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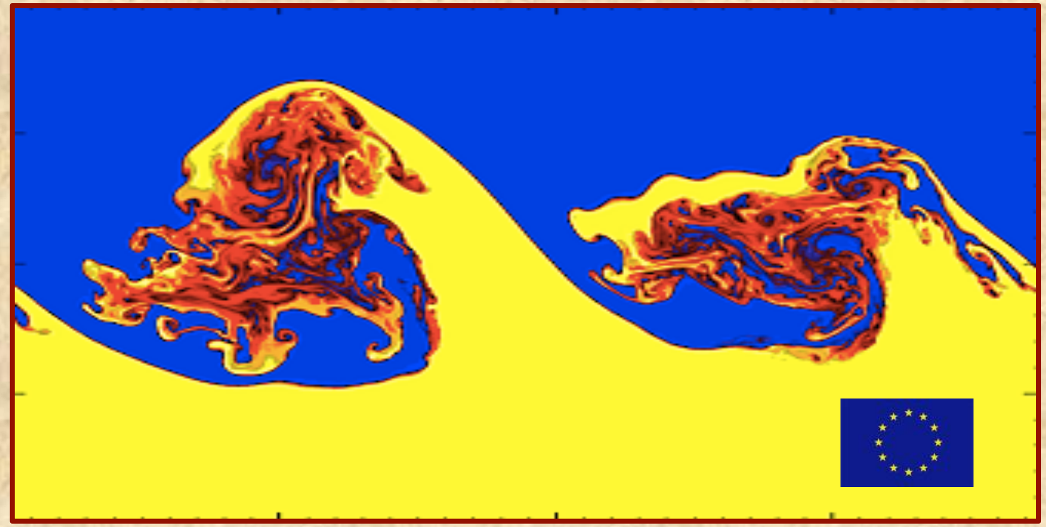
The fluid approach:

Basic instabilities in magnetized plasma



PhD Doctoral School: *Fundamental processes in Space Weather: a challenge in numerical modeling*

A plasma is a

collection of discrete charged particles

globally neutral,

behaving as a collective system,

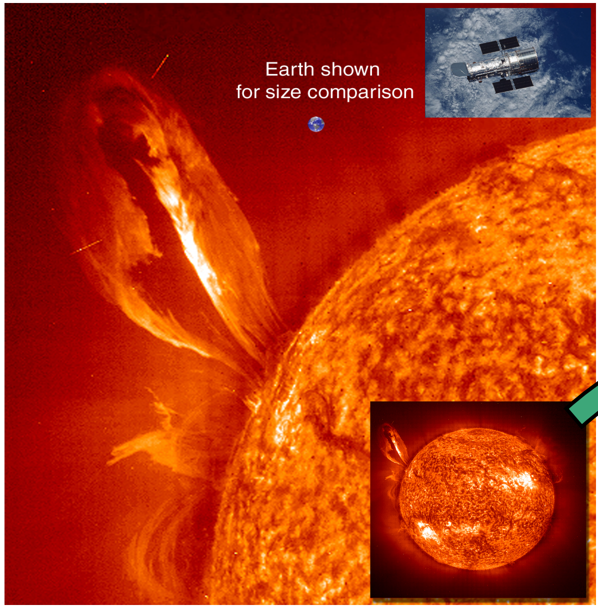
dominated by electromagnetic forces

Collective response of the plasma at the

PLASMA FREQUENCY

$$\omega_{pe} = \sqrt{4\pi n e^2/m_e}$$

When a plasma is disturbed from the equilibrium condition, the resulting space charge fields give rise to collective high frequency oscillations at plasma frequency that tend to restore charge neutrality



