The skeleton of magnetic chaos

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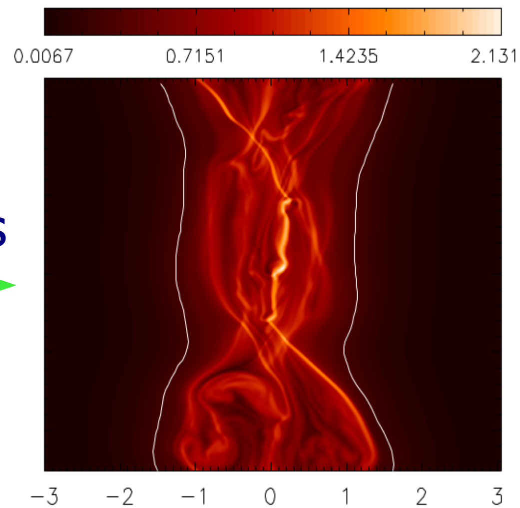
The problem of transport in a chaotic magnetic field

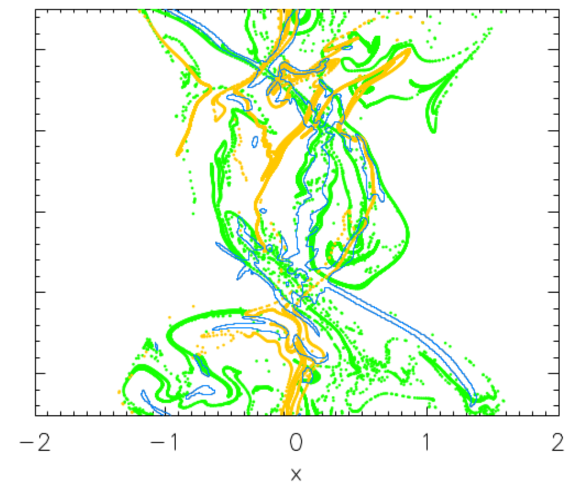
- In a magnetic chaotic field regions characterized by different behavior of the field line motion may coexist.
- How to relate the particle transport to the field line motion is an open problem.
- Here we present our approach based on the concept of Lagrangian Coherent Structures (LCS).
- We consider a model of chaos, derived from the results of a numerical simulation of a magnetic reconnection event, in which we assume electrons move along magnetic field lines.
- Two methods, ridges and most repelling/attracting material lines, borrowed from oceanography studies, have been successively adopted to find the LCS.
- An application to a realistic field configuration will be discussed.

Motivation

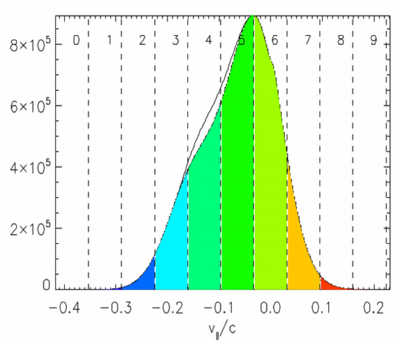
- Test electron simulations show a link between electron distribution and coherent structrures (Borgogno et al, submitted to PoP).

Current sheets

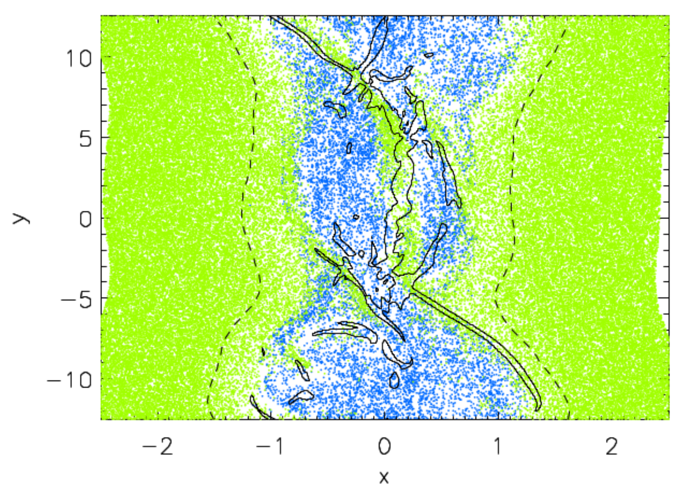


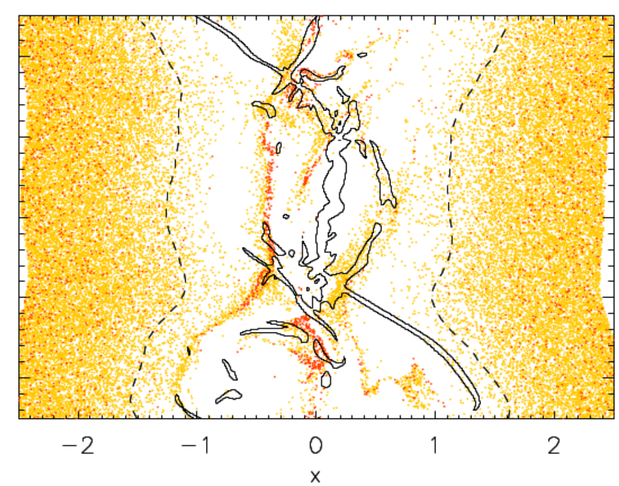


Manifolds



Electron distribution



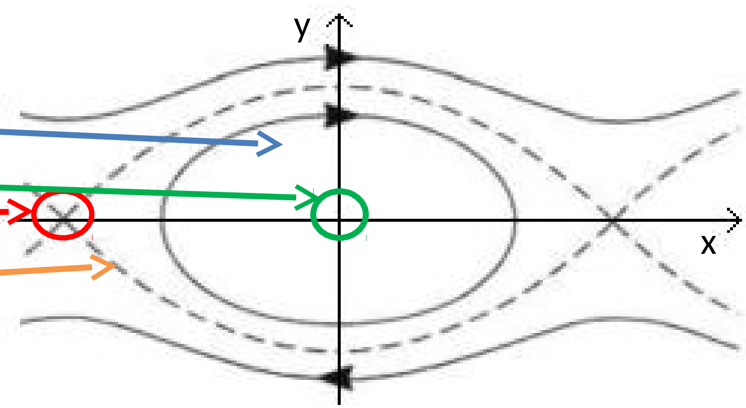


Chaotic magnetic field generated by a reconnection event

- Magnetic reconnection is a relaxation process occurring at resonant surfaces characterized by rational values of q=m/n, where *m* and *n* are the poloidal and toroidal wave numbers

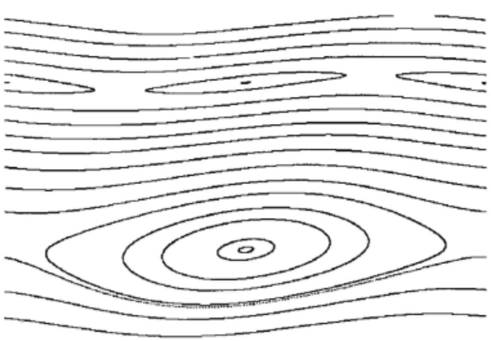
Magnetic field topology

- Magnetic island
- Fixed points: O-point
- X-point
- Separatrix

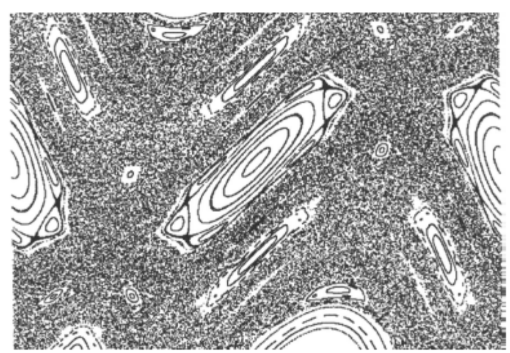


Chaotic magnetic field generated by a reconnection event

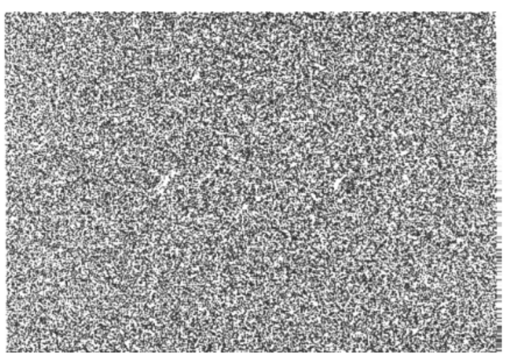
- In 3D configurations there are several resonant surfaces → **Island chains with different helicities**
- Interaction between islands with different helicities → **Magnetic chaos**
- More restrictive case: two close resonant surfaces with $q_1=m/n$ and $q_2=m'/n'$ such that $m'n - n'm = \pm 1$.
- Overlap criterion: $s = \frac{W_1 + W_2}{2\Delta x_{12}} > 1$, W island amplitude, Δx_{12} distance surfaces.



