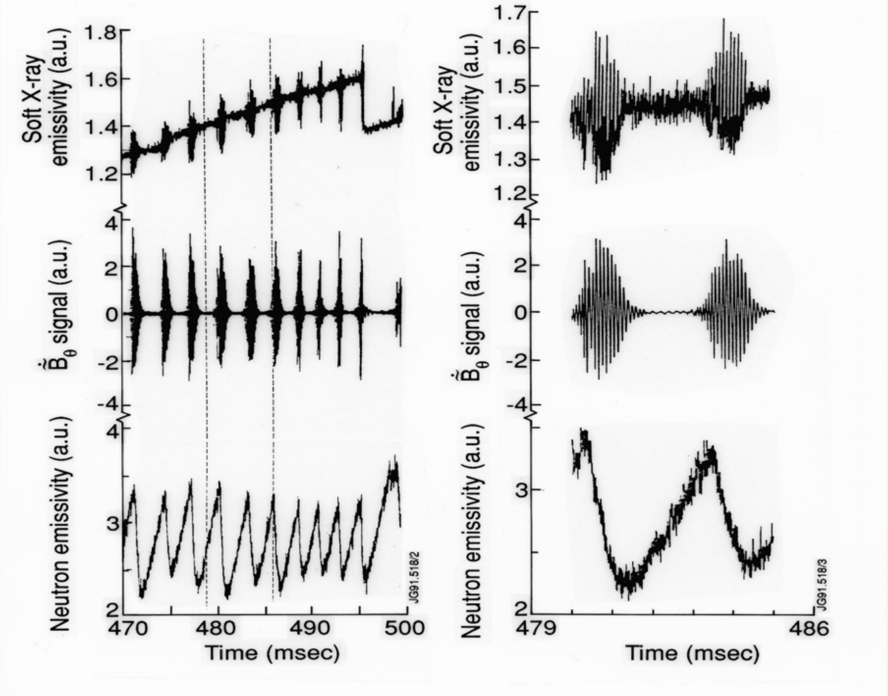
Plasma Physics: Recent results and future perspectives; Pisa, 18 Sept 2017

HYBRID KINETIC-FLUID PLASMA MODELING AND APPLICATIONS



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Francesco and I have co-authored 31 peer-reviewed articles over a period of more than 20 years: 1984-2006

Leitmotif of our research:

dialogue between fluid and kinetic theories

or

The problem of fluid closures: when are they possible, and how

Statement (open to discussion)

Fluid closures are not possible when one (or both) of these two situations arise:

mode-particle resonances

non-local effects

The converse is also true: rigorous fluid closures are always possible, in principle, when the two effects indicated above are not important.

The correct fluid closure is problem-dependent. How does one determine it?

Example of fluid closure: Equation of state for relativistic plasma waves

Equation of state for relativistic plasma waves

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(Received 10 October 1983; accepted 9 March 1984)

It is shown that in a fully relativistic plasma an isothermal equation of state must be assumed in order to obtain from fluid equations the correct dispersion equation of longitudinal plasma waves. This isothermal equation of state contrasts with the adiabatic equation that applies to a non-relativistic plasma. For arbitrary values of the ratio α between the particle rest mass and particle temperature, a polytropic equation of state must be introduced. The dependence of the polytropic index γ on α can be derived from the relativistic Vlasov equation and can be shown to join the adiabatic ($\alpha \gg 1$) and the isothermal ($\alpha \ll 1$) regimes smoothly.

The non-relativistic limit is a textbook calculation (see, e.g., Krall and Trivelpiece)

The fluid derivation yields

$$\omega^2 = \frac{4\pi n e^2}{m_e}\left(1 + \gamma k^2 \lambda_D^2\right)$$

The Vlasov theory *reveals* the correct equation of state to be used in a fluid theory

$$\gamma = 3$$

corresponding to a 1D adiabatic compression.

Why not compute gamma directly in terms of moments of the solution of the linearized Vlasov equation?

The relativistic limit is more tricky, and the result is somewhat surprising

The relativistic fluid derivation yields

$$\omega^2 = \frac{4\pi n e^2}{M(\alpha)} + \frac{9\gamma(\alpha)}{5} k^2 \frac{T}{M(\alpha)}$$

where $\alpha = \frac{m_e c^2}{T}$ ($\alpha \ll 1$: relativistic limit)

$$c^2 M(\alpha) = \mathcal{E} + g(\alpha) T$$

$\mathcal{E}$: average particle relativistic energy ($\mathcal{E} \approx m_e c^2$ for $\alpha \gg 1$)

$$g(\alpha) = c_v(\alpha)\left[\gamma(\alpha) - 1\right]$$

c_v : specific heat at constant volume

$\gamma(\alpha)$: polytropic index for equation of state → $M(\alpha)$ depend on γ !

∴ Assume adiabatic equation of state :

$$\gamma(\alpha) = \gamma_{ad}(\alpha) \qquad \gamma_{ad} = \begin{cases} 5/3 \text{ for } \alpha \gg 1, & (M(\alpha) \approx m_e) \\ 4/3 \text{ for } \alpha \ll 1 \end{cases}$$

But...
$$c_v = \frac{\partial \mathcal{E}}{\partial T} = \left[\gamma_{ad}(\alpha) - 1\right], \quad M(\alpha) = \mathcal{E} + T \rightarrow 4T \text{ for } \alpha \ll 1$$

In the fully relativistic limit, $\alpha \ll 1$, the adiabatic equation of state yields

$$\omega^2 = \frac{4\pi n e^2 c^2}{4T} + \cdots$$ wrong!

