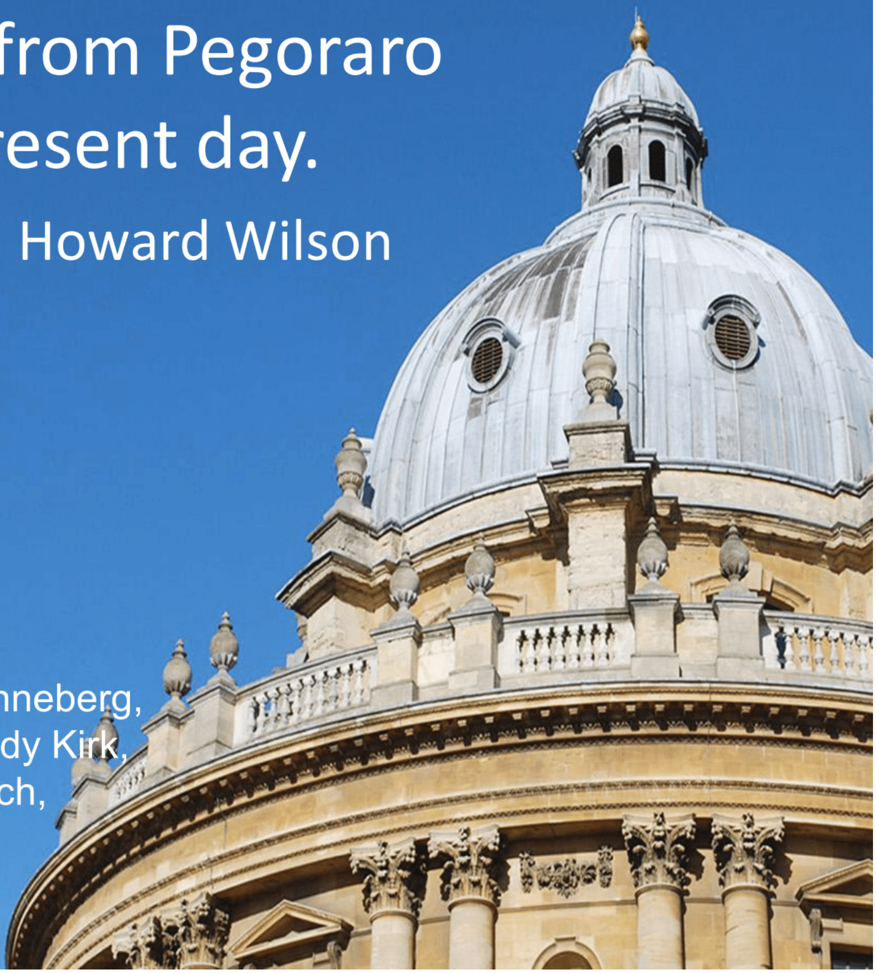
Ballooning modes: from Pegoraro and Schep to the present day.

Steve Cowley, Chris Ham, Howard Wilson

Culham/Oxford/York



Thanks to: Anna Thackray, Sophia Henneberg, Samuli Saarelma, Omar Hurricane, Andy Kirk, Bryan Fong, Mehmet Artun, Colin Roach, James Harrison, Brendan Cowley, David Dickinson, MAST team ……

1965 Culham IAEA

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Ballooning Mode is Born

1. B. Coppi and M. N. Rosenbluth, *in* "Proceedings of the 1965 International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Culham, England, 1965," Papers Nos. CN-21/105 and CN-21/106, International Atomic Energy Agency, Vienna, 1966.
2. R. M. Kulsrud, *in* "Proceedings of the 1965 Conference on Plasma Physics and Controlled Nuclear Fusion Research, Culham, England, 1965," International Atomic Energy Agency, Vienna, 1966, Paper No. CN-21/113.

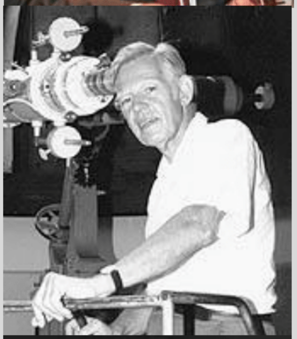
Twisting Slice

Gravitational Resistive Instability of an Incompressible Plasma in a Sheared Magnetic Field

K. V. Roberts and J. B. Taylor

United Kingdom Atomic Energy Authority, Culham Laboratory, Abingdon, Berkshire, England

(Received 13 July 1964; final manuscript received 5 October 1964)



1978-79

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SHEAR AND THE SPATIAL PERIODICITIES OF MODES WITH LONG WAVELENGTHS PARALLEL TO THE MAGNETIC FIELD OF TOKAMAKS

by

Francesco Pegoraro
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Rijnhuizen Report 78-116

ANNALS OF PHYSICS **121**, 1–31 (1979)

Analytical Representation and Physics of Ballooning Modes

B. Coppi and J. Filreis

Massachusetts Institute of Technology, Cambridge, Massachusetts

and

F. Pegoraro

Scuola Normale Superiore, Pisa, Italy

Received September 18, 1978

$$\gamma^2(t) = \frac{t}{\tau_E \tau_A^2}$$

Passing slowly through marginal stability

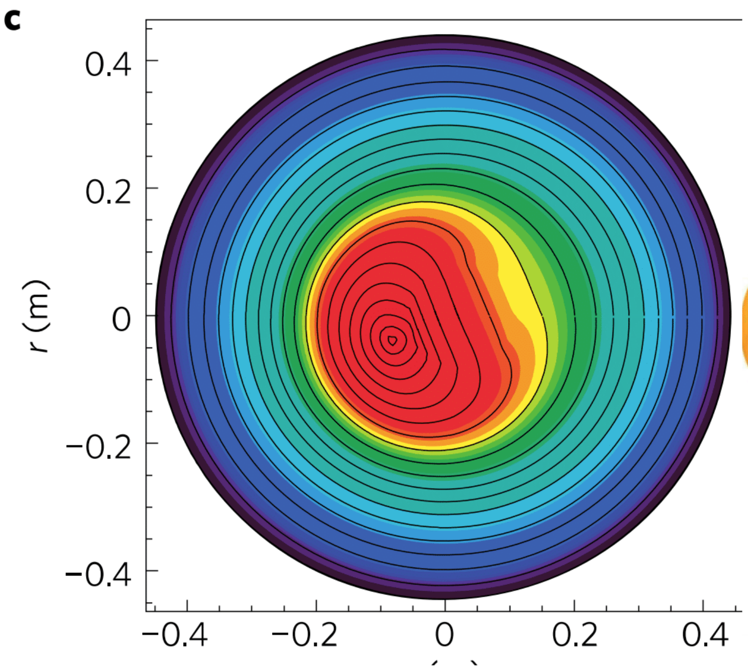
$$\tau_E \gg \tau_A$$ Alfven time

$$\frac{d^2A}{dt^2} = \gamma^2(t)A - \frac{1}{\tau_A^2}A|A|^2 \quad \rightarrow |A|^2 = \frac{t}{\tau_E}$$