s<1 single helicity approximation

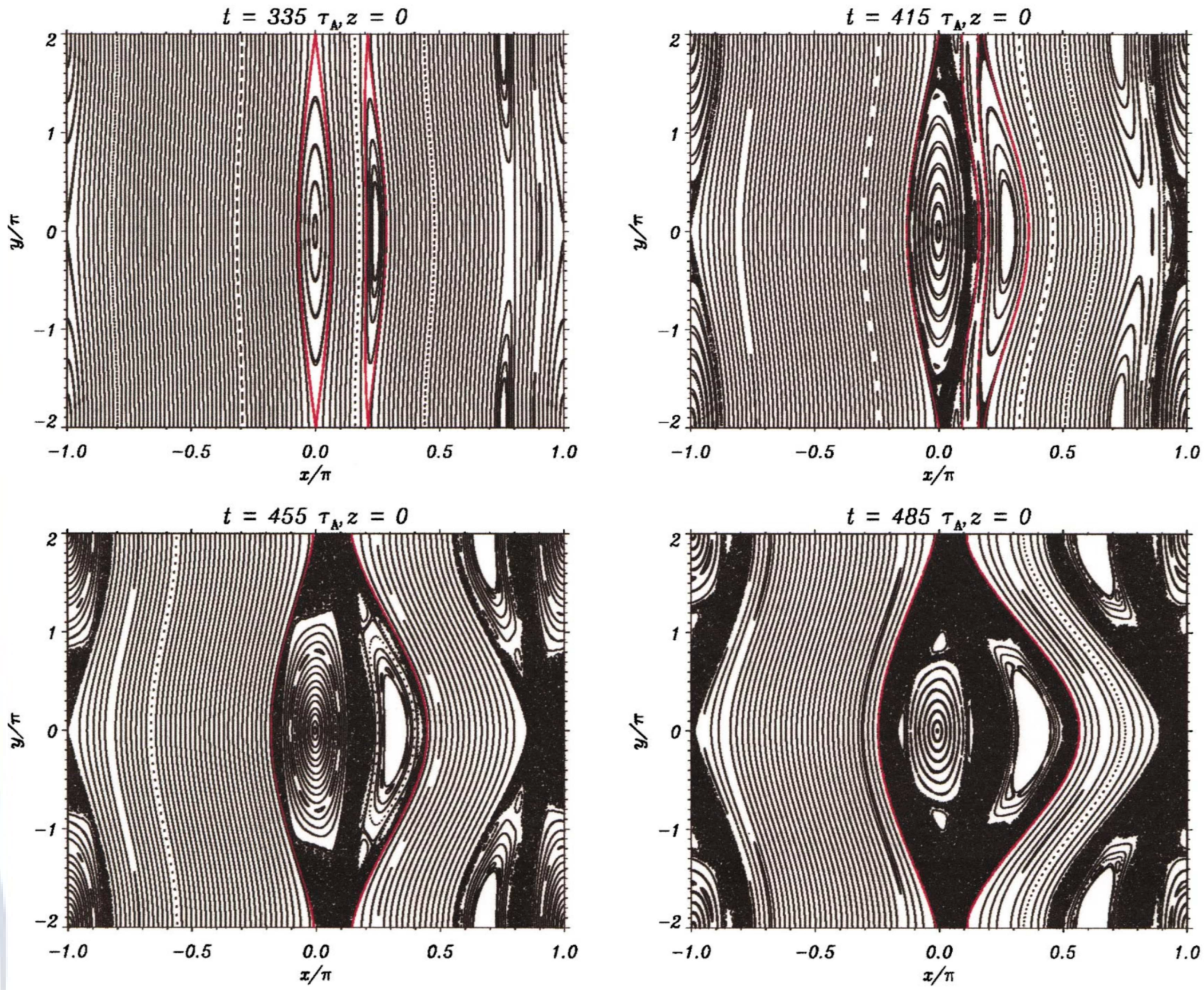


s ~ 1 chaos development



s>>1 completely chaotic domain

3D model for reconnection



- A reconnection event has been induced by a double helical perturbation resonating at two different surfaces.
- As shown in figure, the islands size increase until they start to influence each other leading to a chaotic setting initially enclosed between the islands
- Do some barriers exist influencing the transport in this chaotic sea?

Borgogno et al. PoP 2005

3D model for reconnection

- Magnetic field:

$$\vec{B} = B_0 \vec{e}_z + \nabla\psi \left(\vec{x}, t\right) \times \vec{e}_z$$

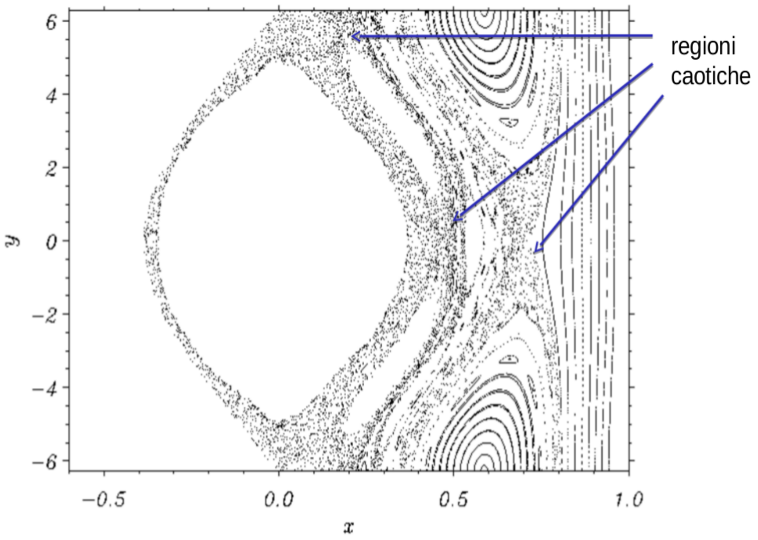
- Magnetic field line equations:

$$\frac{dx}{dz} = -\frac{\partial\psi}{\partial y}; \qquad \frac{dy}{dz} = \frac{\partial\psi}{\partial x}$$

  for fixed time, *t*, *z* plays the role of field line time.

- Poincarè plots give a detailed picture of the chaotic domain of the magnetic field, provided one integrates the Hamilton equations starting from a sufficiently large number of i.c. and for a sufficiently long time.

  NO INFORMATION ON TRANSPORT!



Borgogno et al, PoP 2008

How to identify barriers between different chaotic regions?

- We define the barriers that may form in the chaotic sea as Lagrangian Coherent Structures.
- LCS organize the flows in macroregions whose fluid elements may not exchange and hence act as transport barriers over a finite time interval.
- How to identify these barriers is an open issue and several methods have been proposed.
- We adopted two methods:

1) ridges of the Finite Time Lyapunov Exponent Field (Haller 2001, Shadden 2005-2007)

2) most repelling/attracting material lines

(Haller 2010, 2014)

Ridges of the FTLE field

Empirical definition: If hiking along a ridge one would expect:

- to be locally at the highest point in the field transverse to the ridge. If the hiker stepped to the left or to the right of the ridge he would step down
- for the topography to drop off steepest in the direction transverse to the ridge. The direction the topography decreases most rapidly should be transverse to the ridge.