The correct result, as obtained from Vlasov theory, is

$$\omega^2 \simeq \frac{4\pi n e^2 c^2}{3T} + \frac{3}{5} k^2 c^2$$

∴ The correct equation of state is <u>isothermal</u> : $\gamma(\alpha) \to 1$ for $\alpha \ll 1$

The polytropic index can be computed directly in terms of moments of the perturbed distribution function.

$$\gamma(\alpha) = \frac{1}{3T} \frac{\sum_i \tilde{T}_{ii}}{\tilde{n}}$$

$T_{\mu\nu}$: electron energy momentum tensor

n : electron density

$$\tilde{T}_{\mu\nu} = \int \frac{d^3p}{p_t} p^\mu p^\nu \tilde{f} \quad ; \qquad \tilde{n} = \int \frac{d^3p}{p_t} p_t \tilde{f}$$

A short calculation shows that, for $\omega \gg kc$

$$\gamma(\alpha) \simeq 1 + \frac{2}{3} \frac{I_{4,4}(\alpha)}{I_{4,2}(\alpha)}$$

where $$I_{m,n}(\alpha) = \int_0^\infty dt \, \frac{t^m}{(1+t^2)^{n/2}} \exp\left[-\alpha(1+t^2)^{1/2}\right]$$

(can be expressed in terms of modified Bessel functions)

Early example of non local effects: when the width of the reconnection layer is determined by the ion Larmor radius

Internal kink modes in the ion-kinetic regime

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(Received 13 June 1988; accepted 22 September 1988)

The $m = 1$ kink mode is investigated in the high temperature regime where the width of the singular layer is determined by the mean ion gyroradius. This regime is reached in a number of present-day fusion experiments with strong auxiliary heating. A dispersion relation that contains the full kinetic response of the ions is derived and analyzed. The growth rates are larger than the corresponding ones obtained from fluid theory. Diamagnetic stabilization is weaker than in the fluid case. Ion temperature gradients are shown to be stabilizing at low values of the diamagnetic frequency and destabilizing at large values.

Use fluid theory for the electrons and Vlasov theory for the ions.

Electron perturbed density:

$$\frac{\tilde{n}_e}{n} = \frac{\omega_{*e}}{\omega}\frac{e\tilde{\phi}}{T_e} + \frac{i}{\omega}\frac{d}{dk}\frac{\tilde{J}}{env_A}$$

k: Fourier variable conjugate to $x = (r-r_s)/r_s$

Ion perturbed density:

$$\frac{\tilde{n}_i}{n} = \left(\frac{\omega_{*e}}{\omega} - D\right)\frac{e\tilde{\phi}}{T_e}$$

Ref.: Pegoraro and Schep PF 24, 478 (1981)

where D represents the non-local ion response:

$$D(b) = \frac{\omega_{*e}}{\omega} + \tau\left[1 - \Gamma_0(b)\right] - \frac{\omega_{*e}}{\omega}\Gamma_0(b)\left[1 - \eta_i M(b)\right],$$

$b = \rho_i^2 k^2$

$\tau = T_e/T_i$

$$M(b) = b(1 - \Gamma_0/\Gamma_1), \quad \Gamma_{0,1}(b) = I_{0,1}(b)\exp(-b).$$

$\eta_i = d\ln T_i/d\ln n$

Quasineutrality, $\tilde{n}_e = \tilde{n}_i$, gives

$$D(b)\frac{e\tilde{\phi}}{T_e} = -\frac{\omega_{*e}}{\omega}\frac{d}{dk}\frac{\tilde{J}}{env_A}$$

Non-local ion response is algebric in Fourier space

Ampère's law: $$\frac{\tilde{J}}{env_A} = \tau b \frac{ev_A}{c} \tilde{A}_{\parallel}$$

Ohm's law: $$\left(\frac{\mathcal{E}_\eta}{\tau \rho_i^2} - \frac{\omega_{*e}}{\omega} \frac{d^2}{dk^2}\right) \frac{\tilde{J}}{ecv_A} = \left(1 - \frac{i\omega_{*e}}{\omega}\right) \tilde{E}_{\parallel}$$

where $$\tilde{E}_{\parallel} = -\frac{d}{dk} \frac{e\tilde{\phi}}{T_e} + i\omega \frac{ev_A}{cT_e} \tilde{A}_{\parallel} ;$$ normalized parallel electric field.

Eliminating $\tilde{\phi}$, $\tilde{A}_{\parallel}$ and $\tilde{E}_{\parallel}$, we arrive at the dispersion equation for $\tilde{J}$:

$$\boxed{\frac{d}{dk}\left(1 + \frac{g}{D}\right)\frac{dJ}{dk} - g z_\rho^2 \left(\frac{1}{z_J^4} + \frac{1}{z_A^2 k^2}\right) J = 0}$$

where $g = 1 - \omega_{*e}/\omega$, $\quad z_\rho^2 = \dfrac{\omega}{\tau \rho_i^2 (\omega - \omega_{*i})}$; $\quad z_A^2 = \dfrac{-1}{\omega(\omega - \omega_{*i})}$

Padé approximation for $D(b)$: interpolation formula between the fluid ($\rho_i^2 k^2 \ll 1$) and the large gyroradius ($\rho_i^2 k^2 \gg 1$) regimes:

$$D^{-1} = \frac{1}{\tau + \omega_{*e}/\omega} + \frac{z_\rho^2}{k^2}$$

This form for D allows for analytic solutions of the dispersion equation by means of asymptotic matching techniques.

One interesting limit → "resistive" internal kinks at ideal-MHD marginal stability, neglecting ω_*-effects

$$\frac{\gamma}{\omega_A} = \left[\frac{2(1+T_e/T_i)}{\pi}\right]^{2/7} \left(\frac{\rho_i}{\varepsilon_\eta^{1/3}}\right)^{6/7} \varepsilon_\eta^{1/3} \qquad \varepsilon_\eta = \frac{1}{S}$$

↑

valid for $\rho_i > \varepsilon_\eta^{1/3}$

Growth rate in the ion-kinetic regime larger than the growth rate of the resistive internal kink mode!

This result was later extended to the collisionless regime (PRL 1991)

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Collisionless $m = 1$ Tearing Mode

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(Received 4 September 1990)

The instability of a magnetically confined plasma against macroscopic modes is analyzed in collisionless regimes where magnetic reconnection occurs because of finite electron inertia and the ion gyroradius ρ_i replaces the skin depth d as the width of the mode boundary layer. Growth rates $\gamma/\omega_A \sim (d/r_s)(\rho_i/d)^{2/3}$ are found, with ω_A the Alfvén frequency and r_s the radius of the reconnecting surface. For typical JET parameters, $\gamma^{-1} \sim 50\text{–}100\ \mu s$, which compares favorably with the observed instability growth time of internal plasma relaxations.

$$\frac{\gamma}{\omega_A} = \frac{d_e}{r_s}\left(\frac{\rho_i}{d_e}\right)^{2/3}$$

for $\rho_i > d_e, \varepsilon_\eta^{1/3}$

d_e: electron skin depth

The nonlinear evolution of reconnecting modes in the large ion gyroradius, weakly collisional regimes, was studied after 1991 on the basis of reduced, fluid-like models → the early work by Francesco was fundamental for these developments.

