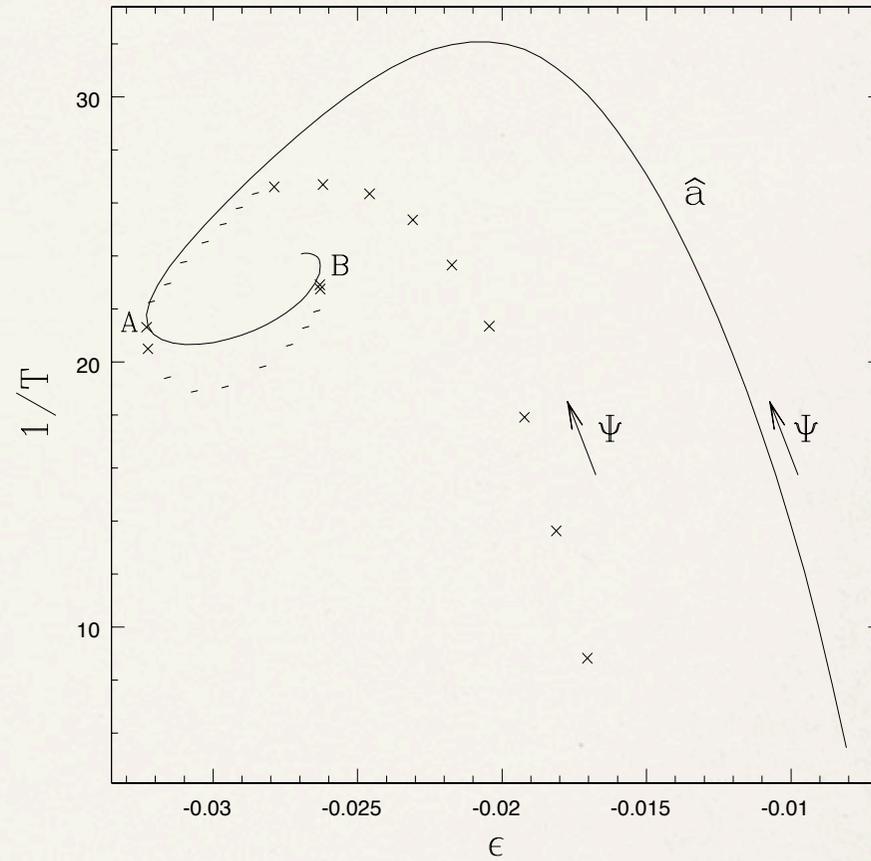
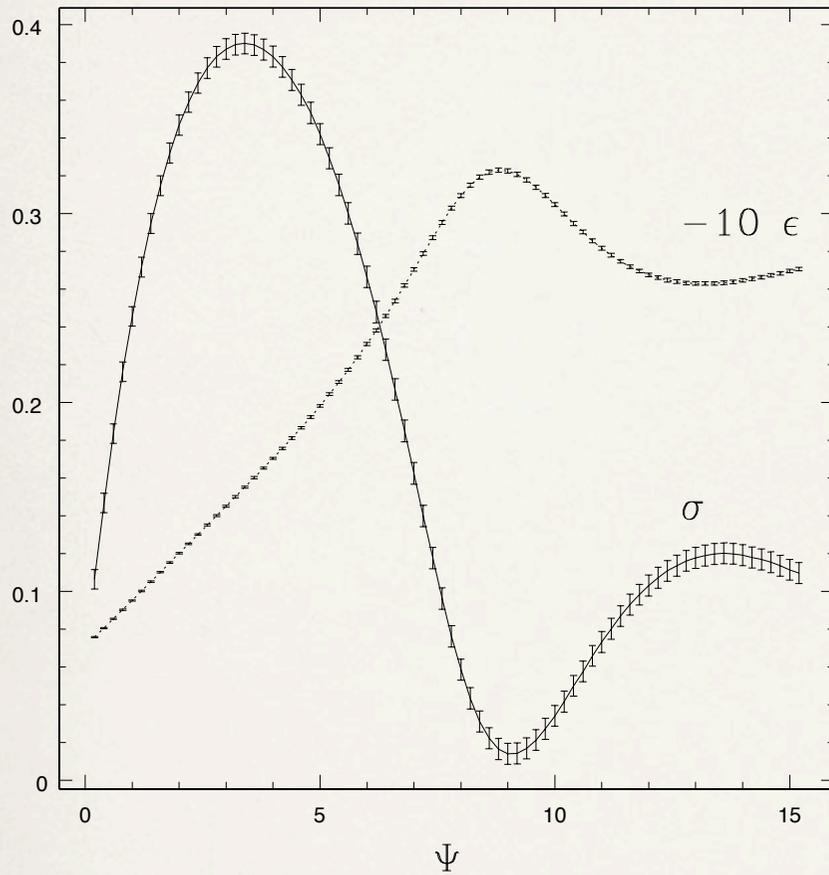
Ebert (1955) Zeit. Astrophys.