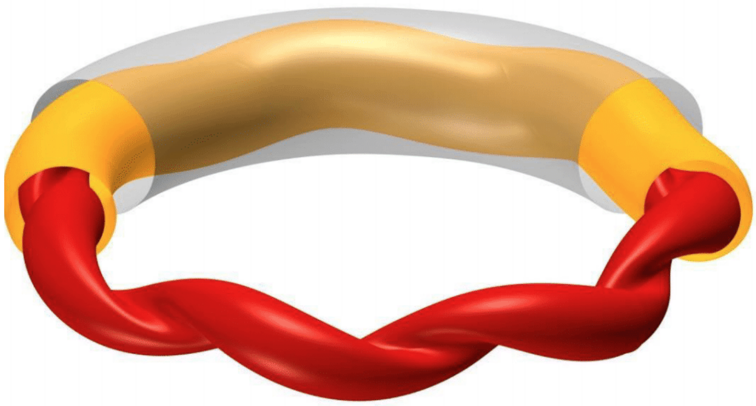
Critical amplitude for nonlinearity $A = \frac{\xi}{L} = (\frac{\tau_A}{\tau_E})^{1/3} \sim 1 - 3\%$

Slow growth

Long Lived Mode, saturated kink modes



Critical Current triggers helical state



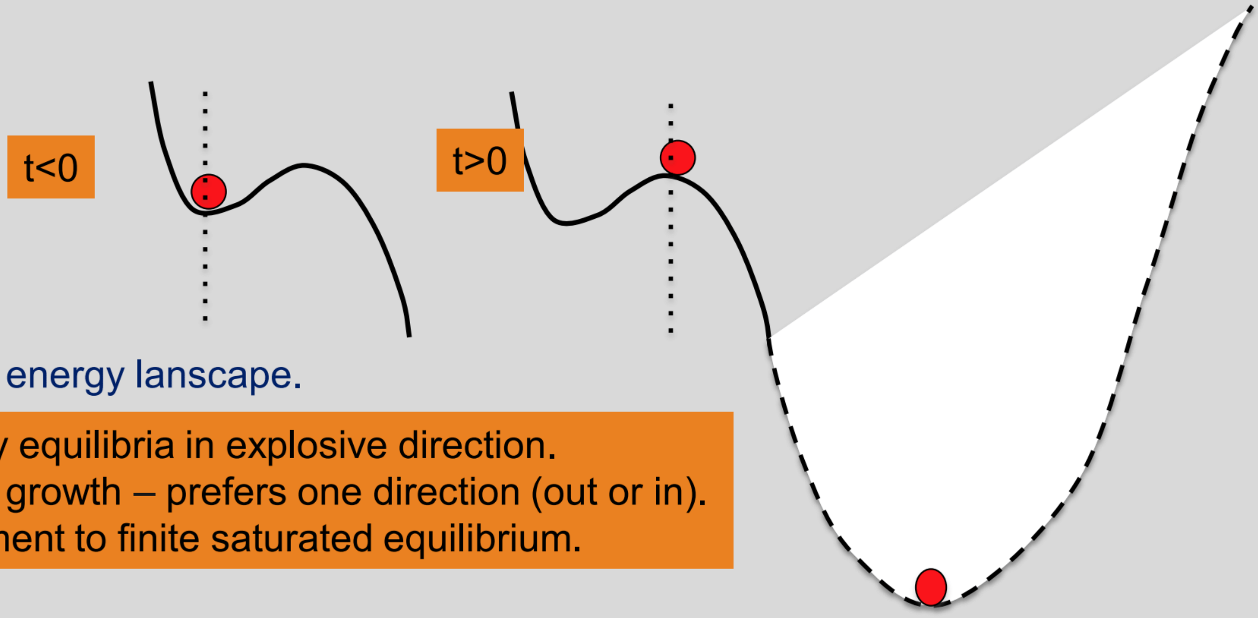
Constant helical flux surfaces define the topology of plasma equilibrium: all measured quantities can be correctly interpreted in terms of the dominant helicity.

R. Lorenzini *et al.*, Nature Physics **5** (2009) 570-574

$$\frac{d^2A}{dt^2} = {}^2(t)A + \frac{A^2}{{}^2_A}$$

$$\gamma(t)^2 \sim \frac{t}{\tau_E \tau_A^2}$$

$$A \quad \frac{6 \; {}_A^2}{(t \quad t_0)^2}$$



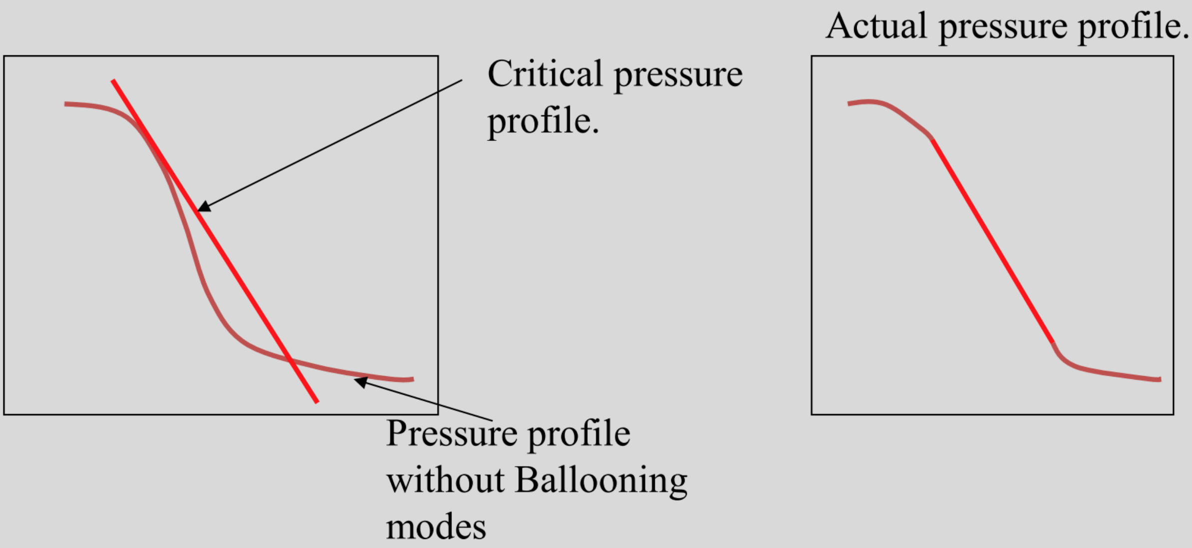
The energy lanscape.