For instance, today we believe that the fast time-scale of the sawtooth crash in tokamak plasmas is no longer an issue, although details of the nonlinear evolution of the m=1 magnetic island are yet not fully understood.

Fast particle effects on MHD modes in tokamak plasmas: the development of a hybrid kinetic-MHD model

Adiabatic invariants of charged particle motion in Tokamak configurations

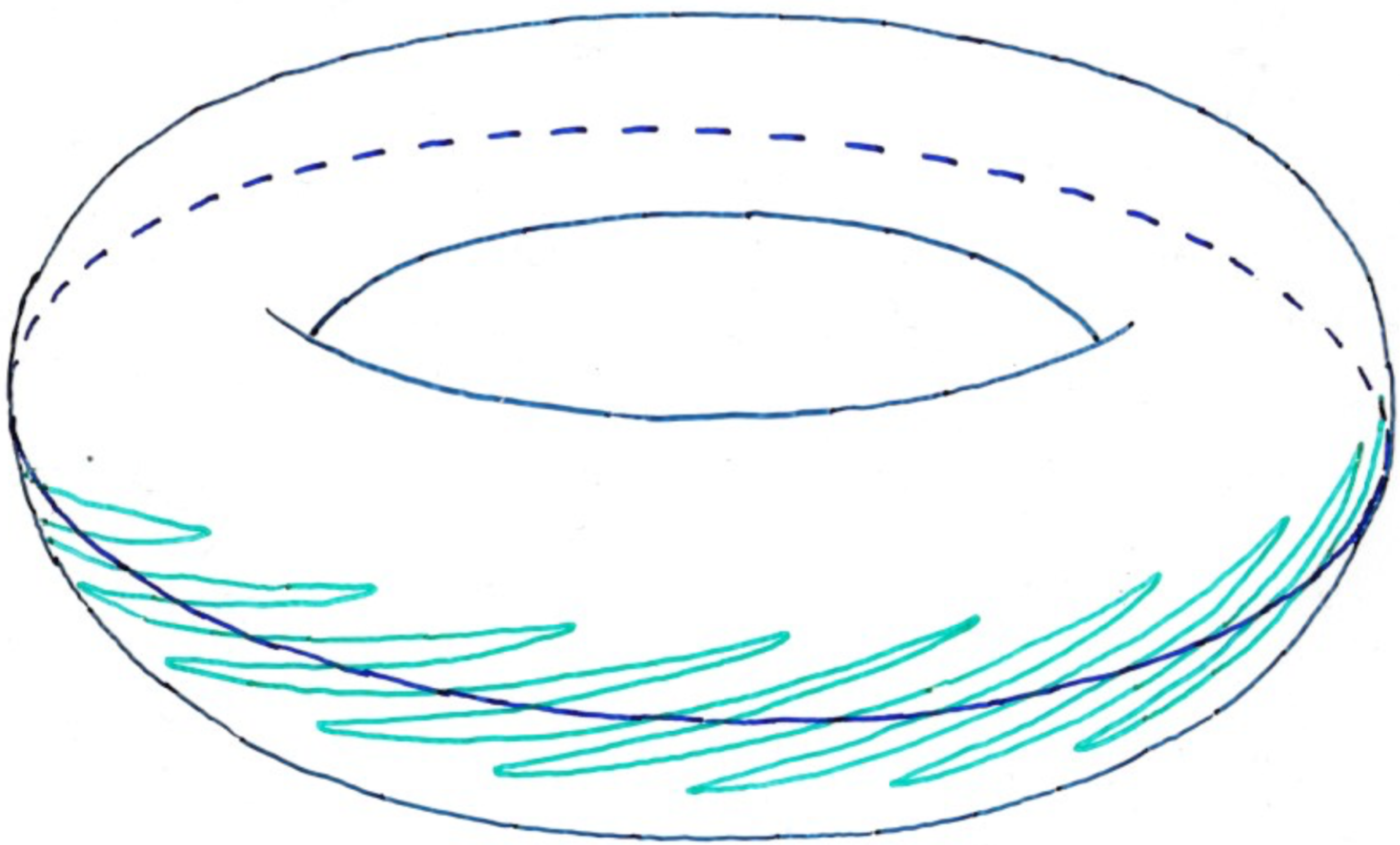
Larmor orbit $\longrightarrow$ $\omega_c = \dfrac{eB}{mc}$ $\longrightarrow$ $\mu_\perp = \dfrac{mv_\perp^2}{2B}$

Banana orbit $\longrightarrow$ $\omega_b = \dfrac{v_\perp}{qR_0}\left(\dfrac{r}{2R_0}\right)^{1/2}$ $\longrightarrow$ $J_\ell = \oint v_\parallel \, d\ell$