Ridges of the FTLE field

- We define fisrt the Finite Time Lyapunov Exponent Field:

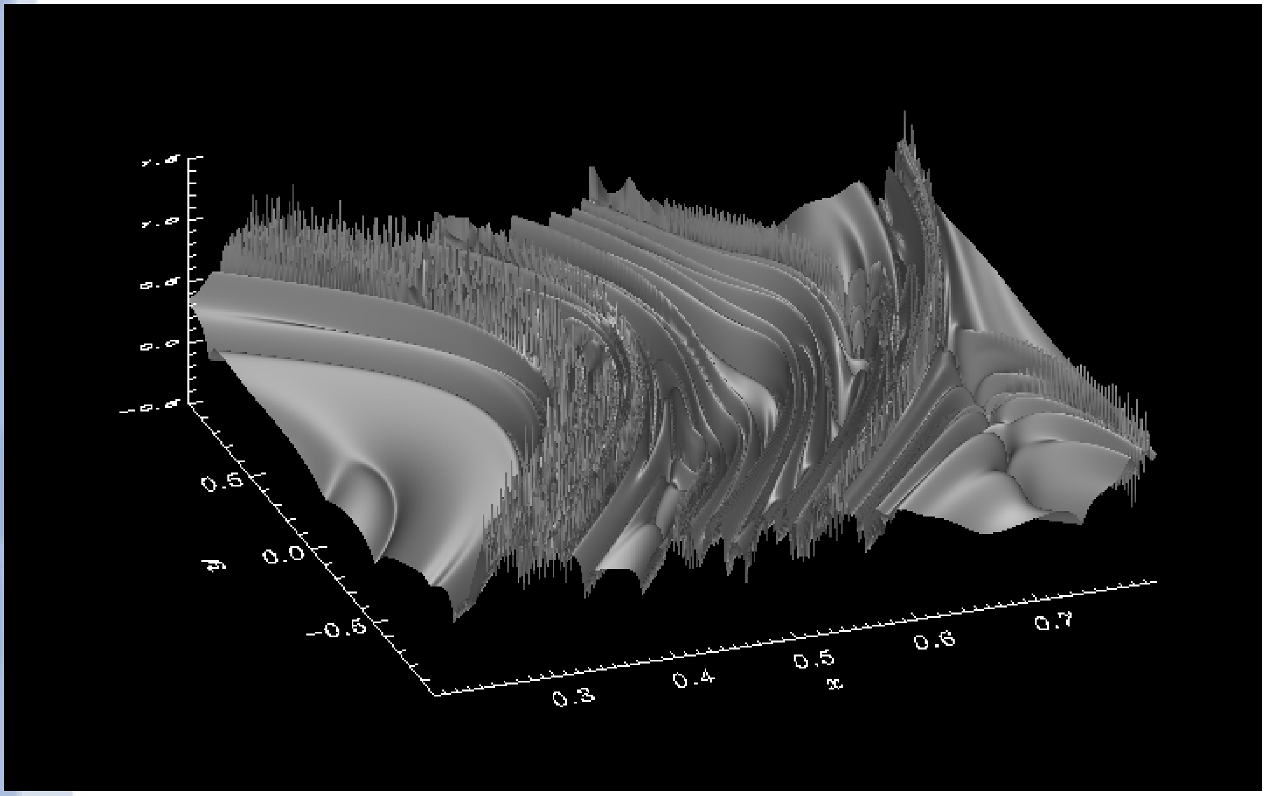
$$\sigma = \sigma_z^Z(x) = \frac{1}{|Z|} \ln \sqrt{\lambda_{\max}}$$

- *Z* plays the role of an effective finite time interval
- λ_{max} is the larget spositive eigenvalue of:

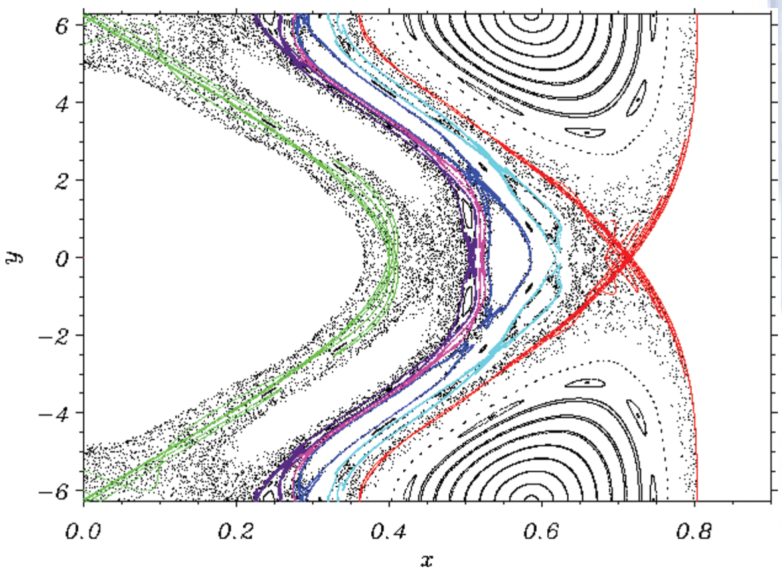
$$M(x,z,Z) = \frac{d\Phi_z^Z(x)}{dx}^{\dagger} \frac{d\Phi_z^Z(x)}{dx}$$

- $\Phi_z^Z(x)$ is the flow map: $x(z) \rightarrow x(z+Z)$
- In the limit $|Z| \rightarrow \infty$ the standard Lyapunov exponent is recovered.
- We now look for the ridges, defined as curves such that the gradient in the FTLE field is along the curve and such that the second derivative of the largest positive FTLE in the direction perpendicular to the curve is minimal.