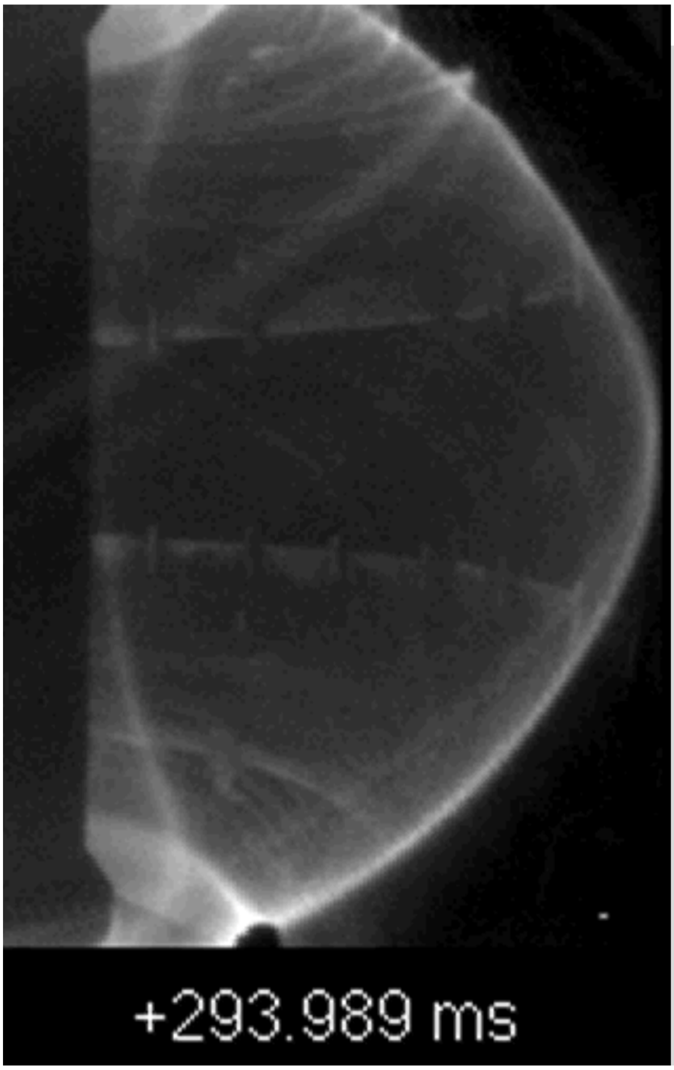
No nearby equilibria in explosive direction.
Explosive growth – prefers one direction (out or in).
Displacement to finite saturated equilibrium.

Transport Critical Gradient

Critical gradient argument – <u>instability pins profile to local marginal stability</u>.
Used in solar convection – tokamak ion temperature gradient modes … etc.

argument for Ballooning modes due to Connor, Taylor and Turner 1984
And for stellarators Kulsrud and Ho 1985





James Harrison

Total duration of eruption about 80 μs.

Rapid 20 μs initial phase.

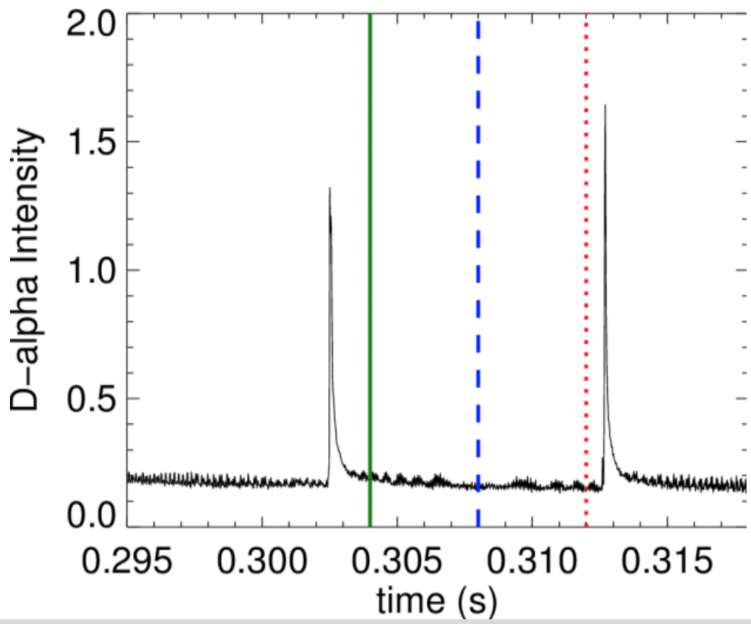
Alfven speed 1 metre per μs (10^6m/s).

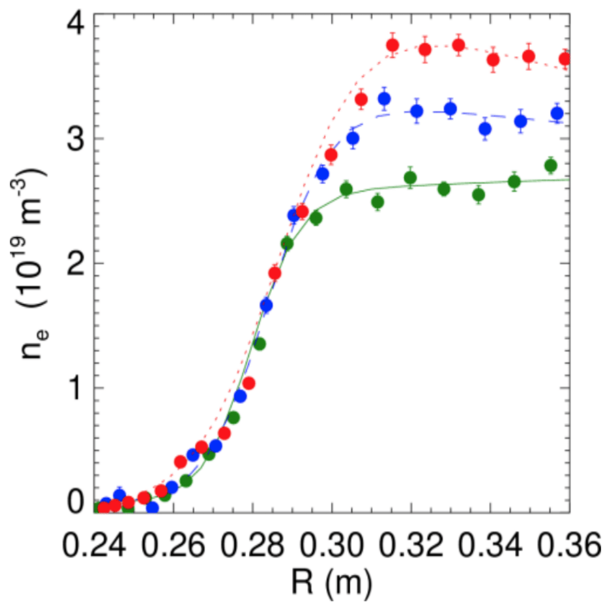
Approximate Field line length top to bottom 5m.

Initial phase is only a few Alfven times.

Leaning against the ballooning boundary?

MAST data: Rory Scannel et. al.

