Nonlinear Waves
Instabilities
Charged Particle Acceleration

S. V. Bulanov

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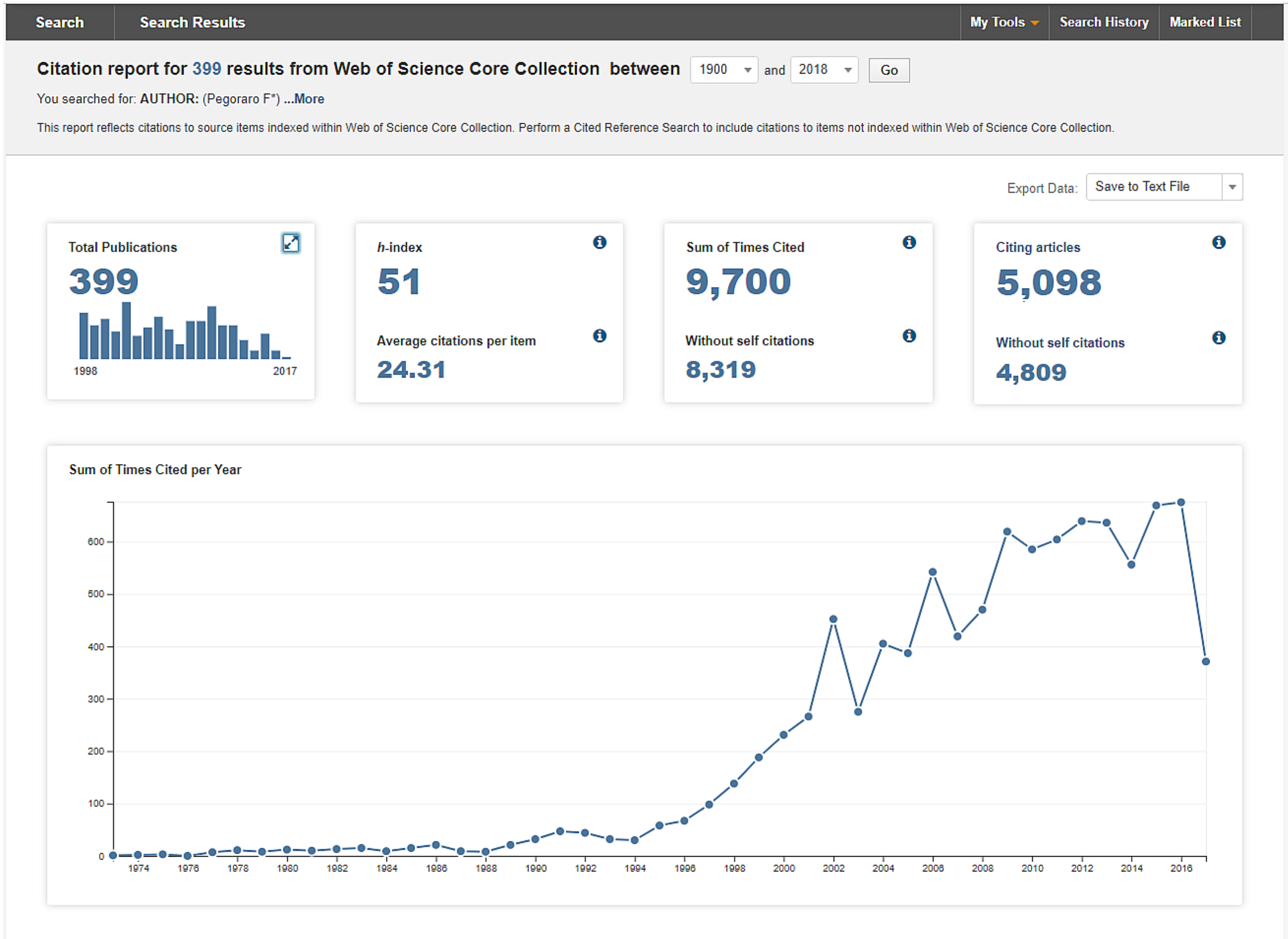
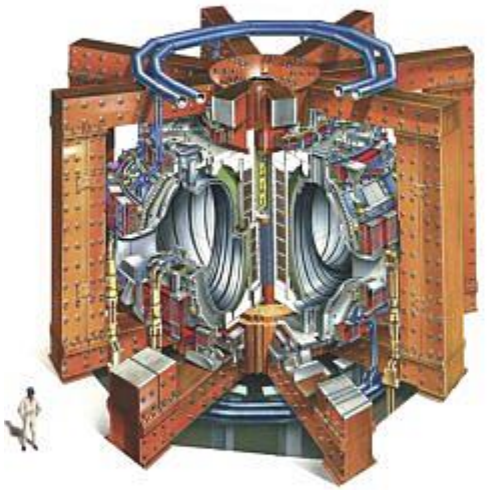
Plasma Physics, Recent Results
and
Future Perspectives
Physics Department, University of Pisa
Pisa, 18 September 2017

Citations



Magnetic Confinement, Magnetic Reconnection, Charged Particle Acceleration, Laser Plasmas, Hamiltonian Systems, etc., etc., etc.

Retrospective



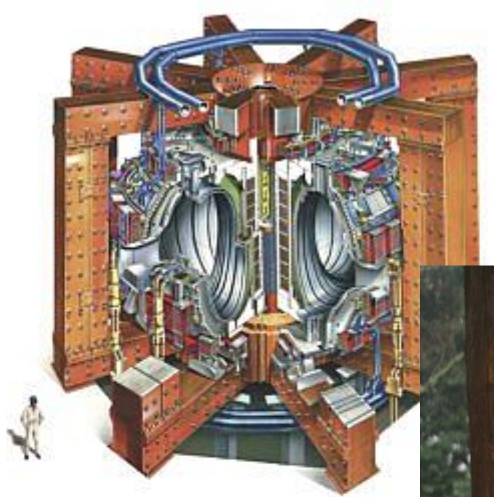
JET

1. <u>Magnetic Reconnection</u>
   a) MHD waves & tearing instability
   b) 3D configurations
   c) EMHD
   d) Charged particle acceleration
   e) Filamentation instability

2. <u>Laser Plasmas</u>
   a) LWFA
   b) RPDA
   c) HHG
   d) Solitons
   e) Vortices
   f) Fast Ignition
   g) Hadron Therapy

ELI-BL

Retrospective



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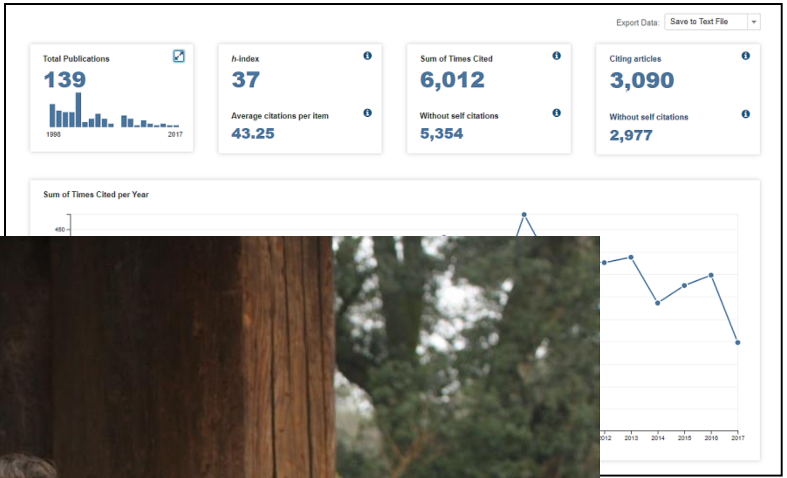

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Magnetic Interaction in Laser Plasmas

Magnetic interaction of self-focusing channels and fluxes of electromagnetic radiation: their coalescence, the accumulation of energy, and the effect of external magnetic fields on them

G. A. Askar'yan and S. V. Bulanov
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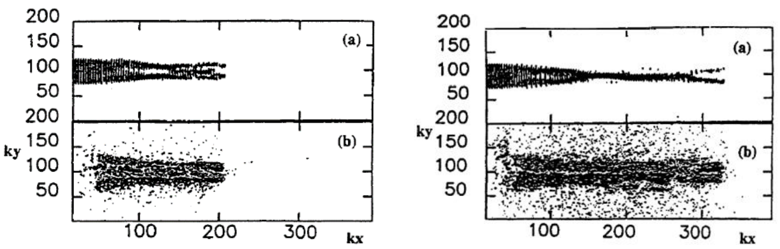
(Submitted 14 July 1994)
Pis'ma Zh. Eksp. Teor. Fiz. **60**, No. 4, 240–246 (25 August 1994)