Lombardi & GB (2001) Astron. Astrophys.

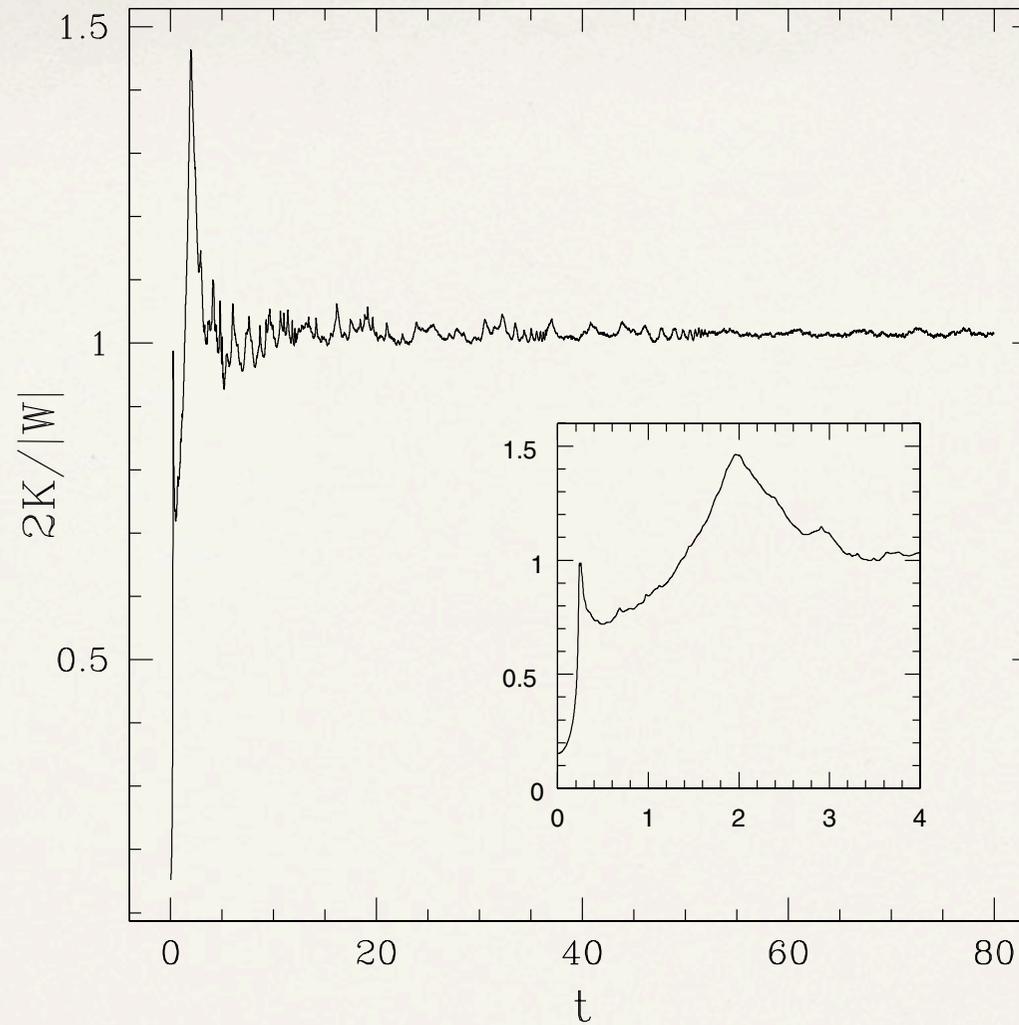
Curiously, if we perform a linear modal analysis of an ideal, inviscid fluid model (assuming infinite thermal conductivity and isothermality) under the boundary conditions listed by Lynden-Bell & Wood (1968) in their study of stellar systems, we recover precisely the same critical points for the onset of instability with respect to radial modes, proving that the time scale of instability is the dynamical time scale (Sormani & GB, 2013).