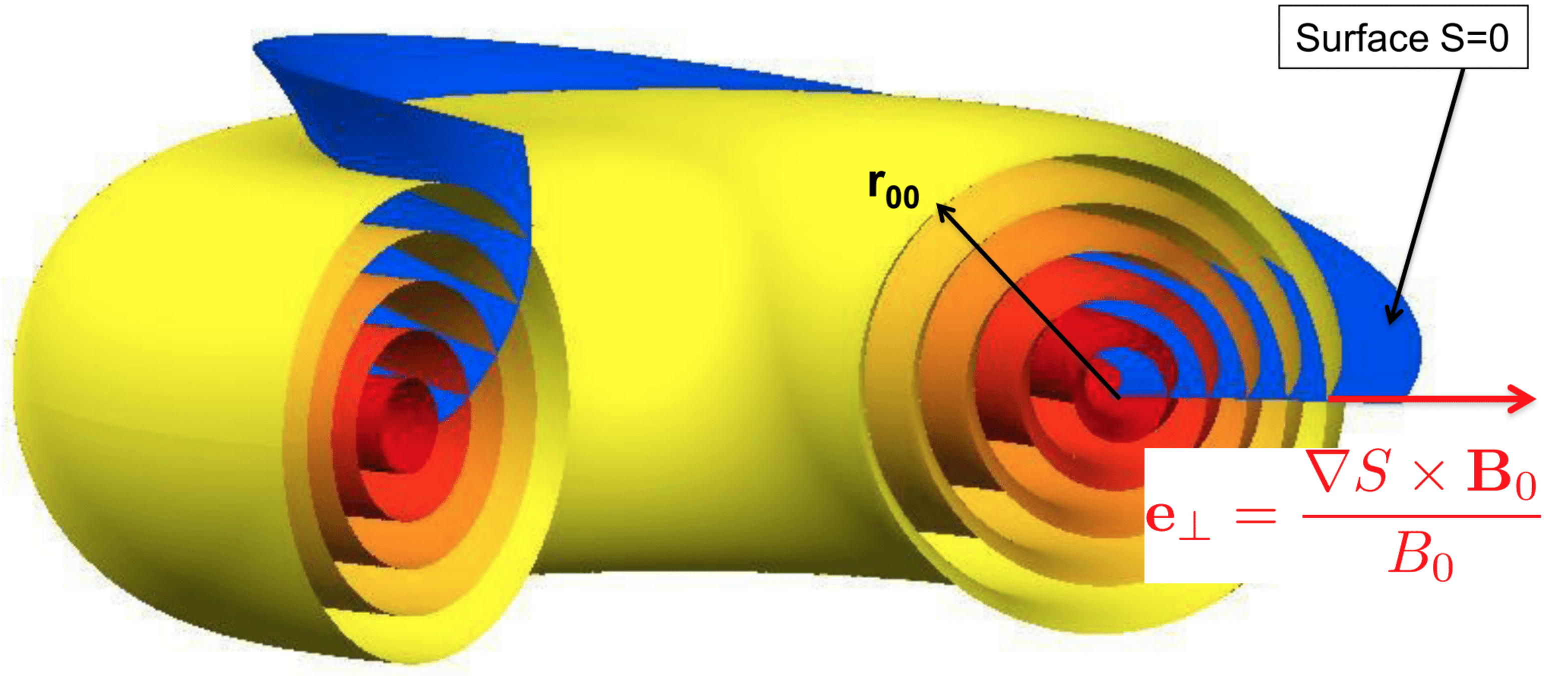




Red line is approximately marginally stable to ideal MHD instability.

What do the ballooning modes do?

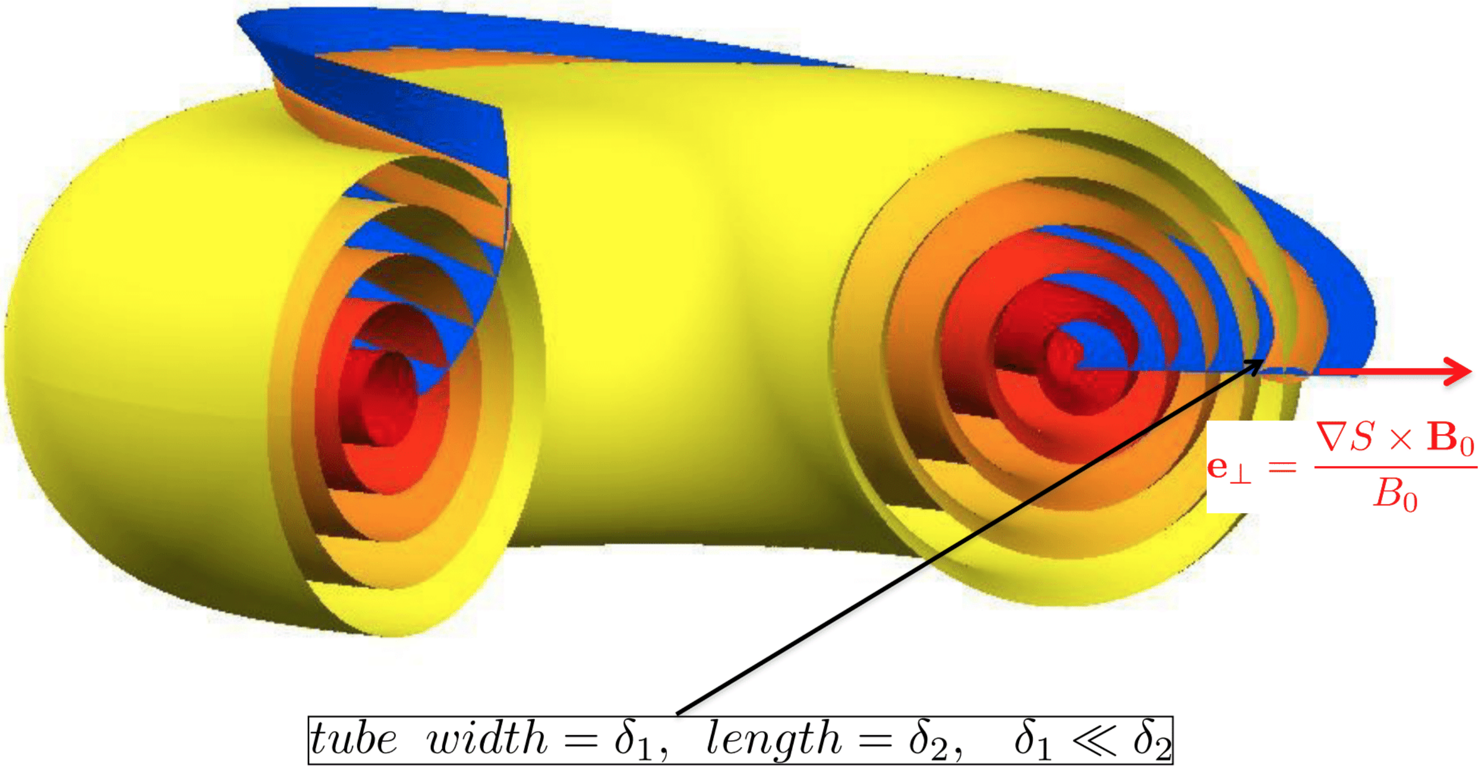
$$\mathbf{B}_0 = -\bar{B}_0 R_0\{f(r)\nabla r \times \nabla S\} \qquad S = \phi - q(r)\theta$$



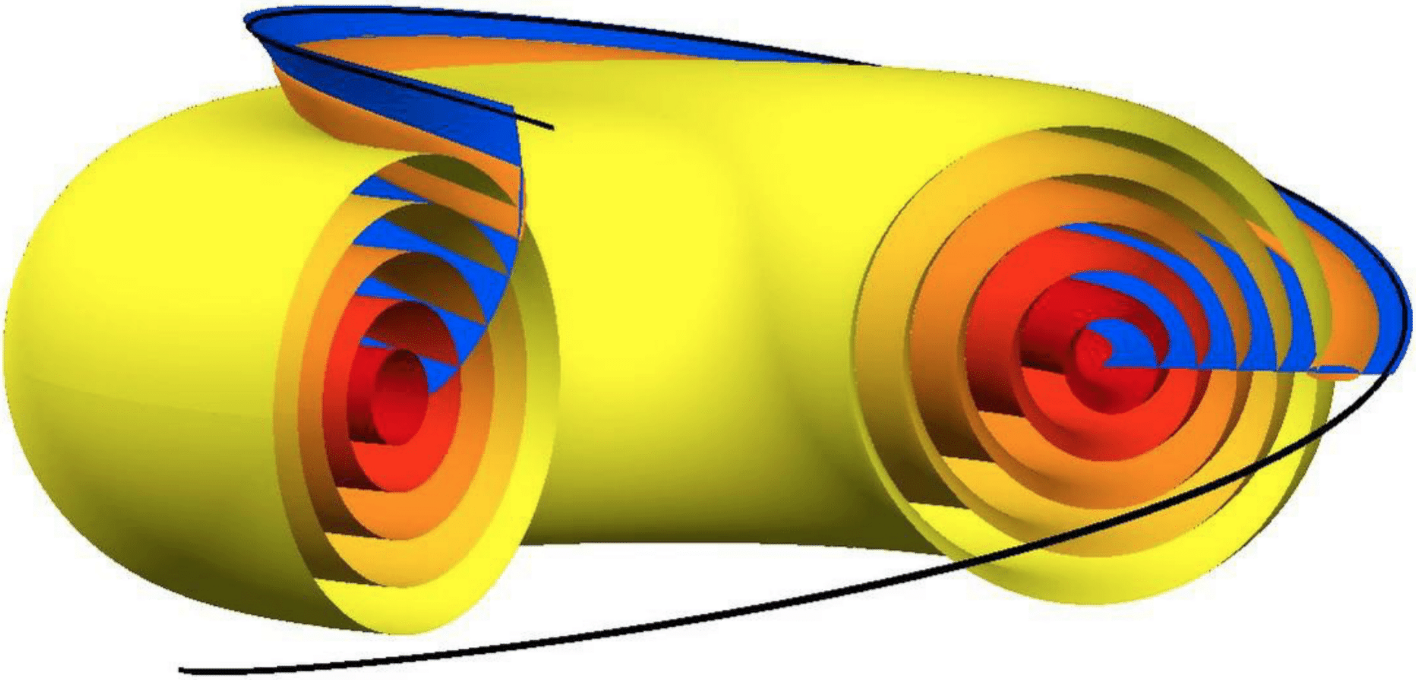
$$r = flux\ surface\ label, \quad \phi = toroidal\ angle,$$