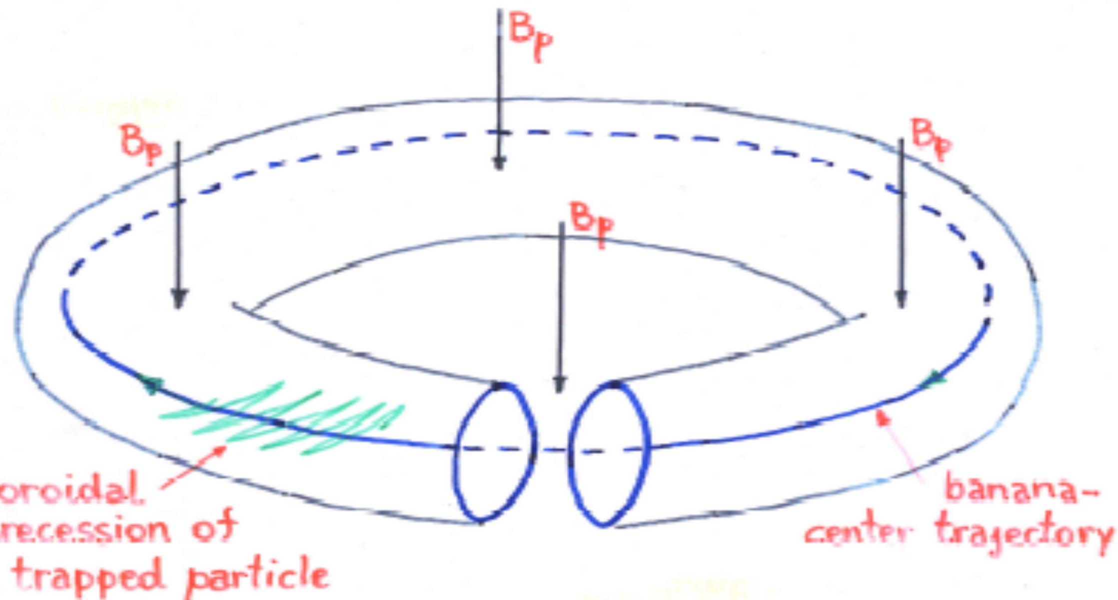
Precessional drift $\longrightarrow$ $\omega_D = \dfrac{v^2}{2\omega_c R r}$ $\longrightarrow$ $\phi = \oint v_D \, d\ell$

Third adiabatic invariant: magnetic flux enclosed by the precessional drift orbit

Precessional drift motion of banana orbits



MECHANISM → adiabatic conservation of the flux through the area defined by the precessional drift motion (third adiabatic invariant).



Condition for third adiabatic invariance:

$$\omega \ll \omega_{Dh}^{(0)} \sim \frac{\mathcal{E}_h}{\Omega_h m_h R r}$$

ω: magnetic perturbation frequency

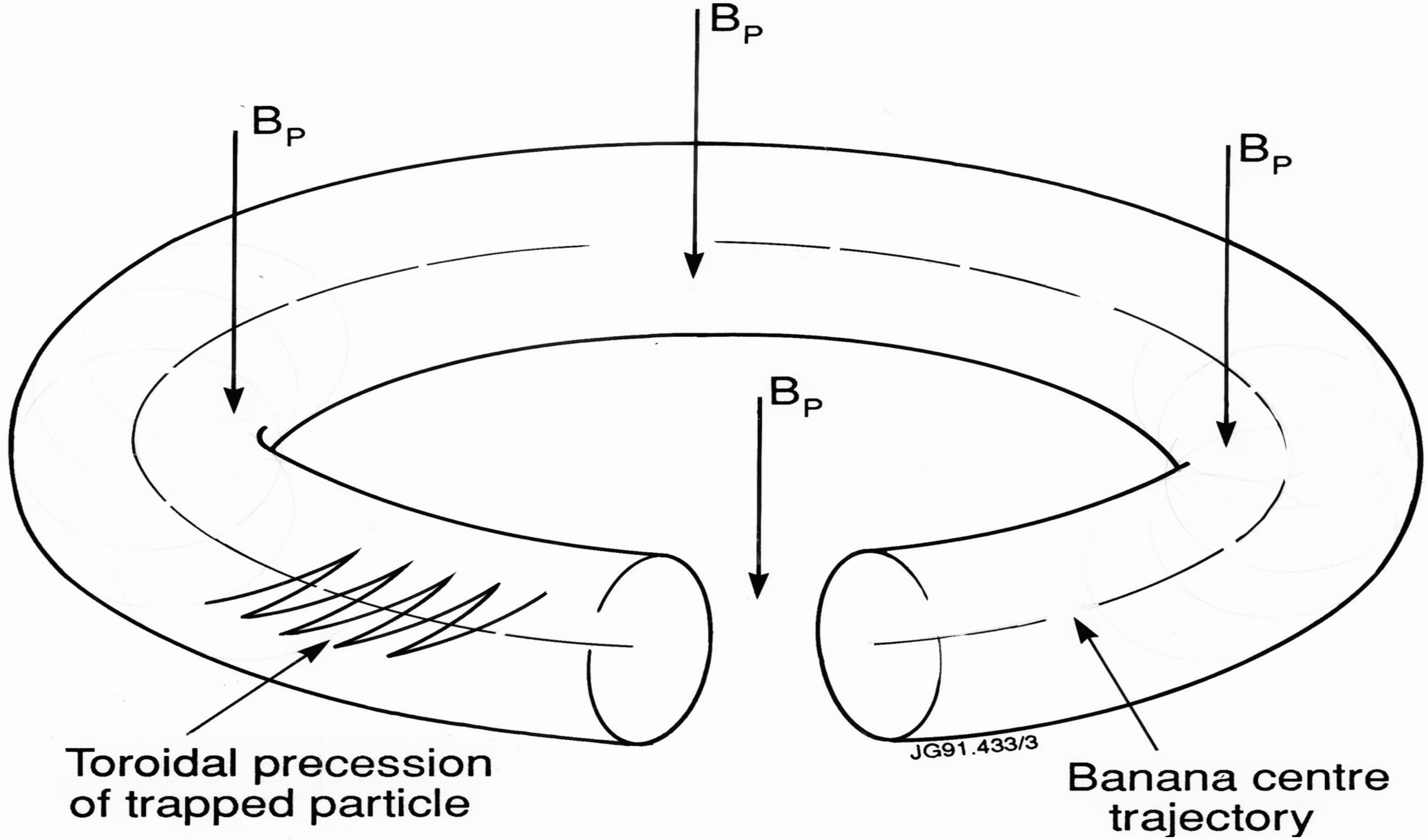
$\omega_{Dh}^{(0)} = (2\pi/\tau_D)$; τ_D → time to complete a toroidal revolution