FTLE field



$\sigma(x, y)$ at $Z = 12L_z$

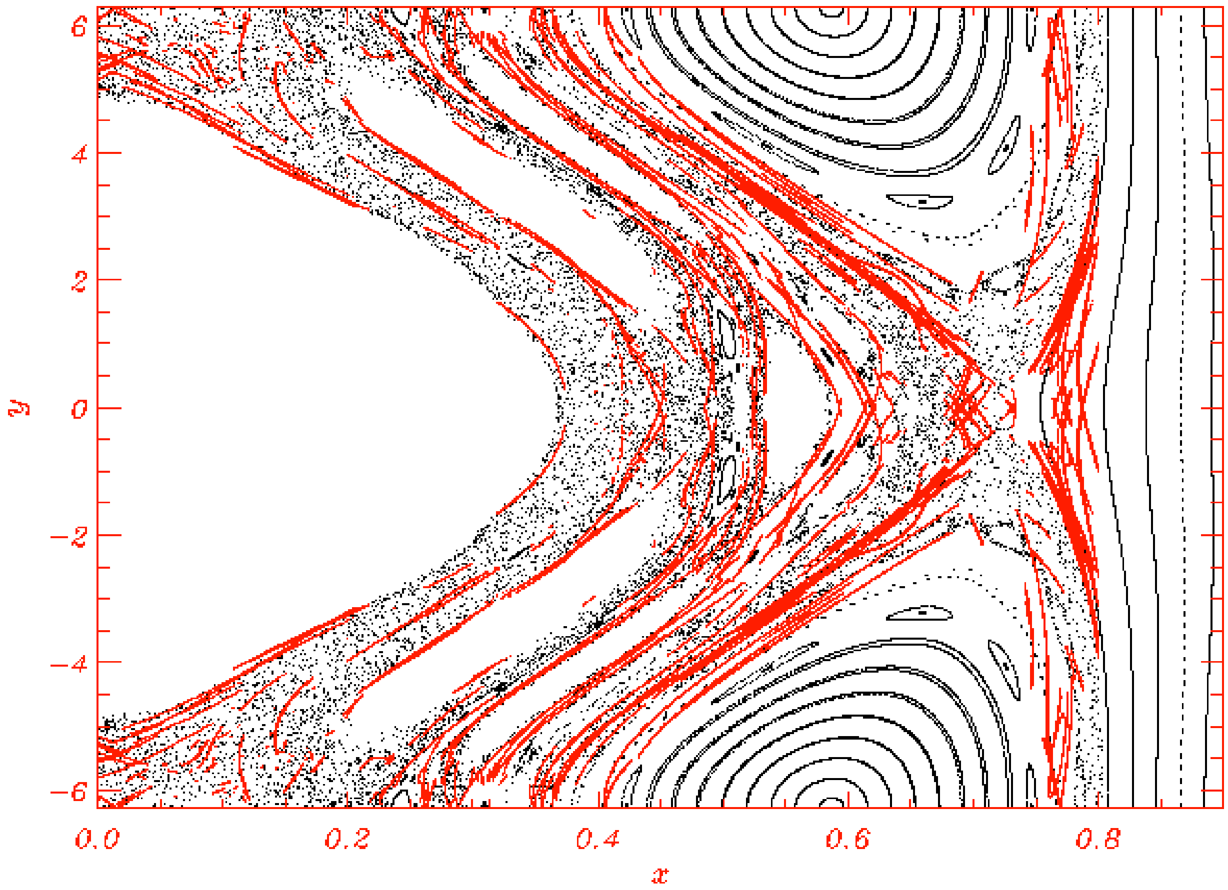


16000X8000 initial conditions uniformly distributed on the range

$$0 < x < 0.8, -2\pi < y < 2\pi$$

Borgogno et al, PoP 2011

RIDGES

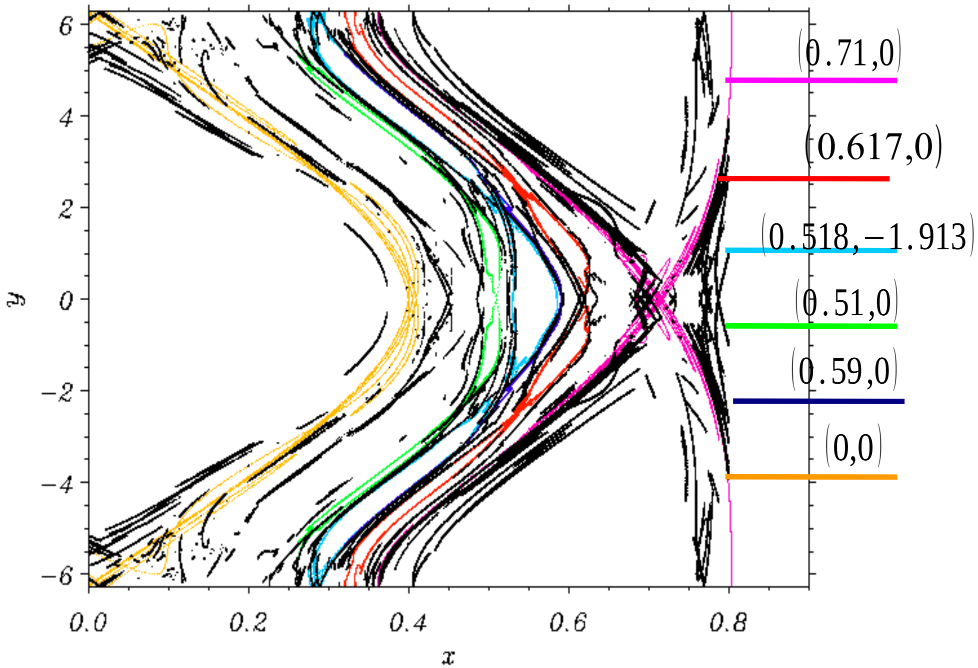


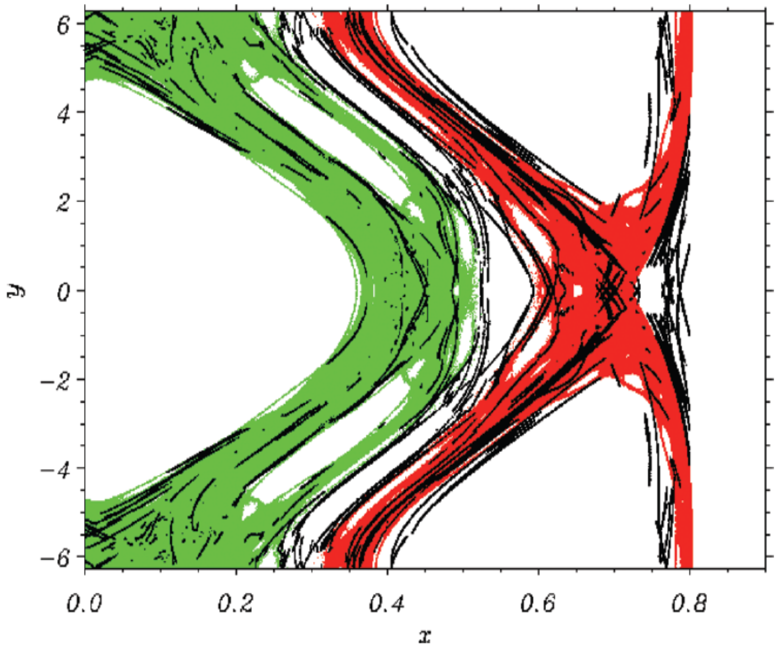
$$Z = 12L_z$$

Borgogno et al, PoP 2011

- Ridges are localised along the boundaries of the chaotic regions
- Well defined structures identified by ridges inside the chaotic domain

Ridges and manfiolds



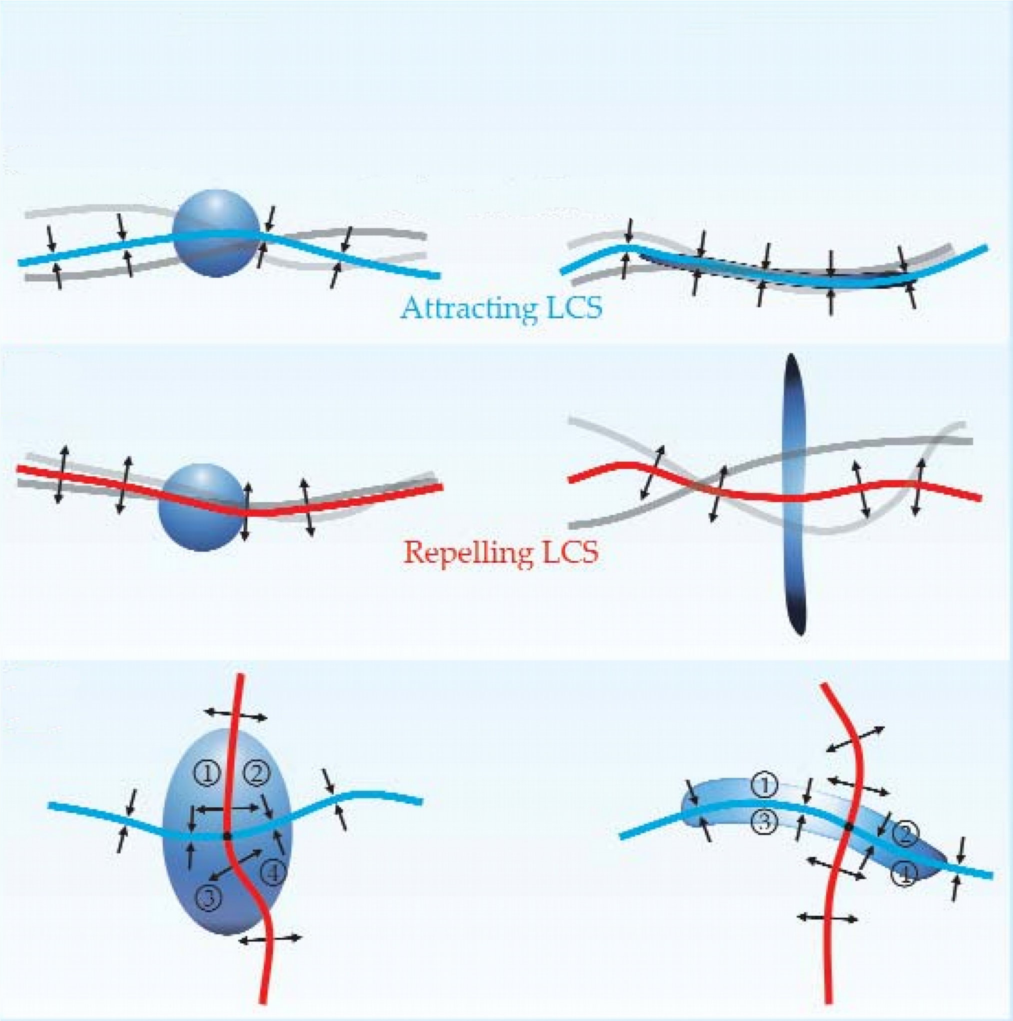


- Ridges (curves in black) lie on the branches of invariant manifolds (curves in color)
- Poincarè plot obtained from 5000 ic iterated for $500L_{z.}$ Both green and red regions are confined by ridges, which confirm the role played by these patterns as magnetic field line transport barriers.