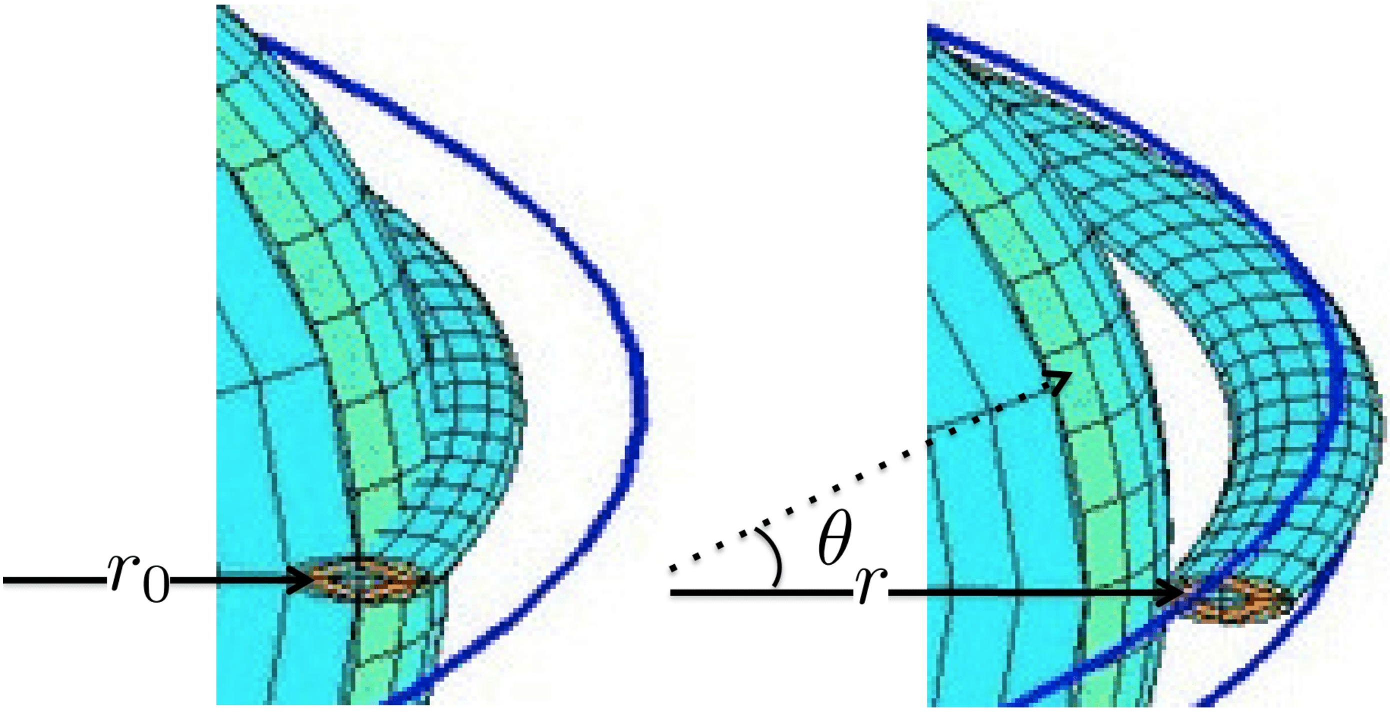
$$\theta = straight\ field\ line\ poloidal\ angle$$

Consider elliptical flux tube aligned along surface S



Elliptical flux tube slides out along surface S parting field lines





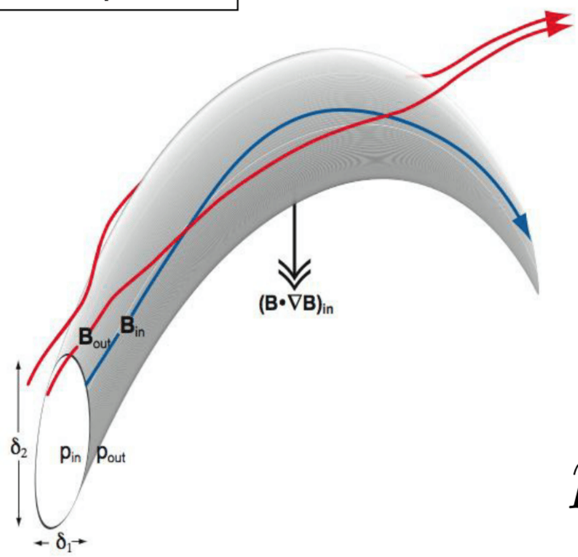
r_0 original flux surface of a field line

$$r = r(r_0, \theta, t),\ \phi = q(r)\theta$$

Equations for perturbed field line

Force across the Tube

Rotated picture



Field lines outside tube effectively unperturbed if

$$\delta_1^2 L \ll \delta_2^3$$

$Linearly \quad \delta_1 \sim Ln^{-1} \quad \delta_2 \sim Ln^{-1/2}$

Force across tube →

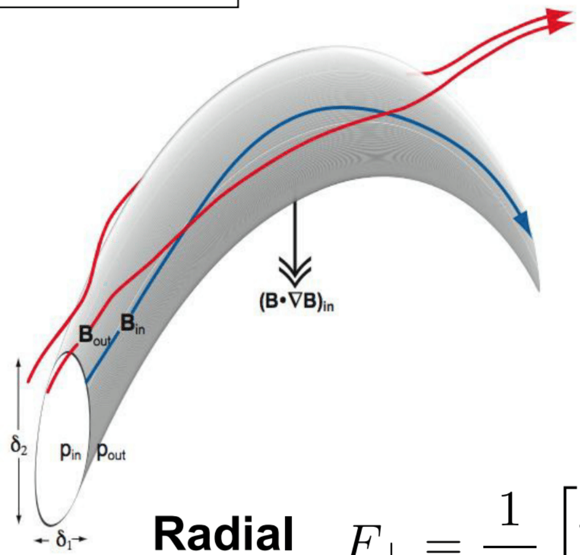
$$p_{in} + \frac{B_{in}^2}{2} = p_{out} + \frac{B_{out}^2}{2},$$