$2\pi/\omega \sim 10^2 \tau_A$

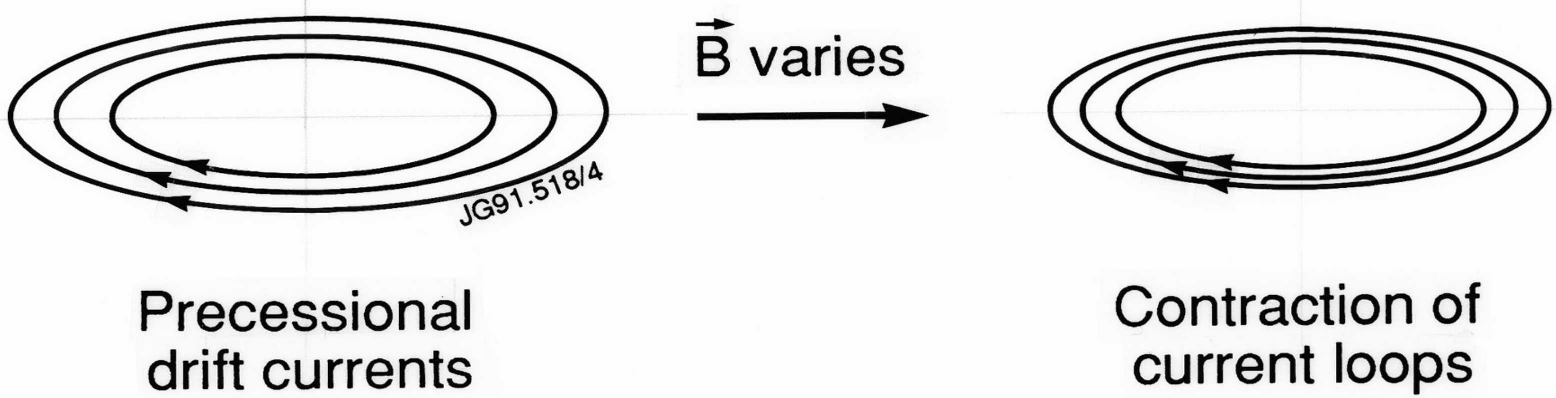
JET: $\sim 10^2 \mu s$

τ_A: Alfvén time

$\tau_D \sim$ few μs for $\mathcal{E}_h \sim 1$ MeV.

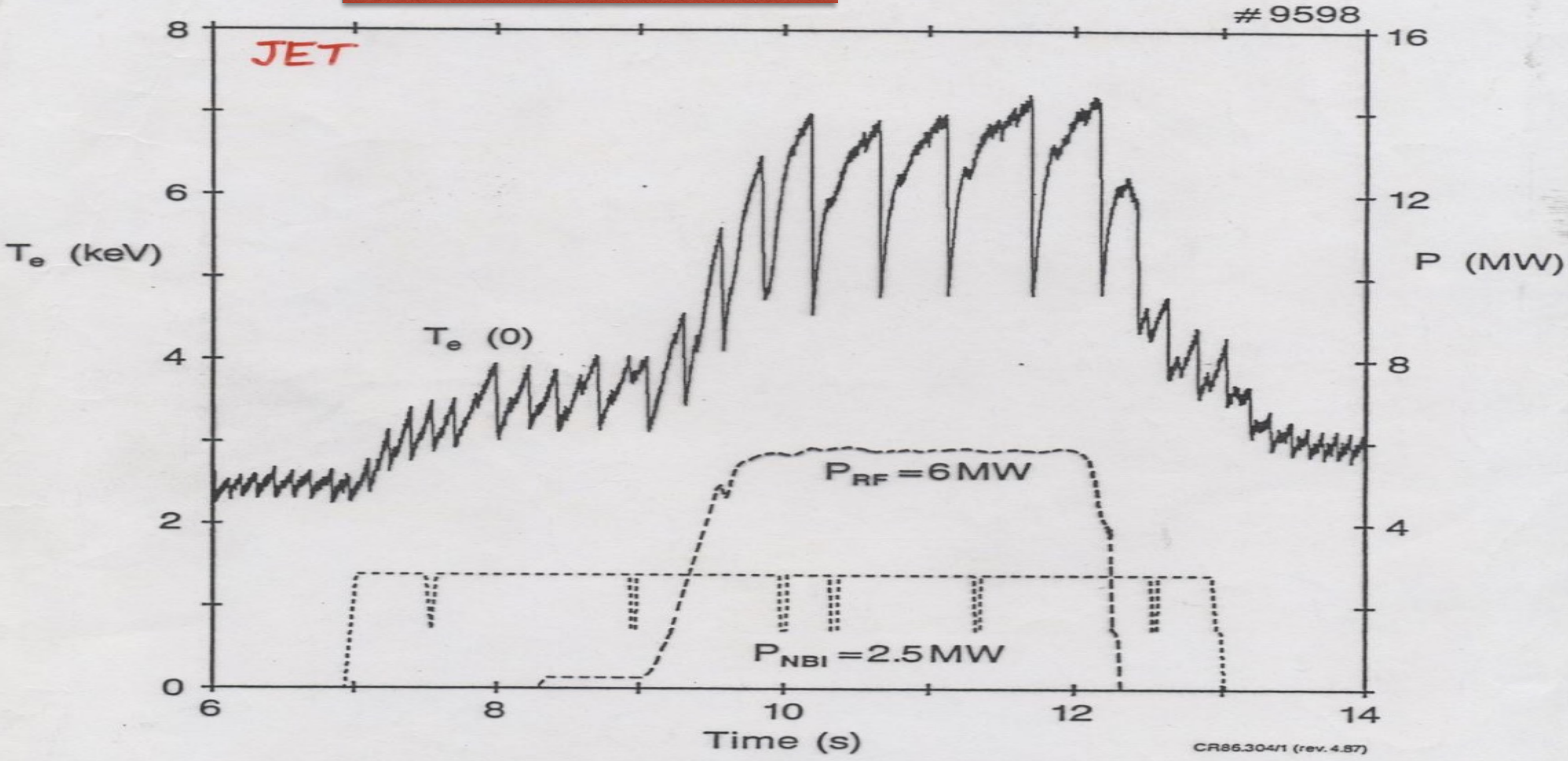


B_P
B_P
B_P
B_P
Toroidal precession of trapped particle
JG91.433/3
Banana centre trajectory