Borgogno et al, PoP 2011

Most repelling/attracting material lines

Graphic definition:



Most repelling material lines

Mathematical definition: A LCS over a finte time interval Z is defined as a material line satisfying in each point the following conditions on the eigenvavlues and eigenvectors of the Cauchy Green strain tensor *M*:

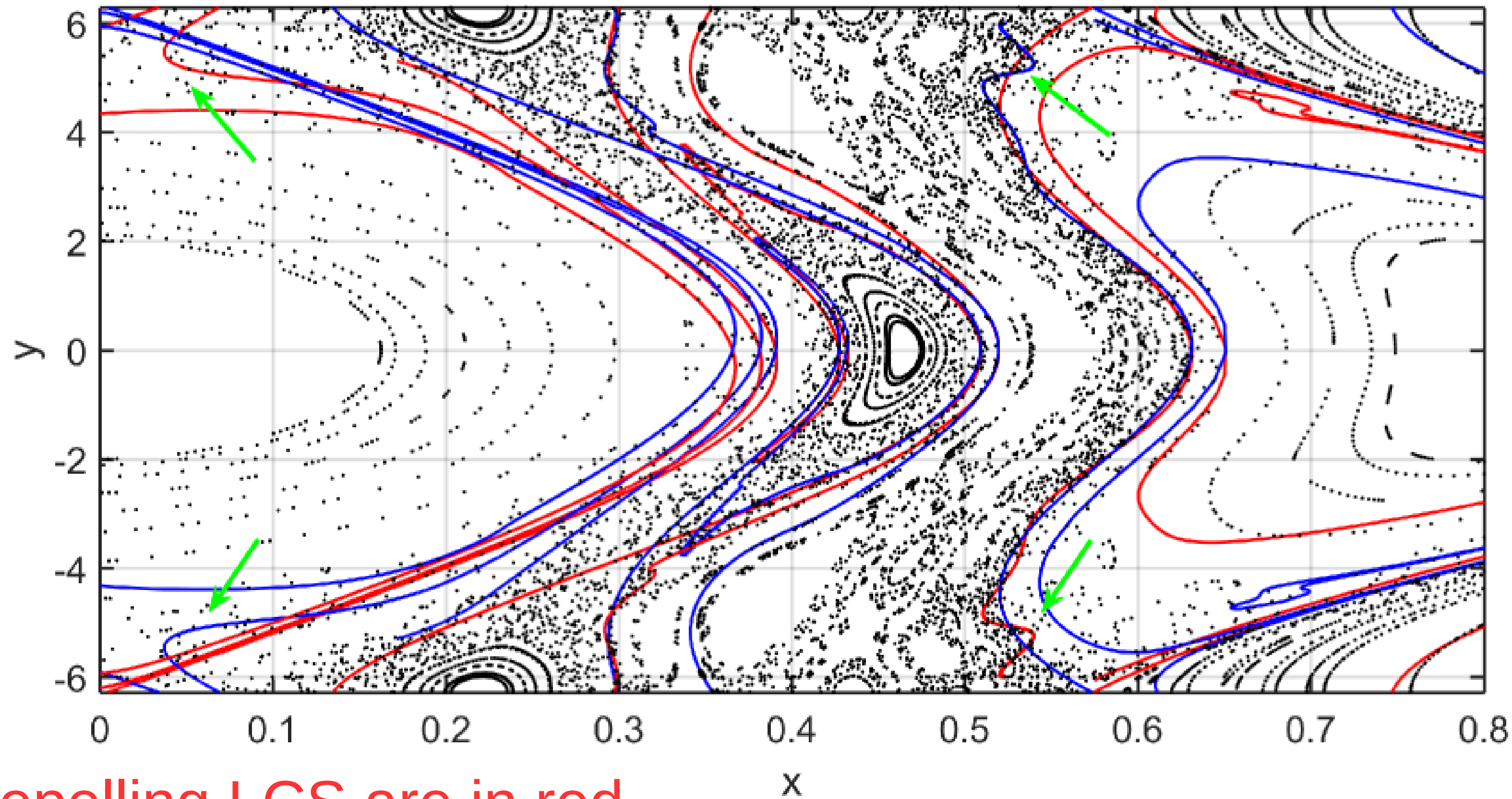
i) $\lambda_{min} < \lambda_{max}\ ;\ \lambda_{max} > 1$

ii) $e_0 = \xi_{min}$ the tangent vector is along the eigenvector associated with the smallest eigenvalue

iii) $\xi_{max} . \nabla \lambda_{max} = 0$ the gradient of the largest eigenvalue is along the curve