From the fact that the $f^{\{\nu\}}$ distribution function can be derived by extremizing the Boltzmann entropy under suitable constraints, it was possible to provide a formal derivation of Lynden-Bell's conjecture on the gravitational catastrophe for stellar systems:

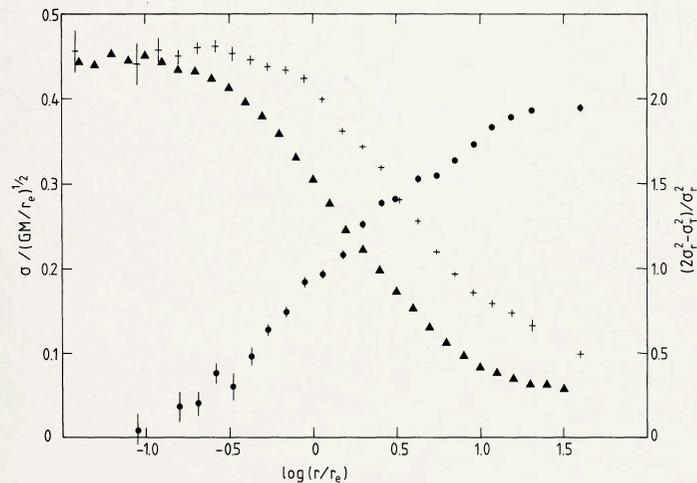


GB & Trenti (2003) *Astrophys. J.*

Incomplete violent relaxation (collisionless systems; see Lynden-Bell 1967)



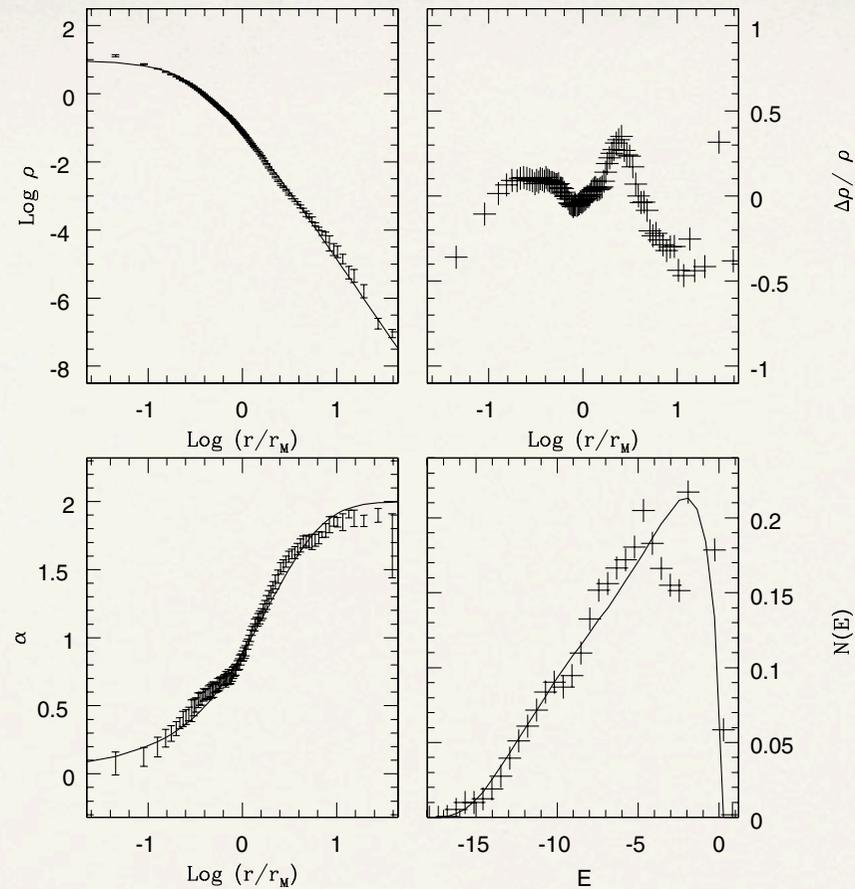
Trenti, GB, van Albada (2005) *Astron. Astrophys.*



van Albada (1982)

With Massimo Stiavelli, we constructed simple distribution functions able to incorporate the qualitative behavior of the results of incomplete violent relaxation at small and large radii [GB & Stiavelli (1984); Stiavelli & GB (1985)].

Surprisingly, the resulting self-consistent models (obtained from the solution of the Vlasov-Poisson system of equations) proved to be an excellent tool to match the observations.....



...about 20 years later, we went back to the general problem and found that the so-called f^{ν} models also provided a very accurate representation of the results of numerical simulations of violent relaxation (Trenti, GB, van Albada, 2005).