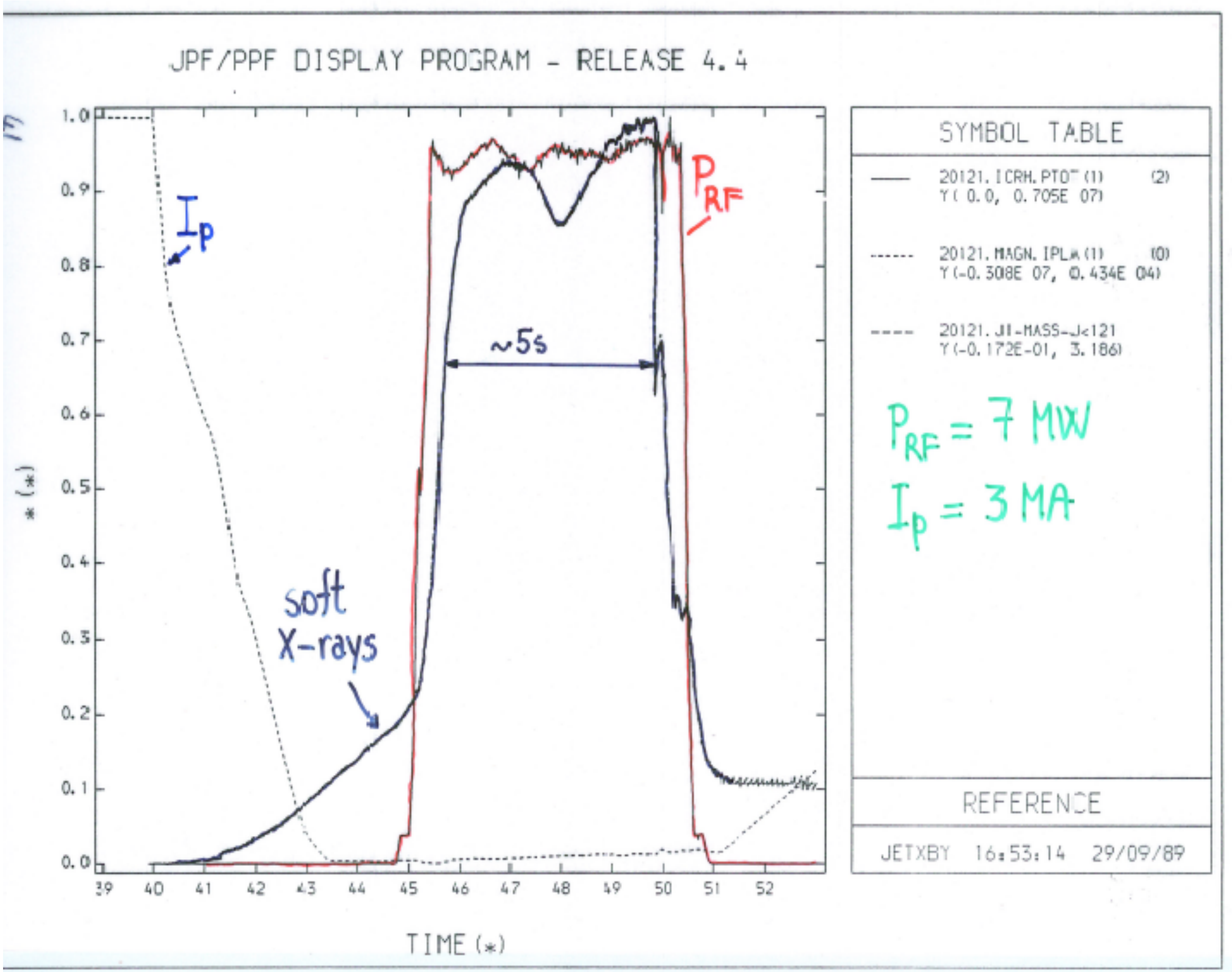
B varies
JG91.518/4
Precessional drift currents
Contraction of current loops

Sawtooth oscillations



Van Goeler et al, PRL 1973
Relaxation oscillations observed in all Tokamak discharges

Monster sawteeth at JET



Break-down of third adiabatic invariance

The criterion $\omega \ll \omega_{Dh}$ can be violated in two situations of interest:

i) fast particles not sufficiently fast

ii) too many fast particles

Violation of third adiabatic invariance

↓

resonant coupling $\omega \sim \omega_{Dh}$

↓

fishbones!