iv) $\xi_{max} . \nabla^2 \lambda_{max} . \xi_{max} < 0$ repulsive LCS

Most repelling/attracting material lines



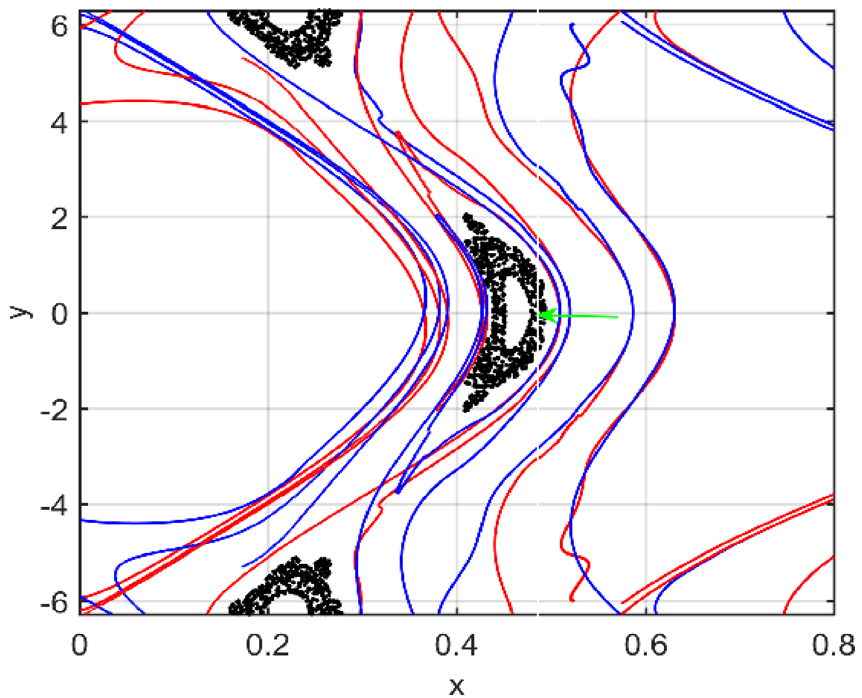
Repelling LCS are in red

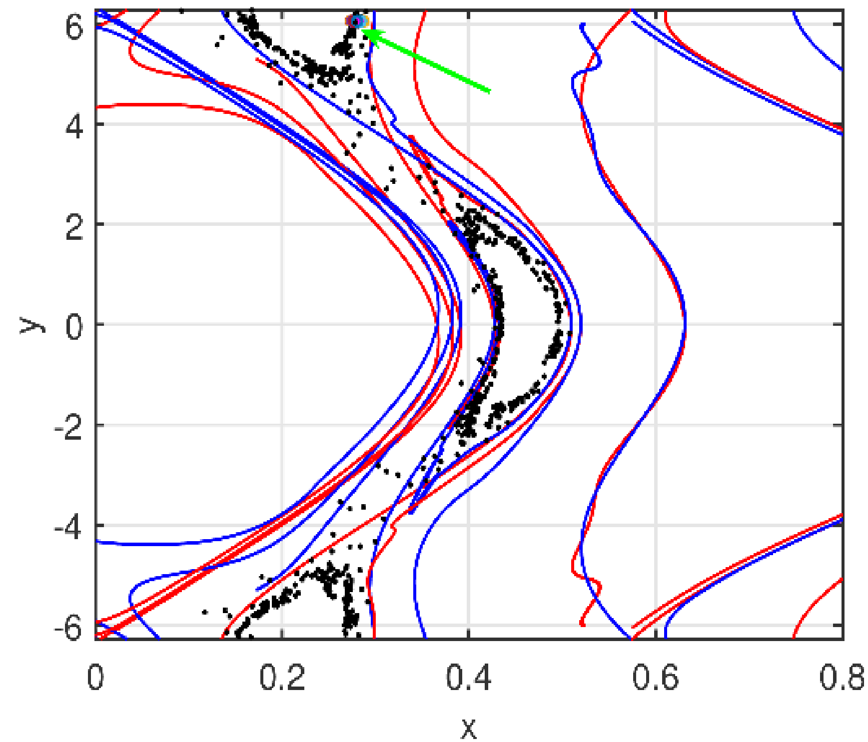
Attractive LCS are in blue

Di Giannatale thesis ready for PoP

Most repelling/attracting material lines

80 L_z





I.c. close to a regular region remain confined

I.c. close to a repelling LCS can escape

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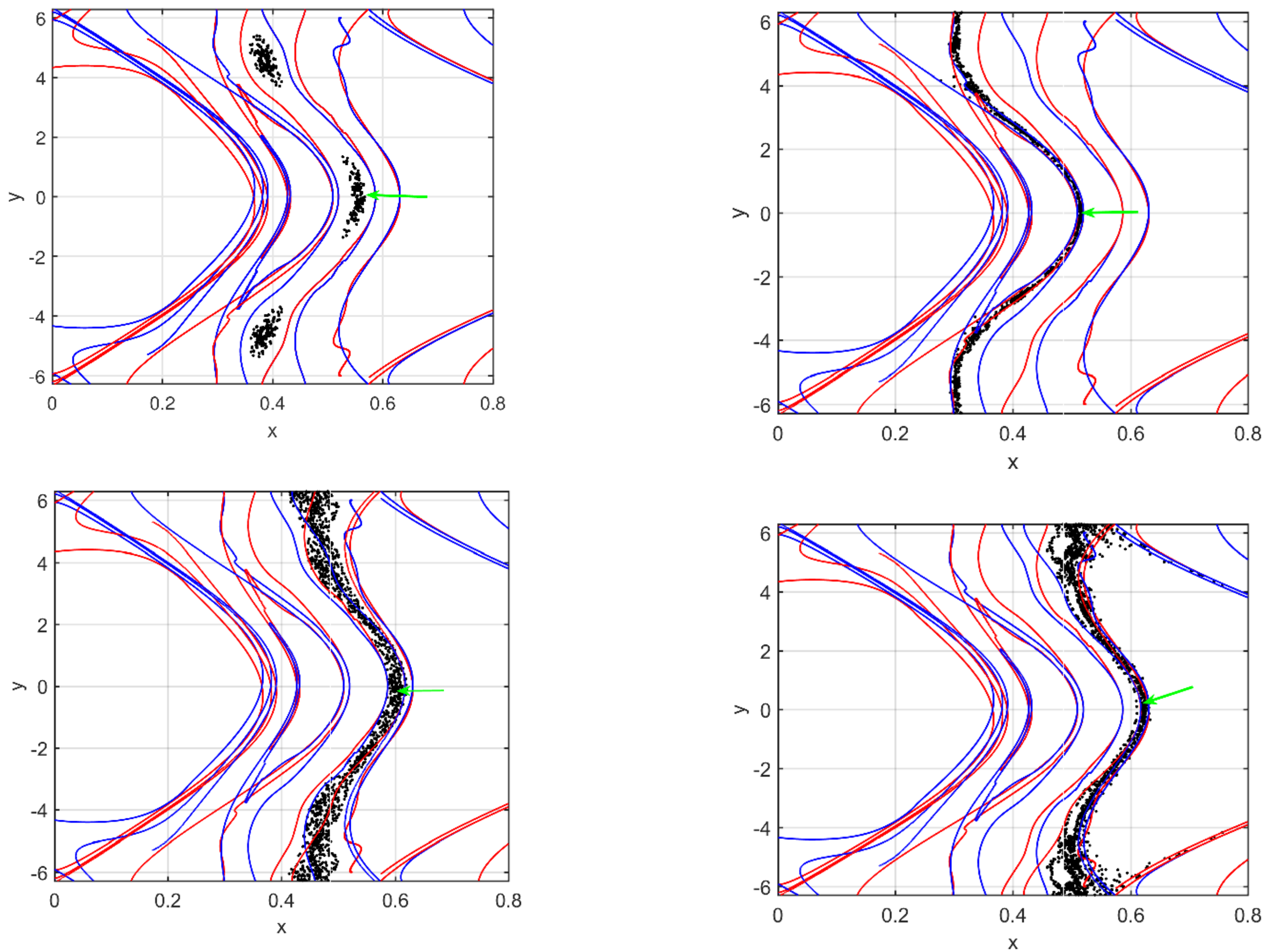
Most repelling/attracting material lines



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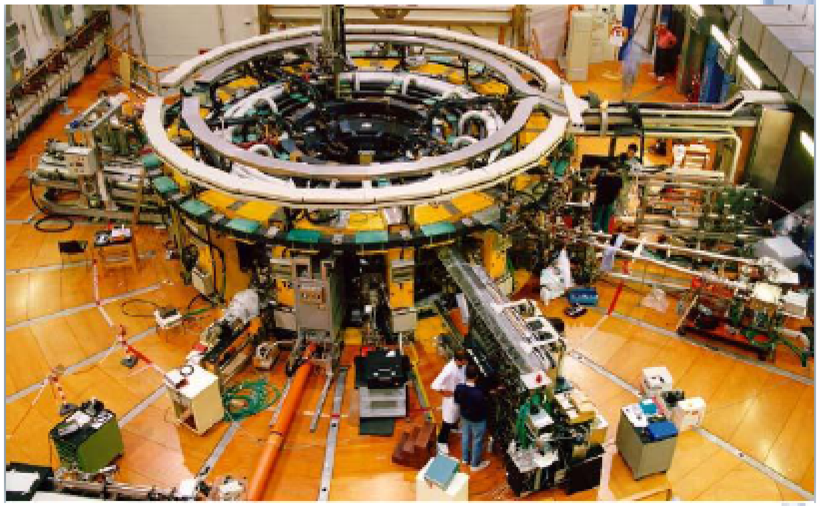
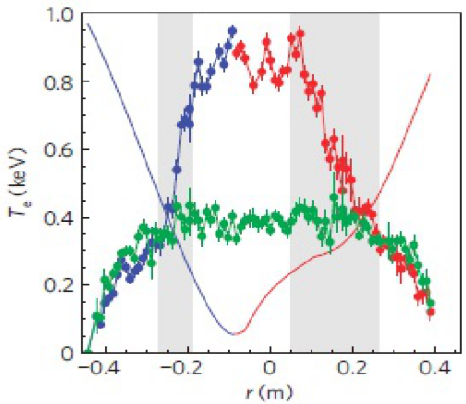
Application to a realistic onfiguration

Increasing the plasma current RFX-mod experiments show (RFX group, Nature 2009):

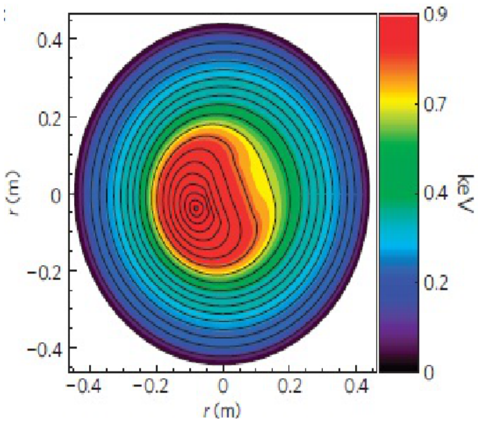
- transition from chaotic configurations to ordered ones, named Quasi Single Helicity (QSH) states;
- consequent formation of a strong transport barrier enclosing high temperature zone.