Comments Plasma Phys. Controlled Fusion
1995, Vol. 17, No. 1, pp. 35–44

Physica Scripta Vol. T63, 280-283 1996

Magnetic interaction and magnetic wake of high intensity laser pulses in plasmas

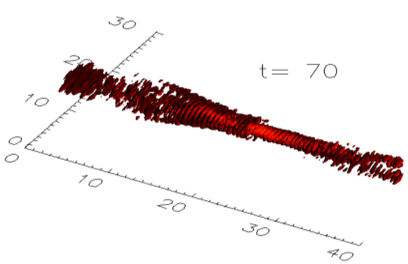
S. V. Bulanov[1,2], T. Zh. Esirkepov[3], M. Lontano[2], F. Pegoraro[4] and A. M. Pukhov[3,5]

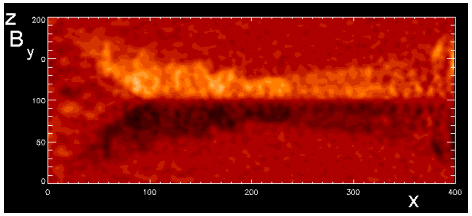


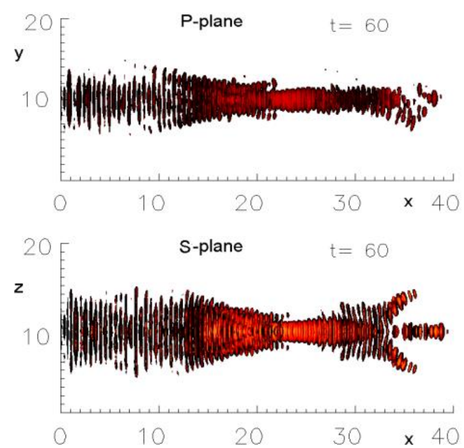
PHYSICAL REVIEW E, VOLUME 65, 045402(R)

Polarization effects and anisotropy in three-dimensional relativistic self-focusing

N. M. Naumova,[1,2] S. V. Bulanov,[2] K. Nishihara,[3] T. Zh. Esirkepov,[4] and F. Pegoraro[5]







Magnetic Interaction in Laser Plasmas

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Magnetic Reconnection in Laser Plasmas

PI

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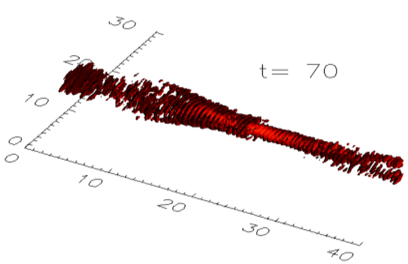
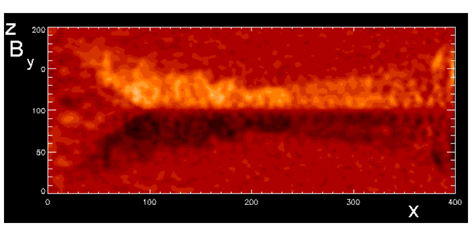
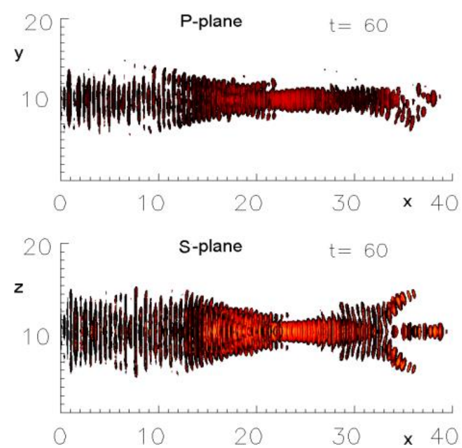
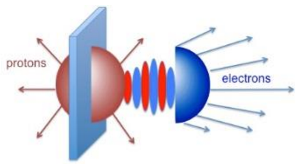
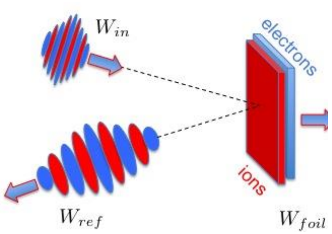