i) Diamagnetic fishbones: $\omega \sim \omega_{*i} \sim \omega_{Dh}$

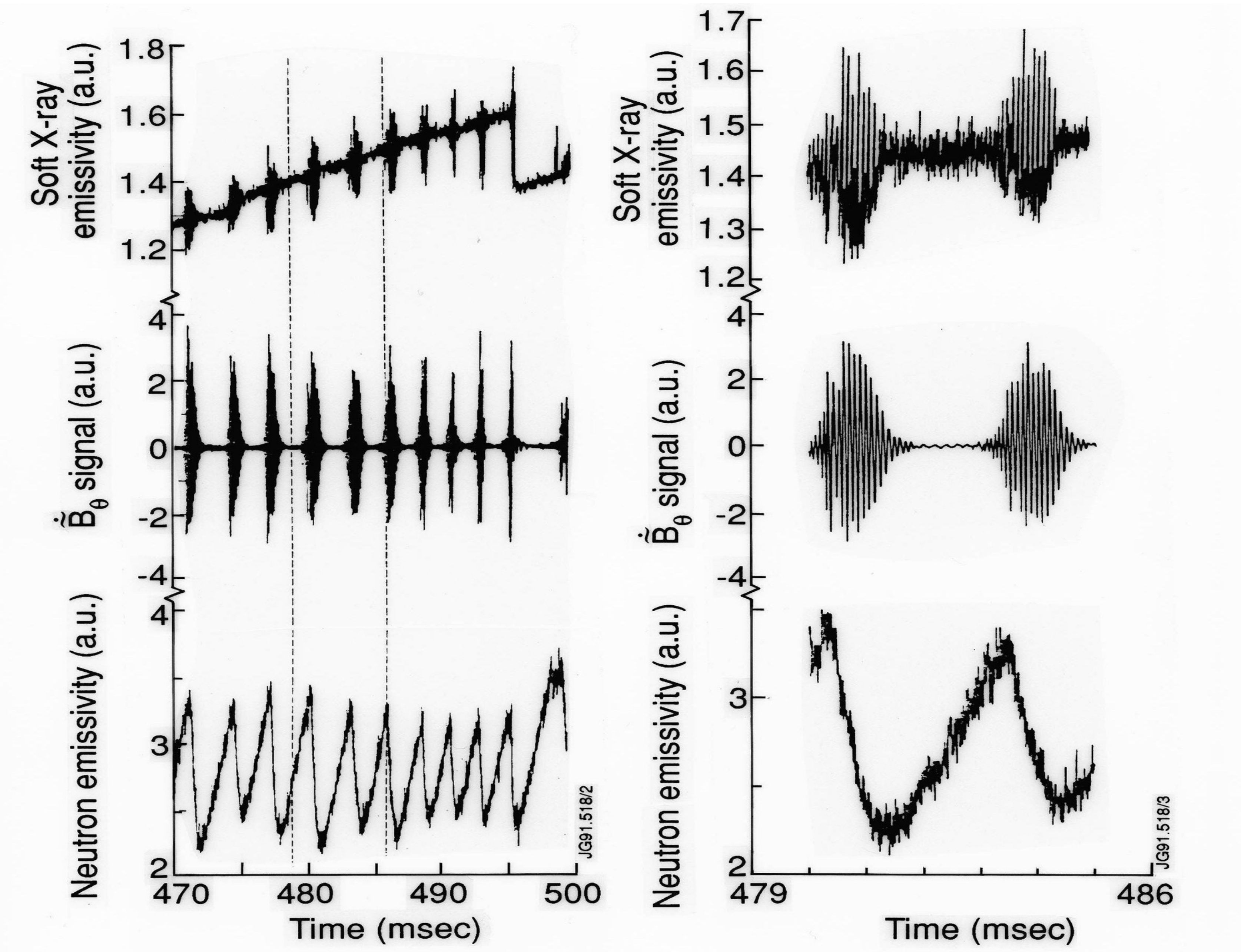
(Coppi & Porcelli PRL '86)

ii) Precessional drift fishbones: $\beta_{ph} > \beta_{crit}$

(Chen, White & Rosenbluth PRL '84) $\omega \sim \omega_{Dh}$ $(> \omega_{*i})$

F. Porcelli, PPCF '91 and other references therein

Fishbones in PBX Beam-injected plasmas



When the third adiabatic invariant is conserved, the relevant "quasi-particle" is the full precessional drift orbit.

This orbit is as large as the system size!

Even for a perturbation that is localized in space, the precessional drift orbit will react as a whole in order to conserve the enclosed magnetic flux

→ non-local response
→impossibility of a fluid closure!!

Hybrid kinetic-MHD model

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \vec{U}) = 0$$

$$\rho_m \frac{d\vec{U}}{dt} = -\nabla p_c - \nabla \cdot \overleftrightarrow{P}_h + \frac{1}{c} \vec{J} \times \vec{B}$$

$$\frac{d}{dt}\left(\frac{p_c}{\rho_m^{\Gamma}}\right) = 0$$

$$\overleftrightarrow{P}_h = p_{\perp h} \overleftrightarrow{I} + (p_{\parallel} - p_{\perp})_h \vec{e}_{\parallel} \vec{e}_{\parallel}$$

$$p_{\perp h} = m_h \int d^3v \, \frac{v_\perp^2}{2} f_h \; ; \qquad p_{\parallel h} = m_h \int d^3v \, v_\parallel^2 f_h$$

$$\frac{\partial f_h}{\partial t} + \bar{v} \cdot \nabla f_h + \frac{q_h}{m_h}\left(\bar{E} + \frac{1}{c} \bar{v} \times \bar{B}\right) \cdot \frac{\partial f_h}{\partial \bar{v}} = 0$$

Vlasov equation

$$\frac{\partial \bar{B}}{\partial t} = \nabla \times (\vec{U} \times \bar{B}) \; ; \qquad \nabla \times \bar{B} = \frac{4\pi}{c} \vec{J}$$

Drift-Kinetics

Solution of the drift-kinetic equation for global plasma modes and finite particle orbit widths

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(Received 19 August 1993; accepted 9 November 1993)

The response of a collisionless plasma to global electromagnetic perturbations of an axisymmetric toroidal equilibrium is derived. By adopting a variational formulation for guiding center motion, the perturbed distribution function is expressed in terms of the linearized guiding center Lagrangian. Finite orbit widths are retained. In particular, the high particle energy limit where mirror-trapped banana orbits are distorted into "potato-shaped" orbits is considered. In this limit, the time scales associated with the drift and bounce motions of a mirror-trapped orbit become comparable, yielding important consequences on plasma stability. Quadratic forms are constructed in the context of kinetic-magnetohydrodynamic (MHD) models of plasmas composed of a thermal component obeying fluid-like equations and a high-energy component described in terms of the collisionless drift-kinetic equation. Relevant applications include improved modeling of energetic ion effects on toroidicity-induced Alfvén gap modes and internal kinks.

Inspired by Antonsen and Lane, 1980; Antonsen and Lee, 1982.

Quadratic form (perturbed energy Integral)

Work done on fast particles by an electromagnetic perturbation ($\partial\vec{B}/\partial t = -c\nabla\times\vec{E}$):

$$\delta W_{hot} = \int d^3x \int_{-\infty}^{t} dt' \; \vec{J}_h \cdot \vec{E} \quad > 0$$

for stabilization

Assumpt. → thermal plasma obeys ideal MHD:

$$\vec{E} + \frac{1}{c}\vec{V}\times\vec{B} = 0, \qquad \vec{V}_\perp = \frac{d\vec{\xi}_\perp}{dt}$$

← displacement of thermal plasma

Use fast particles momentum balance, neglecting inertia and collisions:

$$0 \simeq Z_h e n_h \vec{E} - \nabla\cdot\overleftrightarrow{P}_h + \frac{1}{c}\vec{J}_h\times\vec{B}$$

↓

$$\delta W_{hot} = \frac{1}{2}\int d^3x \; \vec{\xi}_\perp^{\,*}\cdot\left(\nabla\cdot\delta\overleftrightarrow{P}_h\right)$$

Work is done against the pressure of the fast particles.