Time to equalise pressure along tube $\longrightarrow p_{in} = p(r_0)$

Unperturbed field and pressure outside tube $\rightarrow p_{out} = p(r) \quad B_{out} = B_0(r, \theta)$

Force on the Tube

Rotated picture



$$p_{in} + \frac{B_{in}^2}{2} = p_{out} + \frac{B_{out}^2}{2},$$

Radial Force

$$F_\perp = \frac{1}{\mu_0}\left[\mathbf{B}_{in}\cdot\nabla\mathbf{B}_{in} - \nabla\left(\frac{B_{in}^2}{2} + \mu_0 p_{in}\right)\right]\cdot\mathbf{e}_\perp$$

$$= \frac{1}{\mu_0}\left[\mathbf{B}_{in}\cdot\nabla\mathbf{B}_{in} - \mathbf{B}_0\cdot\nabla\mathbf{B}_0\right]\cdot\mathbf{e}_\perp.$$

We can get this from shape and strength of field

"Archimedes" buoyancy force

Ballooning Equation

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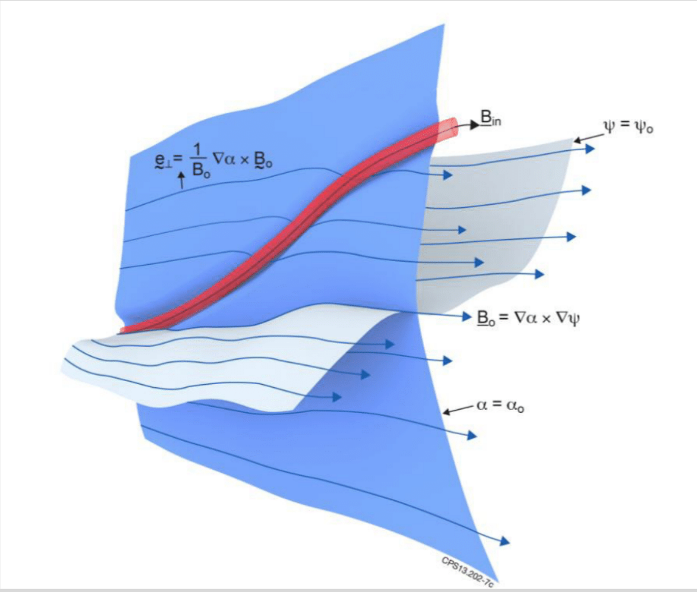


$$l = length\ along\ field\ line\ and \quad \psi = \psi(l, \psi_0, t)$$

$$\mathbf{B}_{in} = B_\| \mathbf{B_0} + B_\perp \mathbf{e}_\perp$$

$$B_\| = B_\|(\psi_0, l) \quad B_\perp = B_\perp(\psi_0, l)$$

Equlibrium

$$B_\|^2 = 1 + 2(p_0(\psi) - p_0(\psi_0)) - B_\perp^2 \frac{|\mathbf{e}_\perp|^2}{B_0^2}$$

$$(\frac{\partial\psi}{\partial l})_{\psi_0} = \frac{\mathbf{B}_{in} \cdot \nabla\psi}{\mathbf{B}_{in} \cdot \nabla l} = \frac{-B_\| B_0}{B_\| \mathbf{B}_0 \cdot \nabla l + B_\perp \mathbf{e}_\perp \cdot \nabla l}$$

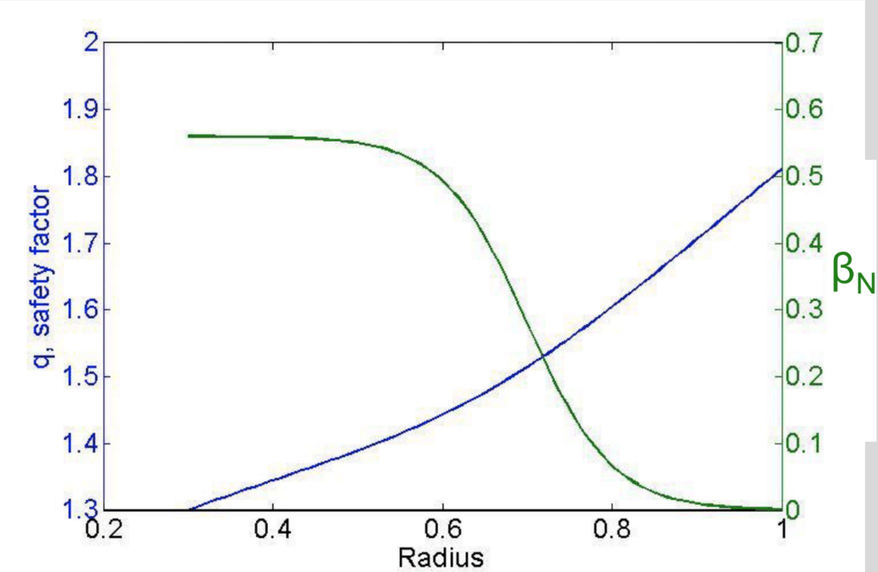
$$F_\perp = \quad (B_\|^2 - 1)(\mathbf{B}_0 \cdot \nabla \mathbf{B}_0) \cdot \mathbf{e}_\perp + B_\| B_0 \mathbf{B}_0 \cdot \nabla \left(\frac{|\mathbf{e}_\perp|^2}{B_0} B_\perp \right) + B_\perp \mathbf{e}_\perp \cdot \nabla \left(B_\perp \mathbf{e}_\perp \right) \cdot \mathbf{e}_\perp = 0$$

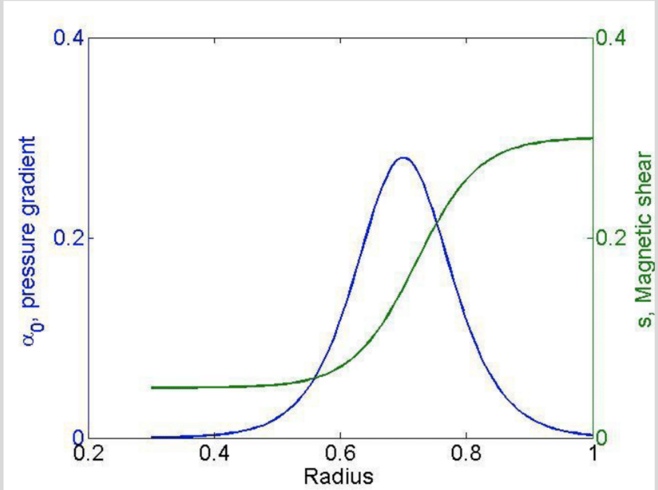
Ballooning Equation,

$$(B_\| \mathbf{B}_0 \cdot \nabla l + B_\perp \mathbf{e}_\perp \cdot \nabla l)|\mathbf{e}_\perp|^2 (\frac{\partial B_\perp}{\partial l})_{\psi_0} = (B_\|^2 - 1)(\mathbf{B}_0 \cdot \nabla \mathbf{B}_0) \cdot \mathbf{e}_\perp + B_\| B_\perp B_0 \mathbf{B}_0 \cdot \nabla \left(\frac{|\mathbf{e}_\perp|^2}{B_0} \right) + B_\perp^2 \mathbf{e}_\perp \cdot \nabla |\mathbf{e}_\perp|$$