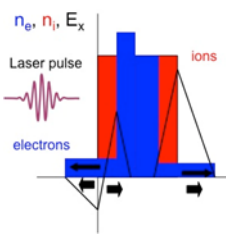


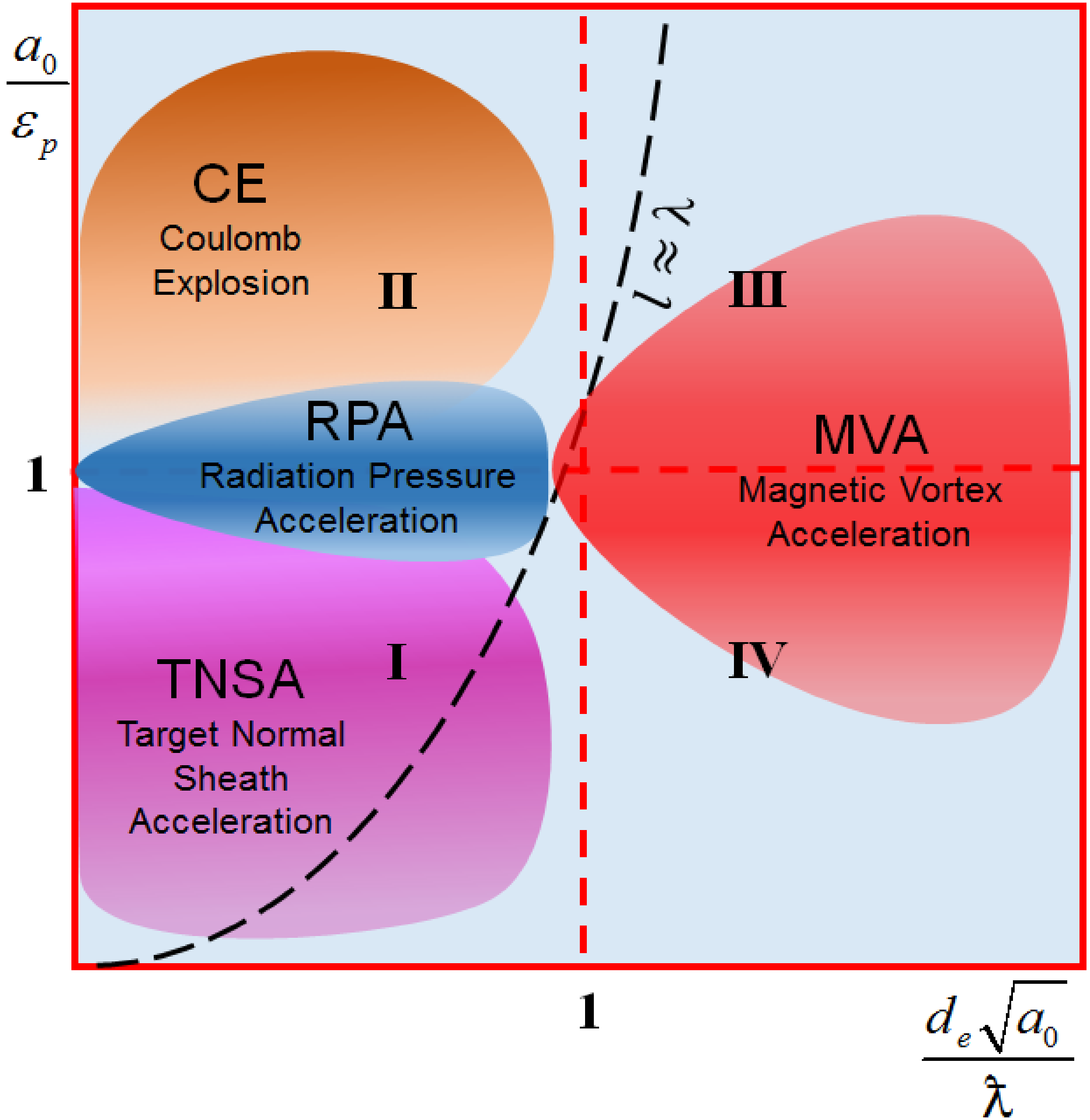


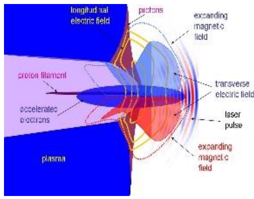
Ion Acceleration Mechanisms



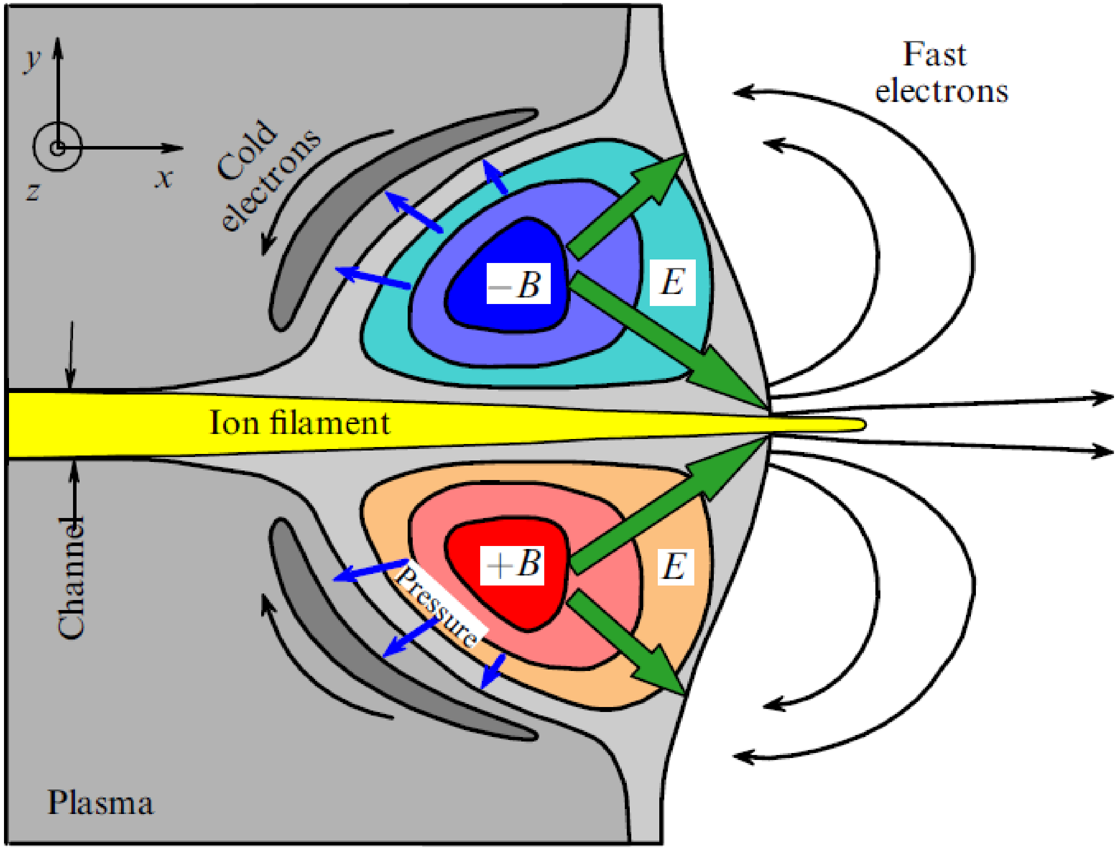








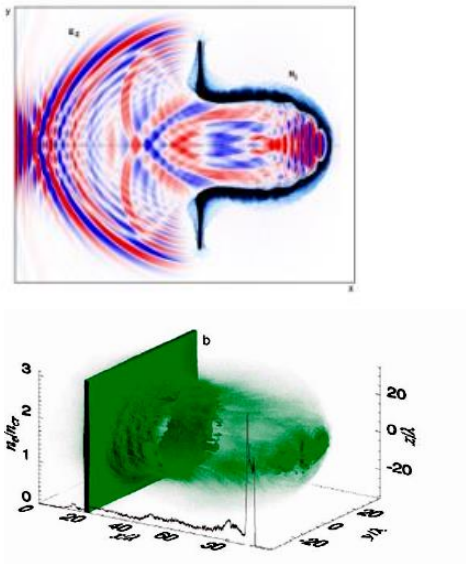
Magnetic Vortex Acceleration Mechanism



Magnetic and electric field configurations at the target's rear side. The pressure of the toroidal magnetic field leads to the formation of a region with a strong electric field which accelerates and focuses the ions.

Phys. Rev. Lett. 98, 049503 (2007)

R-T Stability of RPDA Ion Acceleration



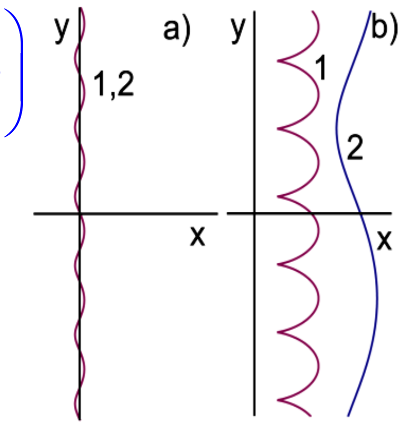
T. Esirkepov et al (2004)

For perturbations we have

$$\frac{\partial}{\partial\psi}\left(\frac{p^{(0)}(\psi)}{m_\alpha c}\frac{\partial x^{(1)}}{\partial\psi}\right)=\frac{R(\psi)}{2\pi}\left(\frac{\partial y^{(1)}}{\partial\eta}+\frac{\partial z^{(1)}}{\partial\zeta}\right)$$

$$\frac{\partial}{\partial\psi}\left(\frac{m_\alpha c}{p^{(0)}(\psi)}\frac{\partial y^{(1)}}{\partial\psi}\right)=-\frac{R(\psi)}{2\pi}\frac{\partial x^{(1)}}{\partial\eta}$$

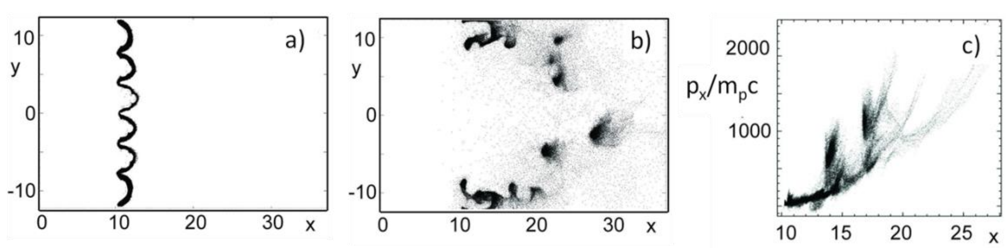
$$\frac{\partial}{\partial\psi}\left(\frac{m_\alpha c}{p^{(0)}(\psi)}\frac{\partial z^{(1)}}{\partial\psi}\right)=-\frac{R(\psi)}{2\pi}\frac{\partial x^{(1)}}{\partial\zeta}$$



where

$$\psi=t-\frac{x^{(0)}(t)}{c},\quad R(\psi)=\frac{E_L^2(\psi)}{2\pi n_0 l_0\omega_0^2}$$

$$\tau_{RT}=\frac{(2\pi)^{3/2}}{\omega_0^{-1}}\frac{\left(R^{(0)}\right)^{1/2}}{6k^{3/2}\lambda_0^2}$$



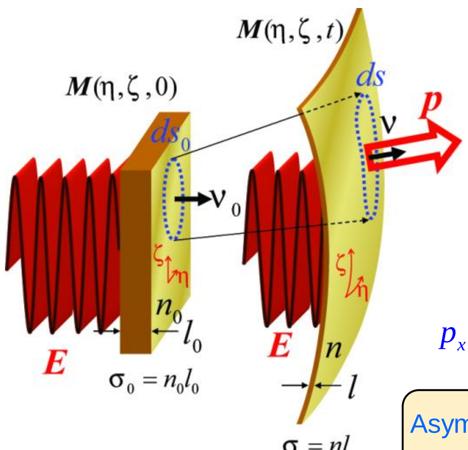
Relativistic theory of RT instability of a thin foil

F. Pegoraro and S. V. Bulanov, *Phys. Rev. Lett.* 99, 065002 (2007); C.A.J.Palmer et al., *Phys. Rev. Lett.* 106, 014801 (2011); S. V. Bulanov, et al., *Phys. Plasmas* 19, 103105 (2012); K. V. Lezhnin, et al., *Phys. Plasmas* 21, 012705 (2014)

‘Unlimited’ Ion Acceleration by RPDA

The energy of the ions accelerated by an intense electromagnetic wave in the radiation pressure dominated regime can be greatly enhanced by a transverse expansion of a thin target. The expansion decreases the number of accelerated ions in the irradiated region increasing the energy and the longitudinal velocity of the remaining ions. In the relativistic limit, the ions become phase-locked with respect to the electromagnetic wave resulting in an unlimited ion energy gain. This effect and the use of optimal laser pulse shape provide a new approach for great enhancing the energy of laser accelerated ions.