Perturbed fast ion pressure

$$\delta p_{\perp h} = \int d^3v \, \mu B \, \delta f_h$$

δf_h is the solution of the drift-kinetic equation

$$\delta f_h = -\xi_\perp \cdot \nabla F_h + (\omega - \omega_{*h}) \frac{\partial F_h}{\partial \mathcal{E}} \sum_{-\infty}^{\infty} {}_p \frac{\Upsilon_p \exp[i(n\omega_{Dh} + p\omega_{bh})\tau]}{\omega - n\omega_{Dh} + p\omega_{bh}}$$

$$\Upsilon_p = -\oint \frac{d\tau}{\tau_b} \mu B \, \vec{\xi}_\perp \cdot \vec{\kappa} \, e^{-ip\omega_b \tau}$$

Fast particle limit : $\omega_{bh}, \; \omega_{*h} \gg \omega_{Dh} \gg \omega$

$$\delta f_h = -\xi_\perp \cdot \nabla F_h - \frac{\partial F}{\partial P_\varphi} \frac{\langle \mu B \, \vec{\xi}_\perp \cdot \vec{\kappa} \rangle}{\omega_{Dh}}$$

$\omega_{Dh} = \langle \dot{\varphi} \rangle_h$

$\langle \cdot \rangle$: bounce average

(zero frequency limit)

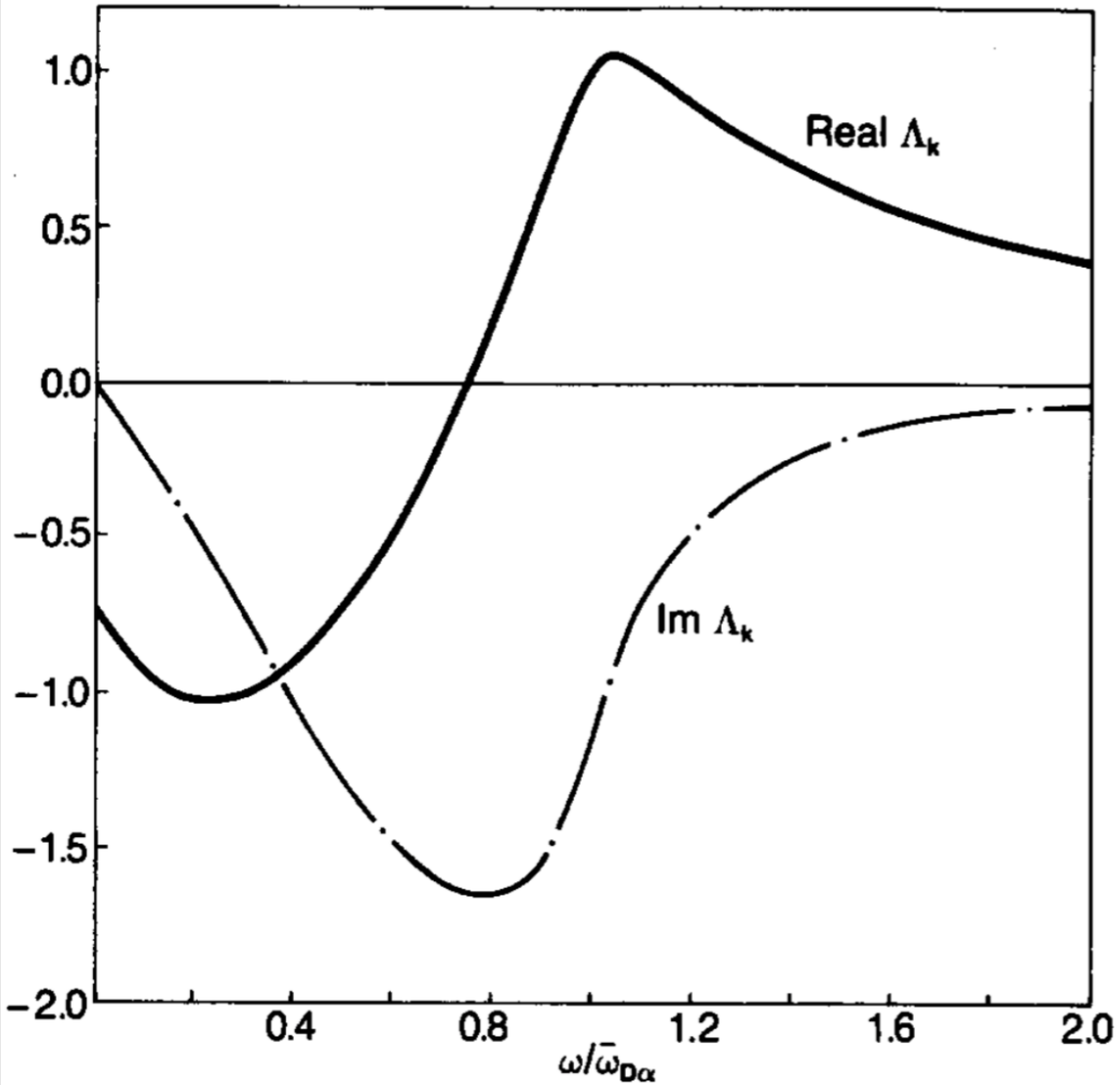

FIG. 1. Real and imaginary parts of $\Lambda_K(\omega/\overline{\omega}_{D\alpha}) \equiv s_0 \beta_{p\alpha}^{-1} \epsilon_0^{-3/2} \lambda_K(\omega/\overline{\omega}_{D\alpha})$, where λ_K is the alpha-particle contribution to the dispersion relation [see Eq. (22)]. A parabolic q profile inside the $q = 1$ surface with $q(0) = 0.8$ has been assumed.

Conclusions

- The things that I understood better in plasma physics have been inspired for the most part by Francesco.
- The theory of magnetic reconnection in the ion-kinetic regime was the key to the development of a model for the sawtooth period and amplitude in realistic tokamak regimes.
- Kinetic-MHD closures were fundamental to the understanding of fast ion stabilization of MHD modes.
- Consideration of the relevant kinetic effects and the analogy between conserved quantities in collisionless magnetic reconnection and phase mixing in Vlasov theory were key points for the understanding of nonlinear reconnection theory in collisionless regimes.