Generalized s - α

We take circular flux surfaces with a narrow region with a small pressure jump.
This models the internal transport barrier – in this talk we take:





$$\beta_N(r) = 2R_0 q^2 \frac{p_0(r)}{\bar{B}_0^2}, \quad \alpha(r) = -\frac{d\beta_N(r)}{dr}, \quad s(r) = rq'(r)/q(r)$$

$$In \;\; layer \;\; \alpha \sim \mathcal{O}(1) \; and \; s \sim \mathcal{O}(1)$$

Generalized s - α

Drag evolution $\nu \mathbf{v} \cdot \mathbf{e}_\perp = F_\perp$ yields nonlinear s-alpha equation for $r = r(r_0, \theta, t)$.

$$\nu' \left(\frac{\partial r}{\partial t}\right) \left[1 + (\alpha \sin\theta - s\theta)^2)\right] = F'_\perp = (\beta_N(r_0) - \beta_N(r)) \left[\cos\theta + \sin\theta(s\theta - \alpha\sin\theta)\right]$$

$$+ \left(\frac{\partial}{\partial\theta}\right)_{r_0} \left(\left[1 + (\alpha\sin\theta - s\theta)^2)\right] \left(\frac{\partial r}{\partial \theta}\right)_{r_0} \right) - \frac{1}{2} \left(\frac{\partial r}{\partial\theta}\right)^2_{r_0} \left(\frac{\partial}{\partial r}\right)_\theta (\alpha \sin\theta - s\theta)^2$$

$$\beta_N(r) = 2R_0 q^2 \frac{p_0(r)}{\bar{B}_0^2}, \quad \alpha(r) = -\frac{d\beta_N(r)}{dr}, \quad s(r) = rq'(r)/q(r)$$

Linearising yields familiar s-α force equation to test for linear instability.

Generalised s - α

We also define an nonlinear potential energy for this $r = r(r_0, \theta, t)$

$$\Delta\mathcal{E} = \int_{-\infty}^{\infty} d\theta \left[\frac{1}{2} \left(\frac{\partial r}{\partial \theta} \right)_{r_0}^{2} \left(1 + (\alpha \sin\theta - s\theta)^2\right) \right]$$

$$- \int_{-\infty}^{\infty} d\theta \left[\left(\mathcal{A}(r, r_0) \cos\theta + \mathcal{B}(r, r_0)\theta \sin\theta - \mathcal{C}(r, r_0)(\sin\theta)^2 \right) \right]$$

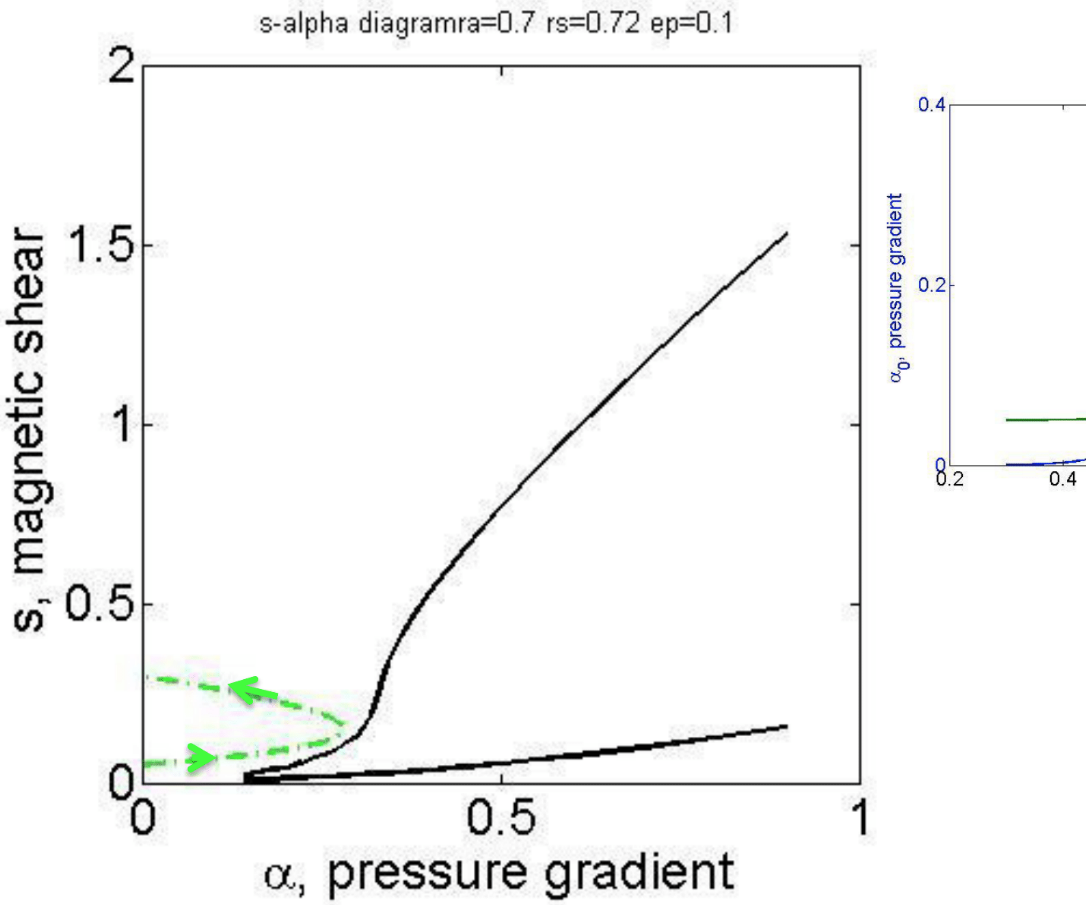
$$\mathcal{A}(r, r_0) = \int_{r_0}^{r} (\beta_N(r') - \beta_N(r_0)) dr'$$