Evolution of thin shell irradiated by a strong electromagnetic wave



Equations of motion

$$\partial_t p_i = \frac{\mathcal{P}}{n_0 l_0}\varepsilon_{ijk}\partial_\eta x_j \partial_\eta x_k$$

$$\partial_t x_i = \frac{c p_i}{\sqrt{m_\alpha^2 c^2 + p_k p_k}}$$

$$i,\ j,k = 1,2,3$$

$$\mathcal{P} = \frac{E_L^2}{2\pi}\frac{c+v}{c-v}$$

$$p_x(t) = m_\alpha c\left(\frac{125 E_L^2 \pi_y^0 \pi_z^0}{48\pi n_0 l_0 m_\alpha^3 c}\right)^{1/5} t^{3/5}$$

Asymptotic solution for ion momentum dependence on time

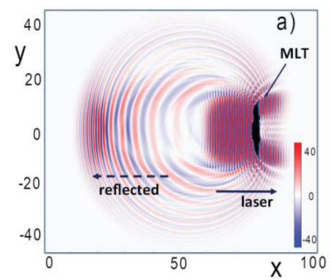
Wave phase: $\psi = \omega_0\left(t - \frac{x}{c}\right) = \omega_0\int_0^t \left(1-\beta(t')\right)dt'$

For $\frac{p_x(t)}{m_\alpha c} = \left(\frac{t}{\tau_k}\right)^k$

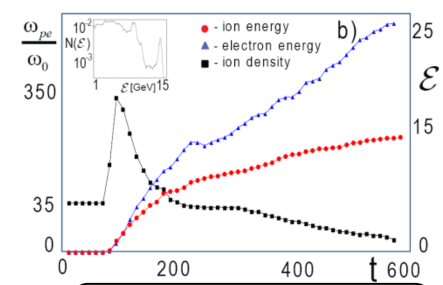
$$\underset{t\to\infty}{\to} \omega_0\tau_k \frac{(t/\tau_k)^{1-2k}}{1-2k} + \frac{\omega_0\tau_k}{\pi^{1/2}}\Gamma\left(\frac{2k-1}{2k}\right)\Gamma\left(\frac{2k+1}{2k}\right)$$

If k<1/2, the phase Is locked

Laser-Mass-Limited-Target interaction in Radiation Pressure Dominated Acceleration regime: PIC computer simulation



Laser pulse, reflected radiation and Mass Limited Target



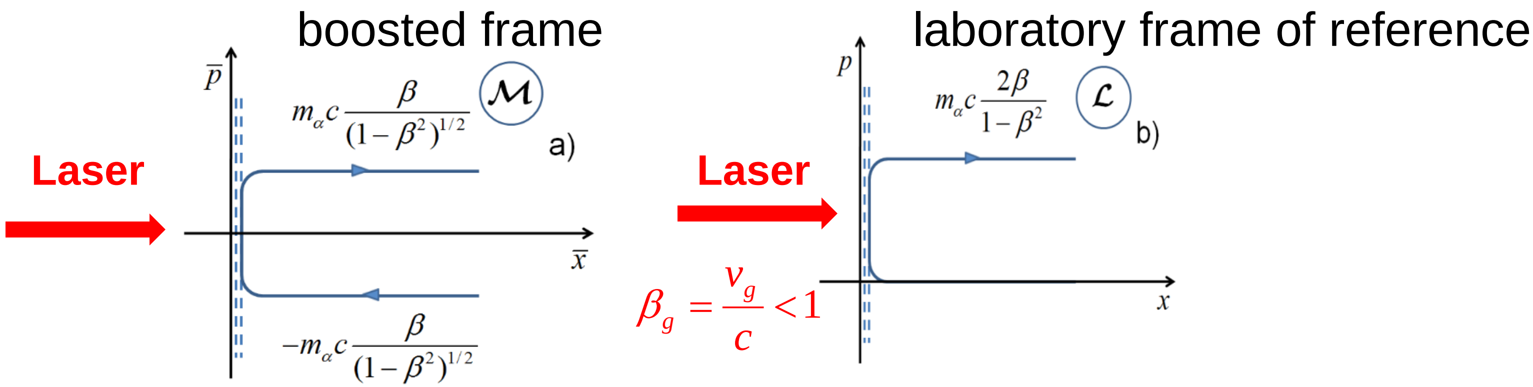
Electron and ion energy and density v.s. time Inset: ion energy spectrum

Phys. Rev. Lett. 104, 135003 (2010)
Phys. Plasmas 17, 063102 (2010)

Wake & Waterfall

赤目四十八滝

Hole Boring by Radiation Pressure



Phase plane of the ions accelerated at the front of the laser pulse.

Velocity of the SW front:

$$\beta = \frac{\left(4\beta_g \mathrm{B}_E^2 + \left(1-\beta_g^2\right)^2 \mathrm{B}_E^4\right)^{1/2} - \mathrm{B}_E^2\left(1+\beta_g^2\right)}{2\left(1-\beta_g \mathrm{B}_E^2\right)}, \qquad \mathrm{B}_E = \left(\frac{E_\perp^2}{2\pi n_0 m_\alpha c^2}\right)^{1/2}$$

$$\mathcal{E}_\alpha = m_\alpha c^2\left(\frac{1+\beta^2}{1-\beta^2}\right) = \begin{cases} \left(\frac{I}{2.5\times 10^{21} W/cm^2}\right)\left(\frac{10^{21} cm^{-3}}{n_0}\right)\mathrm{GeV} & \text{for} \quad \mathrm{B_E} \ll 1 \\ \left(\frac{m_\alpha}{m_p}\right)^{1/2}\left(\frac{I}{5\times 10^{21} W/cm^2}\right)^{1/2}\left(\frac{10^{21} cm^{-3}}{n_0}\right)^{1/2}\mathrm{GeV} & \text{for} \quad \mathrm{B_E} \gg 1 \end{cases}$$

L. O. Silva, et al., Phys. Rev. Lett. 92, 015002 (2004); N. M. Naumova, et al., Phys. Rev. Lett. 102, 025002 (2009); S. V. Bulanov et al., Phys. Plasmas 19, 103105 (2012); A.P.L.Robinson et al., Plasma Phys. Control. Fusion 54, 115001 (2012)

Instability

http://www.akame48taki.com/home.html

赤目四十八滝

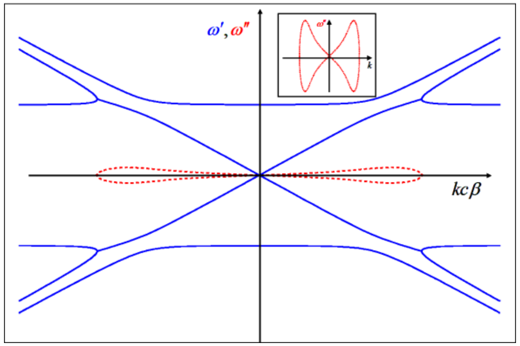
Instability of Counter-Propagating Ion Beams

Dispersion equation for the **Electrostatic Mode** $\omega(k_\parallel)=\omega'(k_\parallel)+i\omega''(k_\parallel)$ for real $k_\parallel$:

$$1-\frac{\omega_{pe}^2}{\omega^2}-\frac{\omega_{p\alpha}^2}{2}\left[\frac{1}{\left(\omega-k_\parallel c\beta\right)^2}+\frac{1}{\left(\omega+k_\parallel c\beta\right)^2}\right]=0$$

$$\max\{\mathrm{Im}[\omega]\}\underset{k_\parallel\approx\omega_{pe}/c\beta}{\approx}\left(\frac{\omega_{pe}\omega_{p\alpha}^2}{4}\right)^{1/3},\quad \mathrm{Im}[\omega]\underset{k_\parallel\to 0}{\approx}\left(\frac{\omega_{pe}}{\omega_{p\alpha}}\right)k_\parallel c\beta$$

$$f_L\propto\exp\left[\frac{\omega''\left(t-\beta x/c\right)-i\left(\omega' t-k_\parallel x\right)}{\sqrt{1-\beta^2}}\right]$$



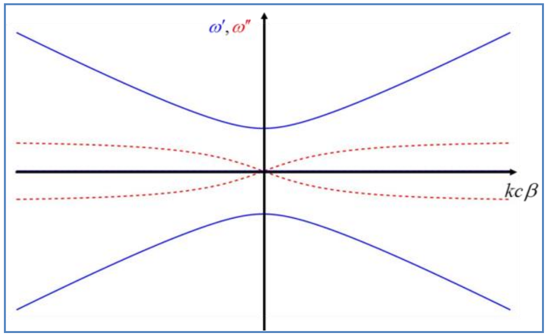
Real (blue) and imaginary parts (dashed, red)
of the frequency vs the wave number for ES mode

Dispersion equation for the **Electromagnetic Mode** $\omega(k_\perp)=\omega'(k_\perp)+i\omega''(k_\perp)$ for real $k_\perp$:

$$\omega^2\left(\omega^2-k_\perp^2c^2-\omega_{pe}^2-\omega_{p\alpha}^2\right)-\left(\omega^2+k_\perp^2c^2\beta^2\right)\omega_{p\alpha}^2=0$$

$$\max\{\mathrm{Im}[\omega]\}\underset{k_\perp\to\infty}{\approx}\omega_{p\alpha}\beta \qquad \mathrm{Im}[\omega]\underset{k_\perp\to 0}{\approx}\left(\frac{\omega_{pe}}{\omega_{p\alpha}}\right)k_\perp c\beta$$

$$f_L\propto\exp\left[\frac{\omega''\left(t-\beta x/c\right)}{\sqrt{1-\beta^2}}-i\left(\frac{\omega' t}{\sqrt{1-\beta^2}}-k_\perp y\right)\right]$$



Real (blue) and imaginary parts (dashed, red)
of the frequency vs the wave number for EM mode

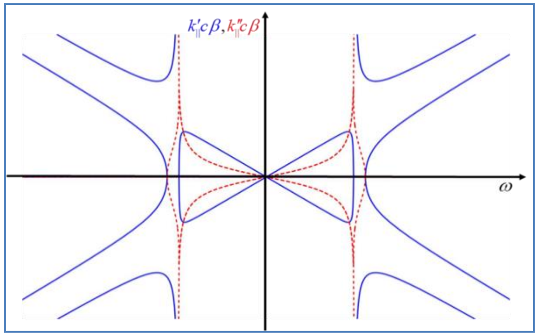
T. Kato, H. Takabe, Phys. Plasmas 17, 032114 (2010); N. J. Sircombe et al., Astron. Astrophys. 452, 371 (2006); R. P. Drake and G. Gregori, ApJ. 749, 171 (2012); F. Fiuza, R. A. Fonseca, J. Tonge, W. B. Mori, and L. O. Silva, Phys. Rev. Lett. 108, 235004 (2012) ; S. V. Bulanov et al., Phys. Plasmas 19, 103105 (2012).