Electrons temp.   vs r



2D map electrons Temp.



Does exist a correlation between the transport and the magnetic barriers?

Application to a realistic configuration: results

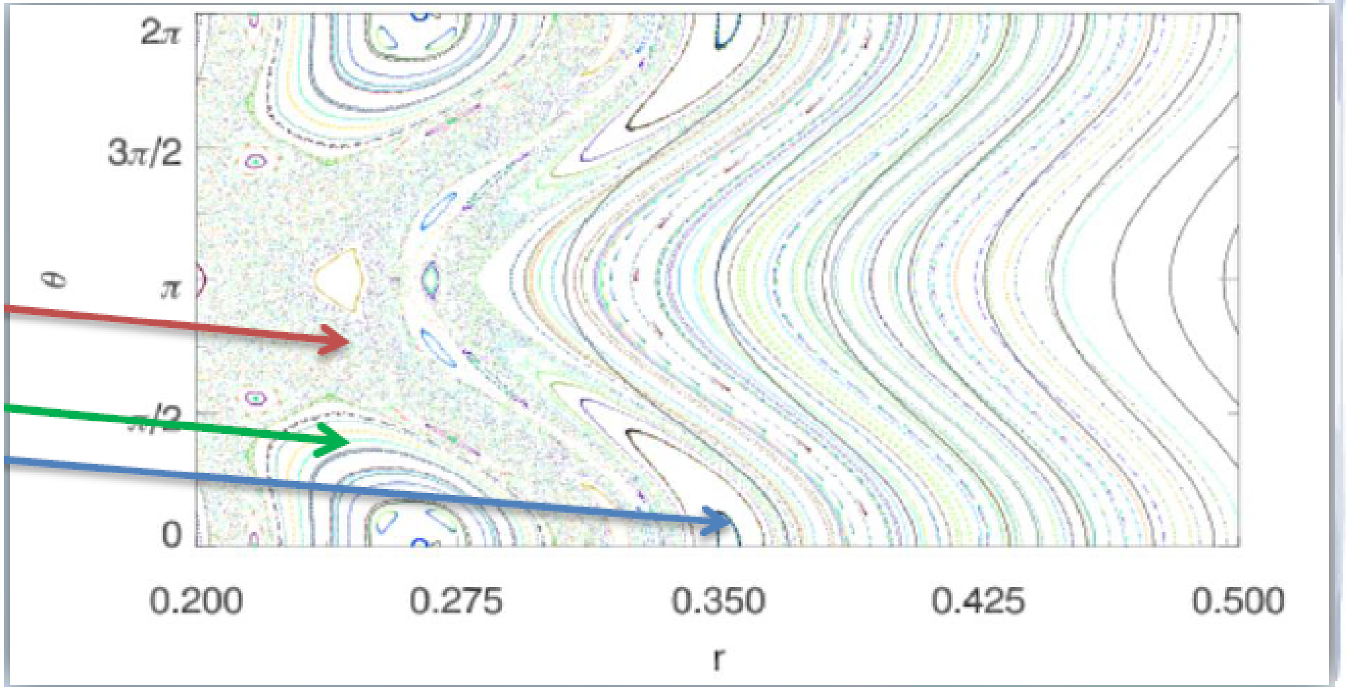
- Numerical simulations of the RFP configuration through SPECYL (3D MHD visco-resistive model) and NEMATO ($d\mathbf{x}/d\tau = \mathbf{B}(\mathbf{x})$) codes.
- **SpeCyl:** 3D MHD visco-resistive numerical simulations
- In the nonlinear phase secondary modes grow and a wide chaotic region develops.
- In the highly nonlinear phase ($t>2000\ \tau_A$) we find a QSH state
- Here we analyze the transition phase ($\approx 600\tau_A$)

NEMATO: Poincaré map

wide chaotic region

m=1 n=-9 mode

m=1 n=-10 mode



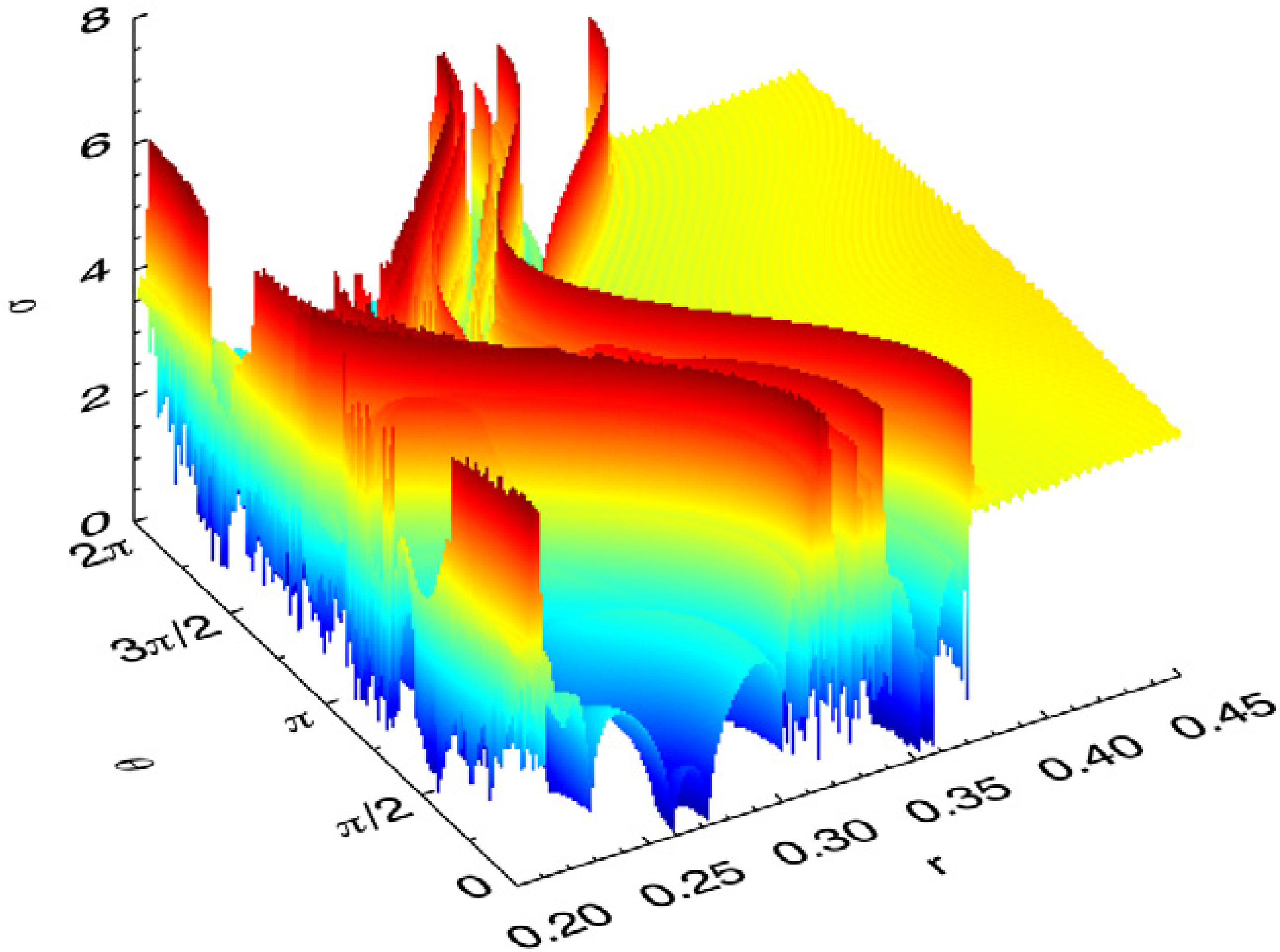
Rubino et al, PPCF 2015

Application to a realistic configuration: results

- Magnetic field line equations:

$$\begin{cases} \frac{dr}{dz} = \frac{B_r}{B_z}, \\ \frac{d\theta}{dz} = \frac{1}{r}\frac{B_\theta}{B_z} \end{cases}$$