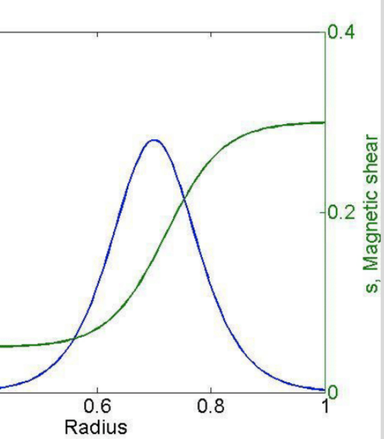
$$\mathcal{B}(r, r_0) = \int_{r_0}^{r} (\beta_N(r') - \beta_N(r_0)) s(r') dr'$$

$$\mathcal{C}(r, r_0) = \frac{1}{2} (\beta_N(r) - \beta_N(r_0))^2$$

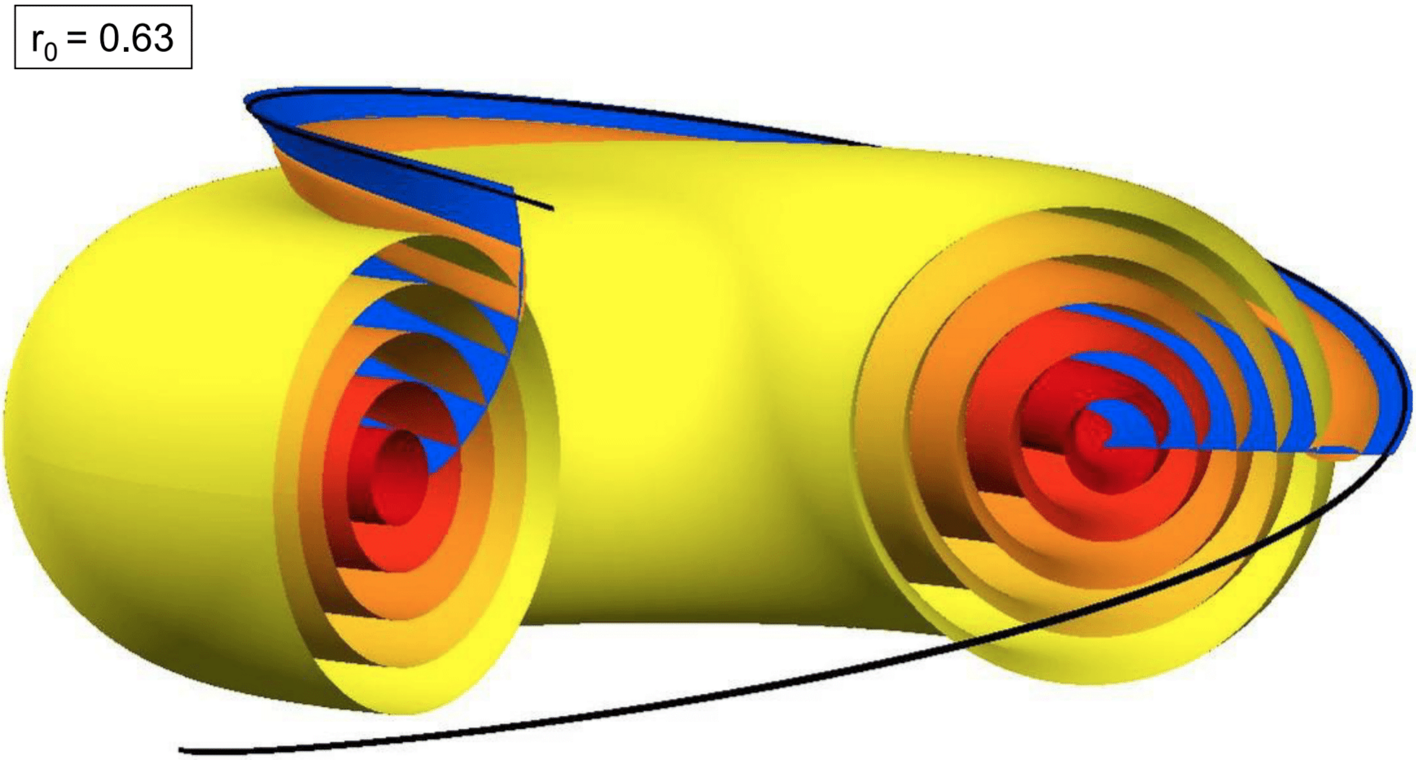
Drag motion minimises this energy. Always locally Downhill.

Stable profile

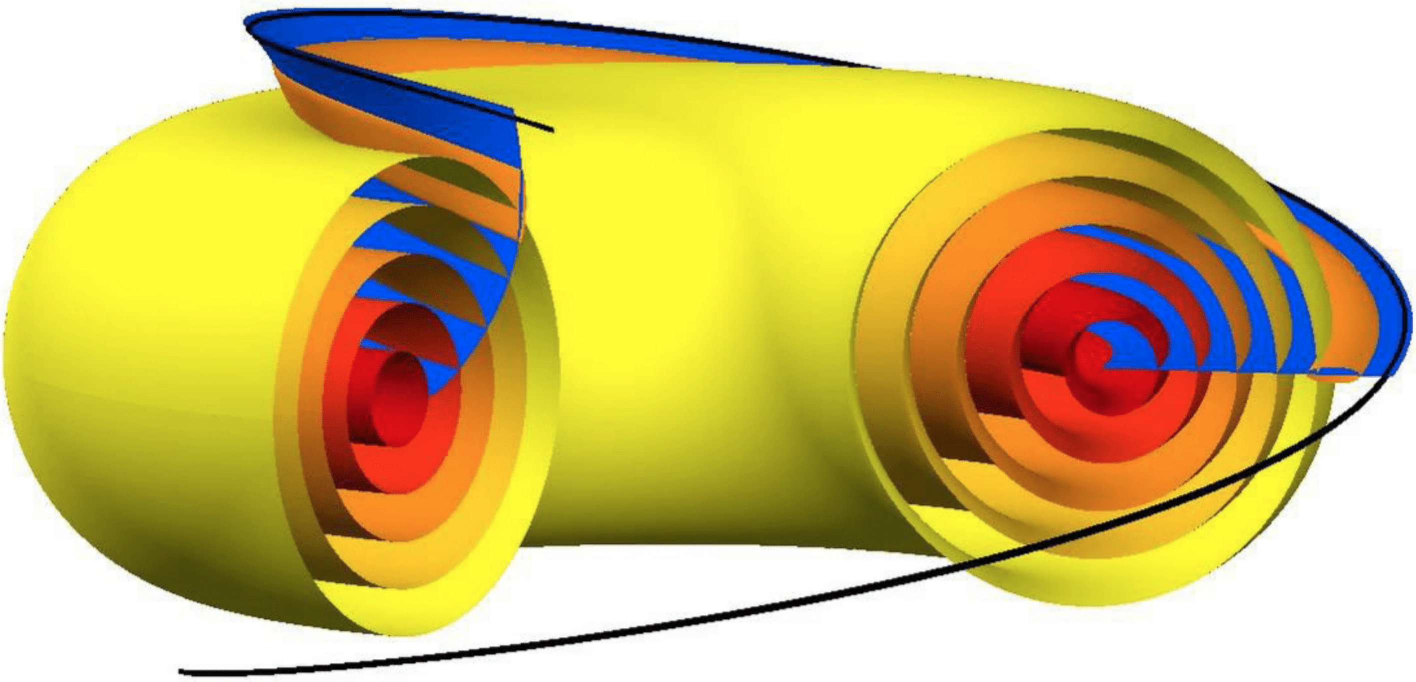




Perturbing a little



Perturbing more



And?

- What affects the tube cross sectional shape.
- We are working on calculation for edge of tokamak – ELMs.
- Transport of heat and particles caused by highly displaced tubes needs study.
- Reconnection of tubes
- **We need to find non disruptive high confinement states.**

Thank you Francesco