Boundary Problem in the Ion Beam Instability

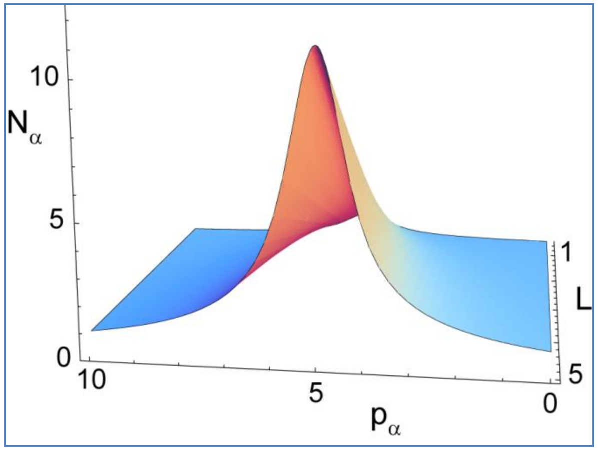
Dispersion equation for the **Electrostatic Mode** $k_{\parallel}(\omega)=k'_{\parallel}(\omega)+ik''_{\parallel}(\omega)$ for real ω :

$$k'_{\parallel}+ik''_{\parallel}=\pm\frac{\omega}{\beta c}\left\{1+\frac{\omega_{p\alpha}^2\pm\omega_{p\alpha}\left[8\left(\omega^2-\omega_{pe}^2\right)+\omega_{p\alpha}^2\right]^{1/2}}{2\left(\omega^2-\omega_{pe}^2\right)}\right\}^{1/2}$$

$$k_{\parallel}\underset{\omega\to 0}{\approx}\pm\frac{\omega}{\beta c}\left(1\pm i\frac{\omega_{p\alpha}}{\omega_{p\alpha}}\right)$$



Real (blue) and imaginary parts (dashed, red) of the wave number vs the frequency for ES mode



PIC Simulation of Collisionless Shock Wave

Laser:

p-pol, super-Gaussian

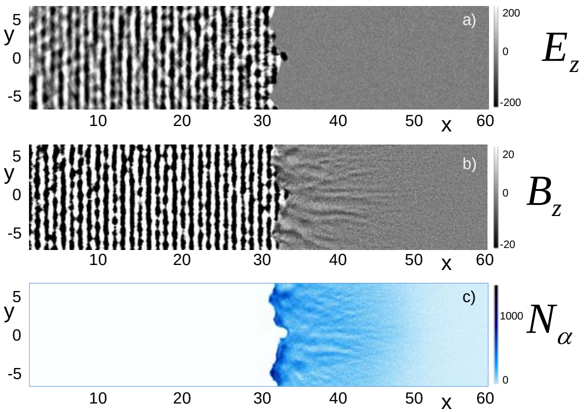
Pulse: $l_x \times l_y = 100 \times 25\lambda^2$

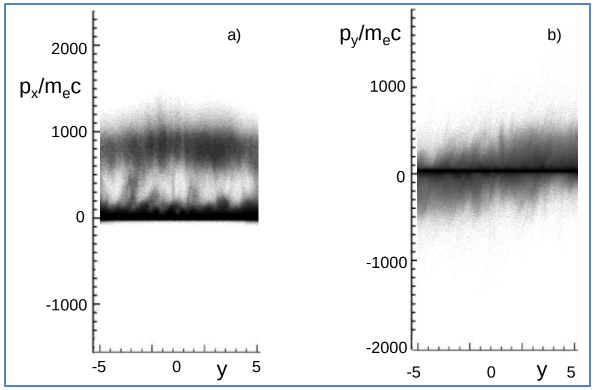
Intensity: $a = 200$

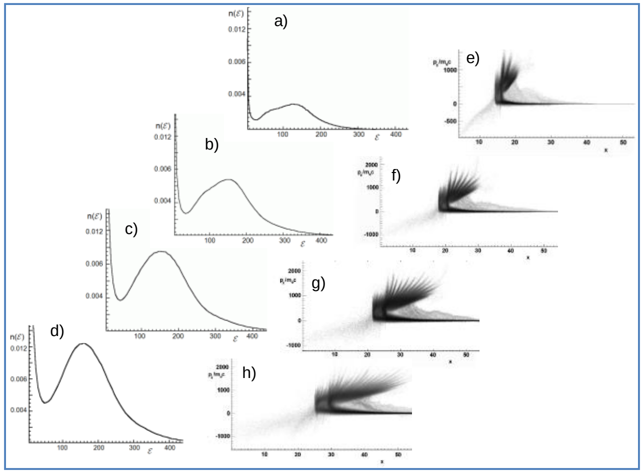
Plasma target:

Density : $n = 256 n_{cr}$

$m_\alpha / m_e = 1836$







Ion phase plane: a) (y, p_x) ; b) (y, p_y) at $t = 100(2\pi/\omega)$

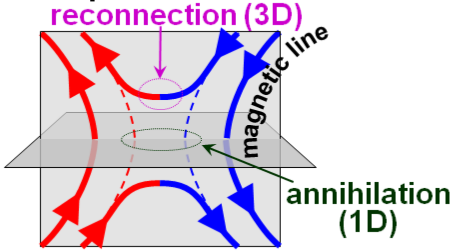
Ion energy spectrum and phase plane (x, p_x) for

$a,e)\ t = 43.5;\ b,f)\ t = 62.5;\ c,g)\ t = 80;\ d,h)\ t = 100$

Electromagnetic Mode ⟷ Magnetic Reconnection

Relativistic Regime of Magnetic Field Annihilation in Laser Plasmas

New relativistic regime of magnetic field annihilation in interaction with plasmas of two parallel ultra-intense femtosecond laser pulses



2D-PIC simulations

Two Gaussian laser pulses, separated by 14 λ.
I=10^{21} W/cm^2, λ=1μm,
$a(t) = a_0 \exp[-(2t/\tau)^2]$
a_0=27, τ = 15 fs, spot 3λ,
Plasma: hydrogen, max density 0.1 n_c

Electron velocity cannot exceed speed of light in vacuum

Electric current density is limited by j_{lim} = enc :

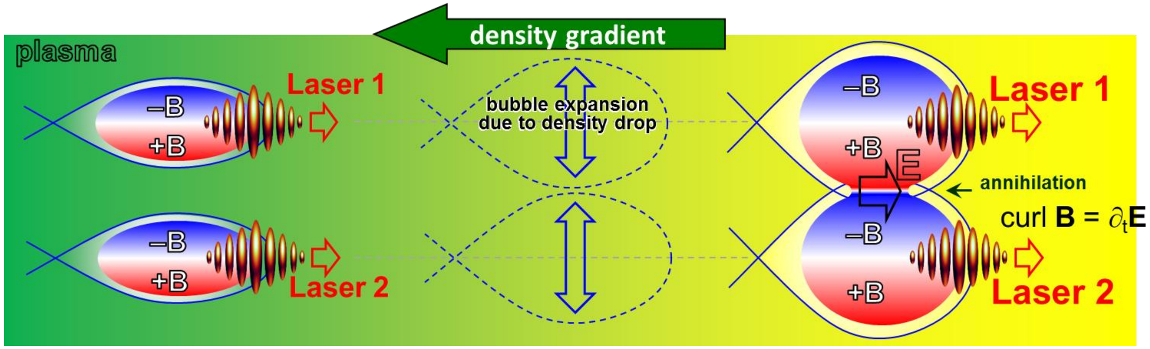
$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{j} + \frac{1}{c}\partial_t \boldsymbol{E}$$