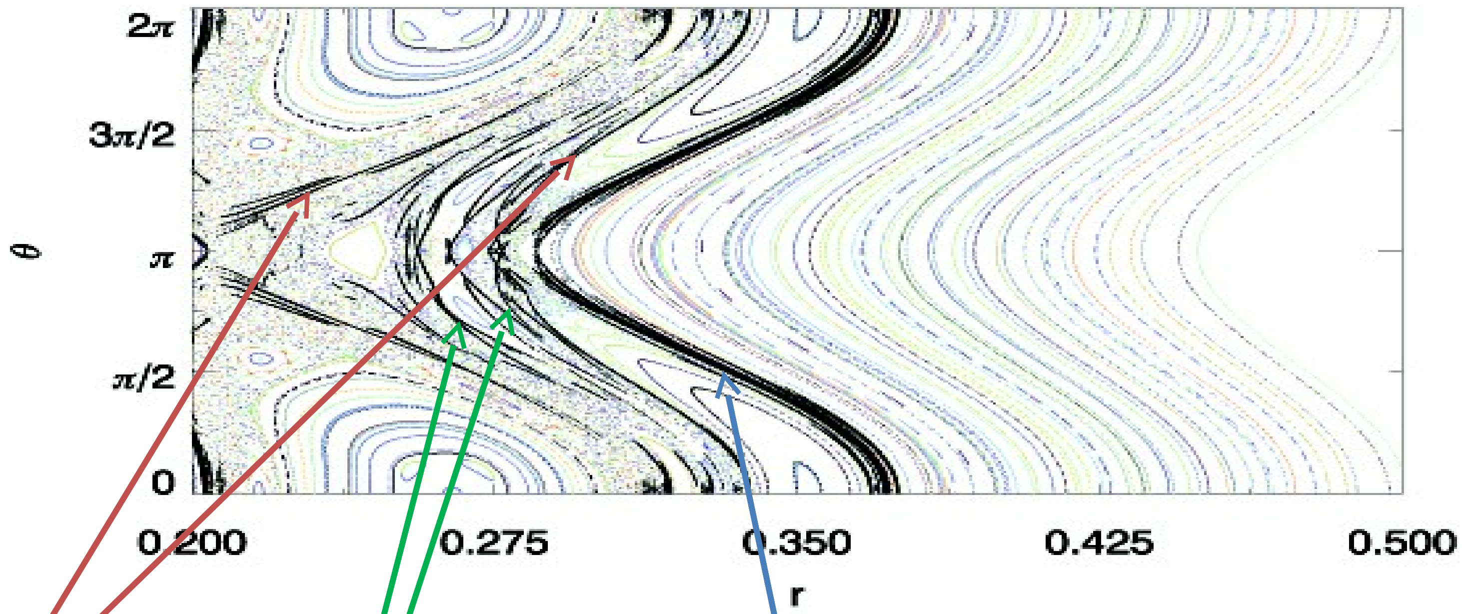
- FTLE evaluated for z=10 L_z, starting from 4096X8400 initial conditions located between 0.2<r<0.45 and 0<θ<2π



- In the chaotic region we observe the presence of ridges
- The FTLE has a regular behavior inside the magnetic islands and where conserved surfaces exist.

Rubino et al, PPCF 2015

Application to a realistic configuration: results

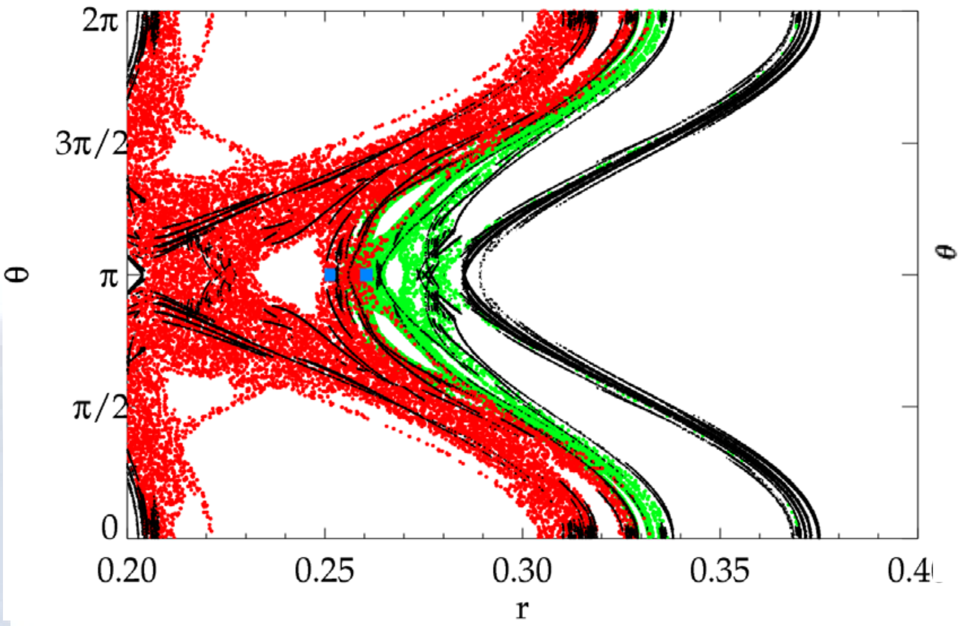


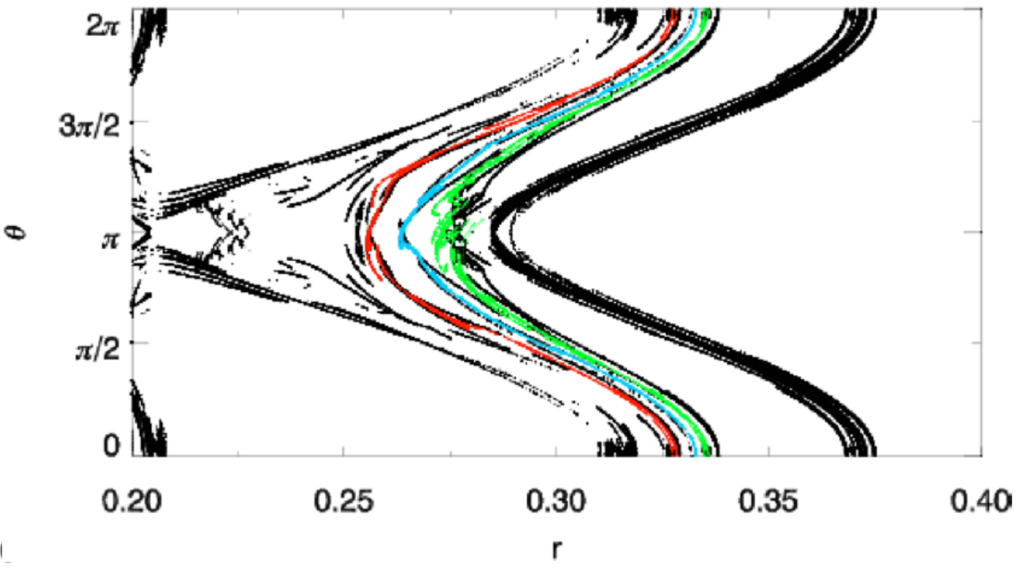
- No LCS are found in regular domains and in highly chaotic regions
- Weak barriers are present at the edge of the magnetic islands
- Strong barrier at the edge of the magnetic flux conserved surfaces

Ridges in the chaotic sea

Rubino et al, *PPCF 2015*

Application to a realistic configuration: results





Validation by Poincarè plot (400 L_z)

Validation by manifolds

Rubino et al, PPCF 2015

Conclusions and future perspectives

- Application to a time dependent case (Pegoraro, Falessi, Di Giannnatale…).

$$\psi_{\mathcal{V}}(x,y,z) \equiv \psi(x,y,z,t=(z-z_o)/V)$$

Main questions:

i) how do the LCS change when the evolution of the magnetic field is considered;

ii) whether and how particles with different velocities can cross LCSs calculated for a different particle velocity (pegoraro@FISMAT October, 2017) .

- Application of the new definition of LCS to the RFX case (Di Giannnatale, Veranda…).
- Test electron simulations can test the goodness of the adopted approximations (Borgogno, Perona…).