The displacement current effects cannot be neglected

This results in fast magnetic field conversion

to electric field, which accelerates charged particles

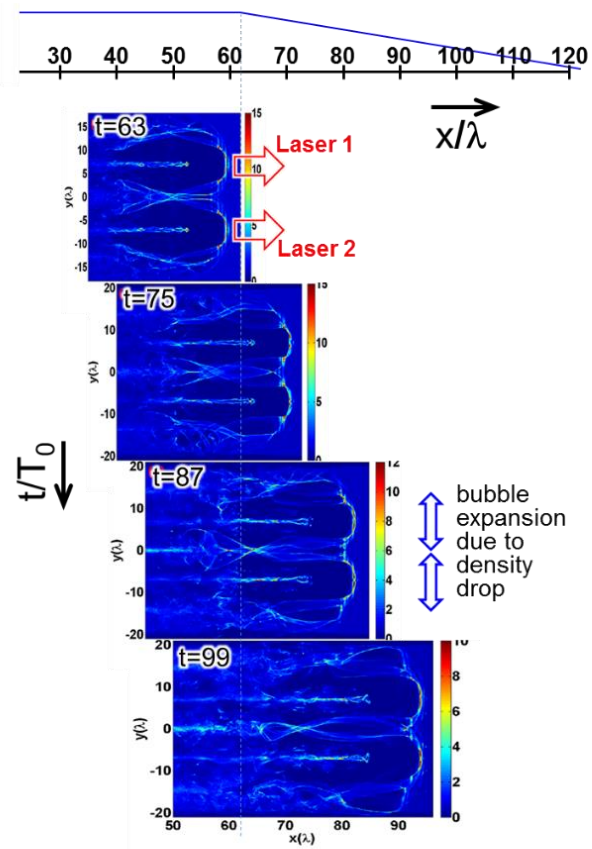
[S. I. Syrovatskii, Sov. Astron. 10, 270 (1966)]



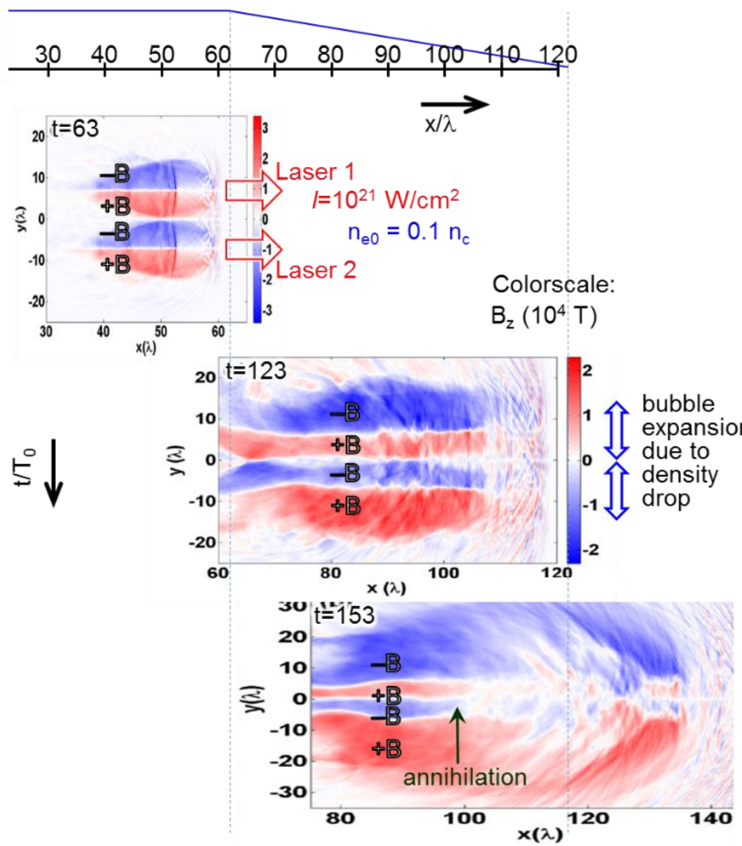
Y. J. Gu, et al, *Phys. Plasmas* 22, 103113 (2015); *Phys. Rev. E* 93, 103203 (2016); *High Power Laser Science and Engineering* 4, e19 (2016)

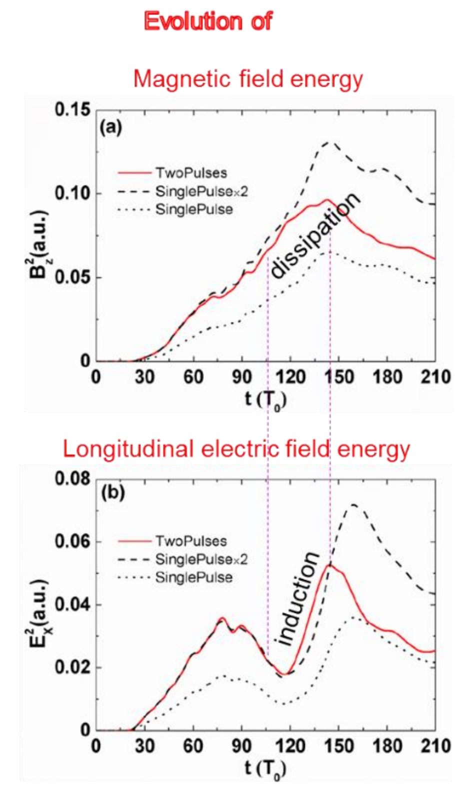
Magnetic field annihilation

Electron density



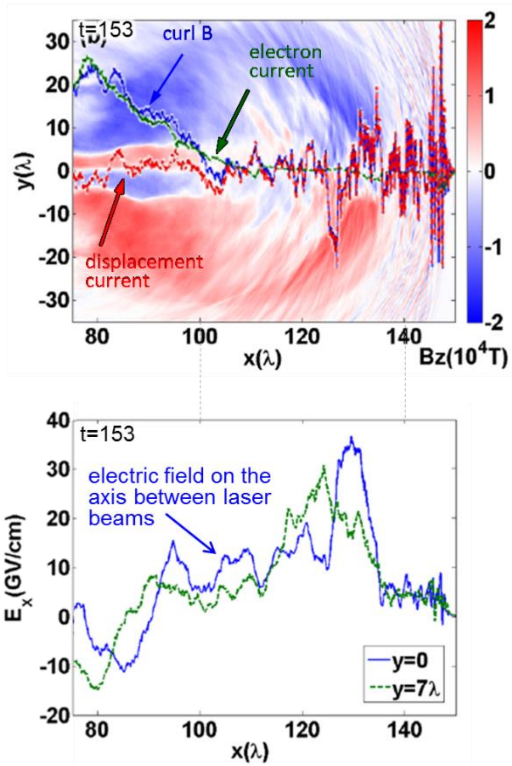
Magnetic field



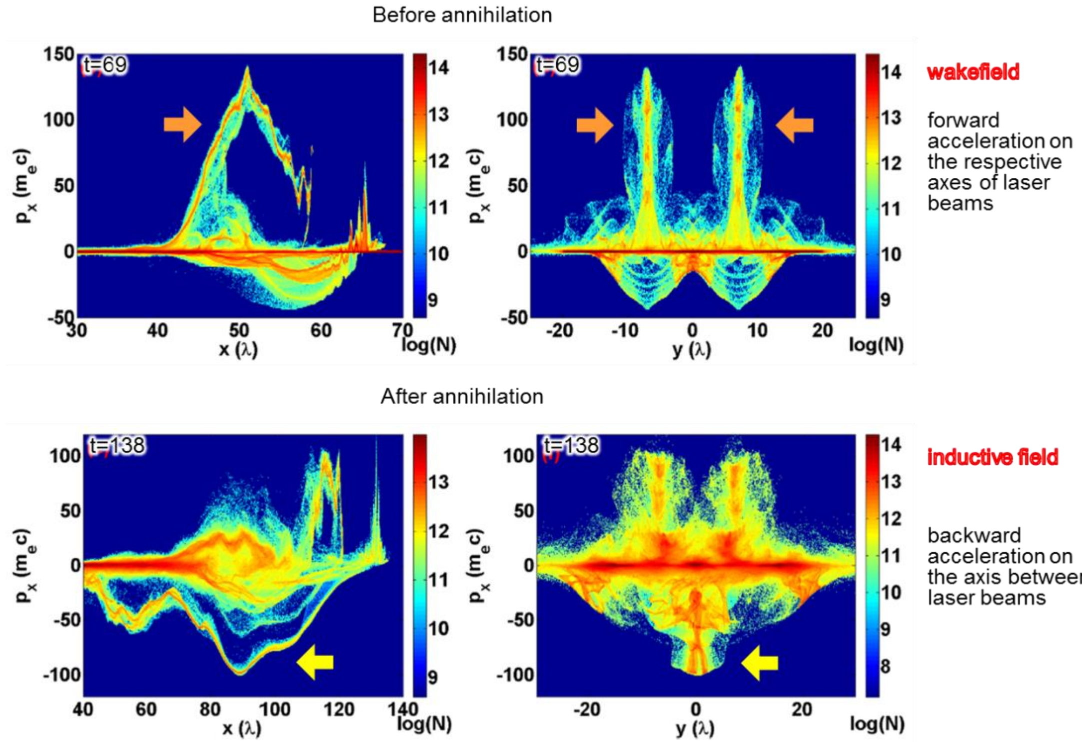


Inductive electric field generation & Electron acceleration

Electric field



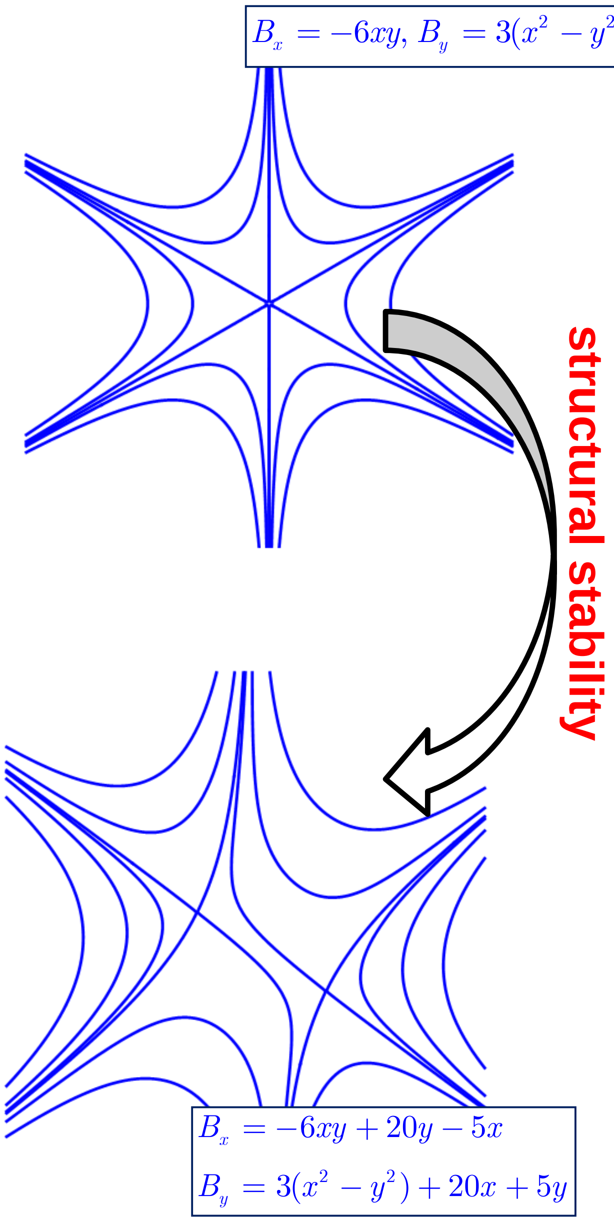
Inductive electric field accelerates electrons – backward

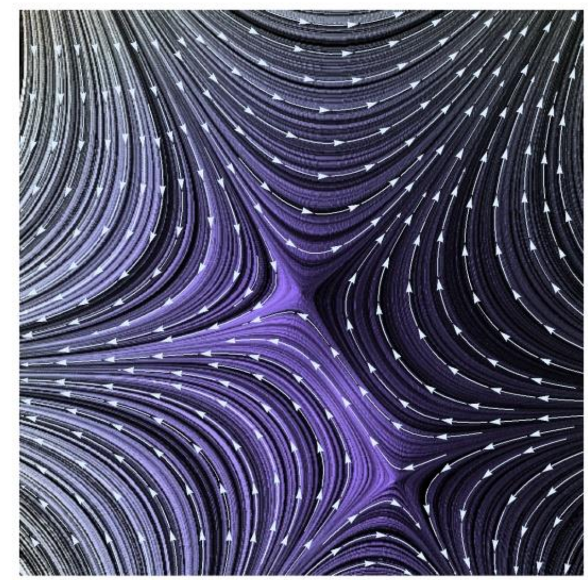


Proposed new regime of magnetic field annihilation in laser plasmas –
efficient charged particle acceleration; relativistic; fast; wide parameter domain.

Important for –
diagnostics, laboratory astrophysics, etc.

On "exponentiation" near null points of magnetic field

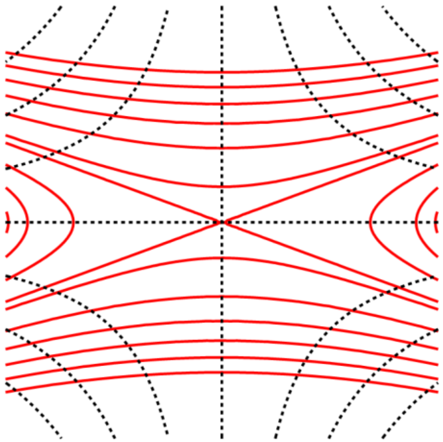




Reasons to consider:

1) Symmetry of Hall reconnection
2) Multiple laser pulse plasma interaction
3) Multiple self-focusing filaments
4) etc

Symmetry of EMHD reconnection



The EMHD equations (in normalized variables) :

$$\partial_t(\mathbf{B}-\Delta\mathbf{B}) = \nabla\times[(\nabla\times\mathbf{B})\times(\mathbf{B}-\Delta\mathbf{B})]$$

In linear approximation the EMHD describes **the whistler waves.**

Magnetic field in 2D configuration

$$\mathbf{B}(x,y,t) = \nabla\times(A_{||}(x,y,t)\,\mathbf{e}_z) + B_{||}(x,y,t)\,\mathbf{e}_z$$

EMHD equations

$$\frac{d}{dt}(A_{||} - \Delta A_{||}) = 0; \quad \frac{d}{dt}(B_{||} - \Delta B_{||}) + \{B_{||},\Delta B_{||}\} = \{A_{||},\Delta A_{||}\}$$

where $d/dt = \partial_t + (\mathbf{v}_\perp\cdot\nabla_\perp)$ and $\{f,g\} = \partial_x f\partial_y g - \partial_x g\partial_y f$ - Poisson brackets

Magnetic field pattern in the x,y plane is given by $A_{||}(x,y,t) = \mathrm{const}$

Electron velocity field for quadruple magnetic field $B_{||} = w\,x\,y$: $\mathbf{v}_\perp(x,y) = \nabla\times B_{||} = w(x,-y)$

Solution of the EMHD equations :

$$A_{||}(x,y,t) = A_0[\exp(-2wt)x^2 - \exp(2wt)y^2 + 4\sinh(2wt)]$$

Magnetic field lines move with respect to magnetic separatrices, i.e. they do reconnect.

This is a simplest example of the magnetic field line reconnection in collisionless plasmas.

Physical mechanism of the reconnection in this case is due to **finite electron inertia**

Nonlinear pile-up of magnetic field near the 0-points in EMHD

The formal solution of EMHD equations is given by the Cauchy formula obtained:

$$B_i(\boldsymbol{x},t) - \Delta B_i(\boldsymbol{x},t) = \frac{\partial x_i}{\partial x_j^0}\left(B_j(\boldsymbol{x}^0,t) - \Delta B_j(\boldsymbol{x}^0,t)\right)$$

The Lagrange variables x_i^0 and the Euler coordinates x_j are related by

$$x_i = x_i^0 + \xi(x^0,t)$$

where $\xi(x^0,t)$ is displacement of electron fluid element from its initial position x_i^0.

From Maxwell equations, $-4\pi e n_e \mathbf{v}/c = \nabla\times\mathbf{B}$, we obtain for $\xi(\mathbf{x^0},t)$ $\quad(\mathbf{v} = \partial_t\xi)$:

$$\partial_t\xi_i = \varepsilon_{ijk}\left|\,\partial x_j^0 / \partial x_l^0\right|\left|\,\partial B_l(\mathbf{x^0},t) / \partial x_k^0\right|$$

Self - similar solution $\qquad B_i(\mathbf{x},t) = B_{ijk}(t)x_j x_k$

corresponds to **null point of third order** (intersection of three separatrices)

Fluid velocity of electron component is a linear function of the coordinates :

$$\partial_t\xi_i = W_{ij}x_j \quad \text{i.e.} \quad x_i = M_{ij}(t)x_j^0, \qquad B_{ikl}(t) = M_{ij}(t)B^{(0)}_{imn}M^{-1}_{mk}(t)M^{-1}_{nl}(t)$$

where deformation matrix $M_{ij}(t)$ obeys equation (ODE)

$$\dot{M}_{ij} = -2\varepsilon_{ikl}M_{lm}M^{-1}_{nk}A^{(0)}_{mnj}$$

Phys. Fluids B 4, 1408 (1992)

Topology Magnetic Field near 3D null points

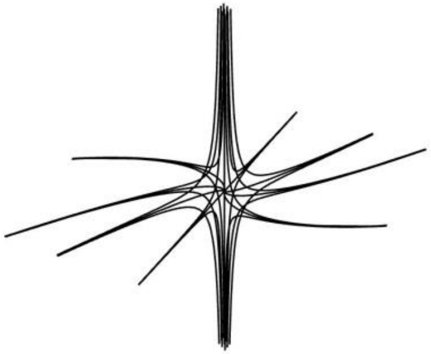
Near null point the magnetic field has a form

$$B_i = B_{ij} x_j + B_{ijk} x_j x_k + \ldots$$

with $B_{ij} = \partial_j B_i \big|_{x=0}$ and $B_{ijk} = \partial_j \partial_k B_i \big|_{x=0} / 2$

If $B_{ij} \neq 0$ for 3D null point of curl-free magnetic field gradient matrix is

$$B_{ij} = \frac{1}{2} \mathrm{diag}\{a^{(2)}, b^{(2)}, -(a^{(2)} + b^{(2)})\}$$

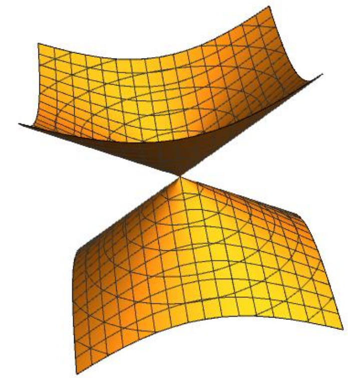


Topology Magnetic Field near 3D null points

Curl-free vector field is equal to the gradient field: $\mathbf{B} = \nabla F$ with the potential $F(x,y,z)$.

For the second order magnetic field we have

$$F^{(2)}(x,y,z) = \frac{1}{2}[a^{(2)}(x^2 - z^2) + b^{(2)} \;\; y^2 - z^2 \;\;]$$

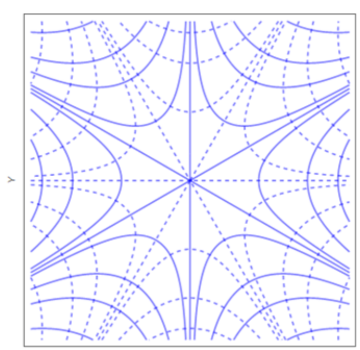


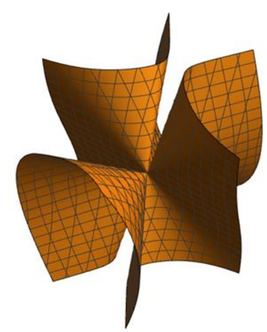
$F^{(2)}(x,y,z) = 0$

The third order field is given by

$$F^{(3)}(x,y,z) = \frac{(a^{(3)} + b^{(3)})}{3} z^3 - z(a^{(3)}x^2 + b^{(3)}y^2) + \frac{(c^{(3)} + d^{(3)})}{3} x^3$$

$$- x(c^{(3)}y^2 + d^{(3)}z^2) + \frac{(e^{(3)} + f^{(3)})}{3} y^3 - y(e^{(3)}z^2 + f^{(3)}x^2)$$





$F^{(3)}(x,y,z) = 0$

On the “exponentiation”

According to A. Boozer the "exponentiation" is an important property of the magnetic field topology showing where the magnetic reconnection can occur.

Equations for magnetic field lines are

$$\frac{dx_i}{ds} = B_i$$

where s is the parameter.

The distance between two points on the curve δx_i $(x_i = x_{0,i} + \delta x_i)$ can be found from

$$\frac{d(x_{0,i} + \delta x_i)}{ds} = B_i(x_{0,i} + \delta x_i) \approx B_i(x_{0,i}) + \delta x_j \partial_j \left. B_i \right|_{x_{0,i}}$$

which gives the equation

$$\frac{d\delta x_i}{ds} = \delta x_j \partial_j \left. B_i \right|_{x_{0,i}}$$

On the “exponentiation” near 2D null line

$$\partial_j B_i = \begin{pmatrix} \partial_x B_x & \partial_y B_x & \partial_z B_x \\ \partial_x B_y & \partial_y B_y & \partial_z B_y \\ \partial_x B_z & \partial_y B_z & \partial_z B_z \end{pmatrix}$$

Following to A. Boozer, we consider the hyperbolic magnetic field given by

$$\boldsymbol{B} = \left(\nabla \times A_z \boldsymbol{e}_z + B_z \boldsymbol{e}_z \right)$$

with

$$A_z(x, y) = h\ xy$$

This gives for $\partial_j B_i = \mathrm{diag}\{h, -h, 0\}$ and for δx_i

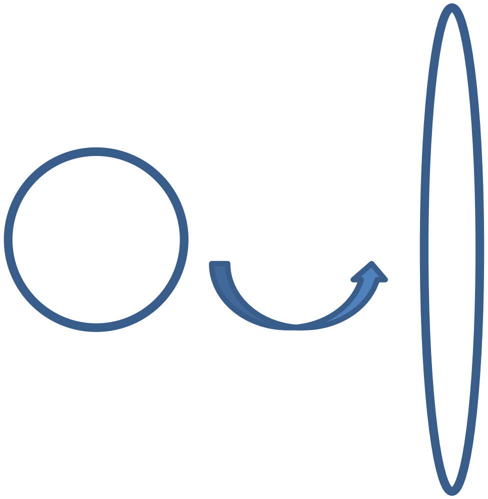
$$\delta x = \delta x_0 e^{hl/B_z}$$

$$\delta y = \delta y_0 e^{-hl/B_z}$$

$$\delta z = \delta z_0 + l$$

Here l is the magnetic field line length

$$l = \int \sqrt{\sum_i \left(\frac{dx_i}{ds} \right)^2} ds \approx B_z s$$



This is an indication on the current sheet formation

On the “non-exponentiation” near 3D null point

For 3D null point of curl-free magnetic field given by the gradient matrix

$$B_{ij} = \frac{1}{2}\mathrm{diag}\{a^{(2)}, b^{(2)}, -(a^{(2)} + b^{(2)})\}$$

equation for δx_i takes the form

$$\frac{d\delta x}{ds} = a^{(2)}\delta x$$

$$\frac{d\delta y}{ds} = b^{(2)}\delta y$$

$$\frac{d\delta z}{ds} = -(a^{(2)} + b^{(2)})\delta z$$

If $a^{(2)} > b^{(2)}$ than the magnetic field line length

$$l \approx \frac{1}{a^{(2)}} e^{a^{(2)}s}$$

This yields non-exponential dependence of δx_i on the magnetic field line length

$$\delta x = \delta x_0 \; a^{(2)} l$$

$$\delta y = \delta y_0 (a^{(2)} l)^{b^{(2)}/a^{(2)}}$$

$$\delta z = \delta z_0 (a^{(2)} l)^{-(a^{(2)} + b^{(2)})/a^{(2)}}$$

On “exponentiation” near high-order null points

For the null point of magnetic field of the 3rd order given by

$$\boldsymbol{B} = \nabla \times A_z \boldsymbol{e}_z + B_z \boldsymbol{e}_z$$

with

$$A_z(x, y) = h(x^3 - 3xy^2)$$

the gradient matrix is

$$\partial_j B_i = \begin{pmatrix} -6hy & -6hx & 0 \\ -6hx & 6hy & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

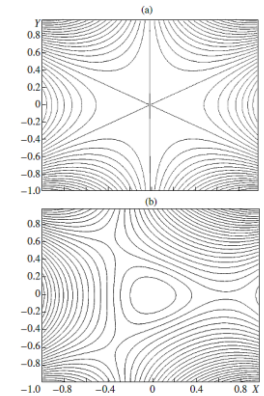
Along the field line $y = \sqrt{(x^3 - A/h)/3x}$.

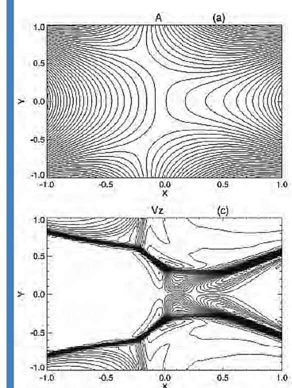
On the separatrix $(y = \pm x/\sqrt{3})$ we have

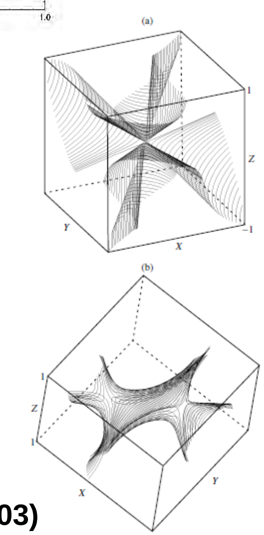
$$x = \frac{x_0}{1 \pm 6x_0 s/\sqrt{3}} \approx \frac{x_0}{1 \pm 6x_0 l/B_z\sqrt{3}}$$

and

$$\begin{pmatrix} \delta x \\ \delta y \end{pmatrix} \propto \exp\left(\pm \frac{2l(1 \pm l\,x_0/3B_z)}{\sqrt{3}x_0 h B_z} \right)$$







PPR (2000); Dokl. Phys. (2003)

Finale

Homeric Iliad (Thetis to Achilles):

Swift-Footed, Shepherd of the People, Achilles was to choose between a long inglorious life and a short but glorious life

Finale

Francesco Pegoraro has chosen a long glorious scientific life.

We wish him keep going along this line!

Thank you for listening to me !

ELITRANS H2020 Project on the ELI roadmap

Transition from distributed implementations towards integrated and unified operation