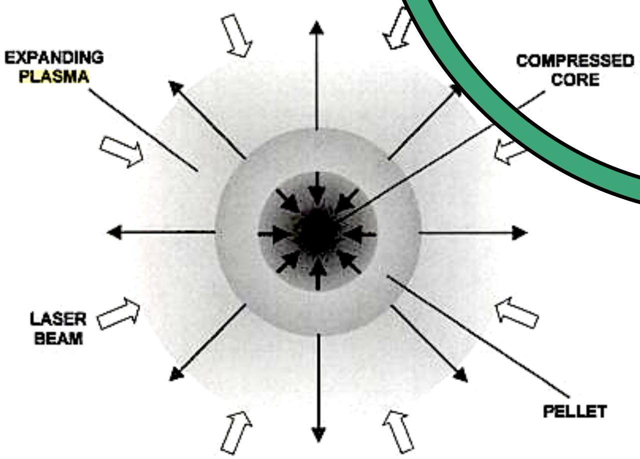
Natural and laboratory plasmas

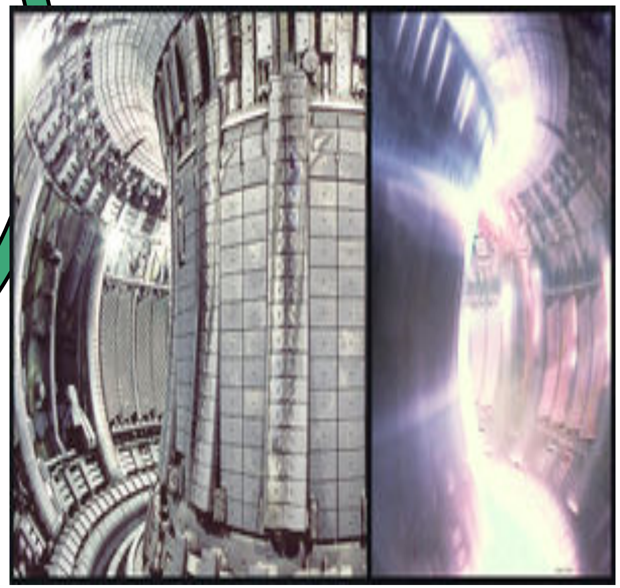
Complex and fascinating systems

For many observed phenomenon

same physical mechanism

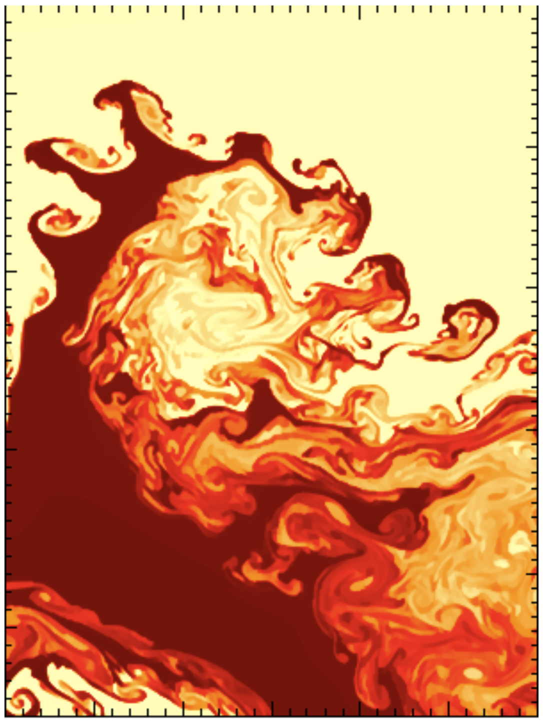
Differences and Similarities

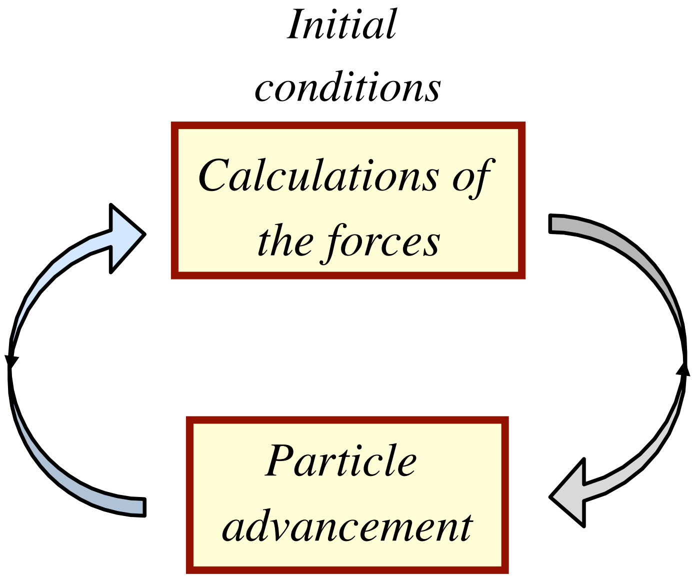




Problem: how to model a plasma ?

Too many particles for a N-body description even for modern super-computing systems





Computationally too heavy !

"NEEDS" FOR A CONTINUOUS DESCRIPTION

High temperature, tenuous plasmas usually found in space and in the laboratory can be considered as collisionless

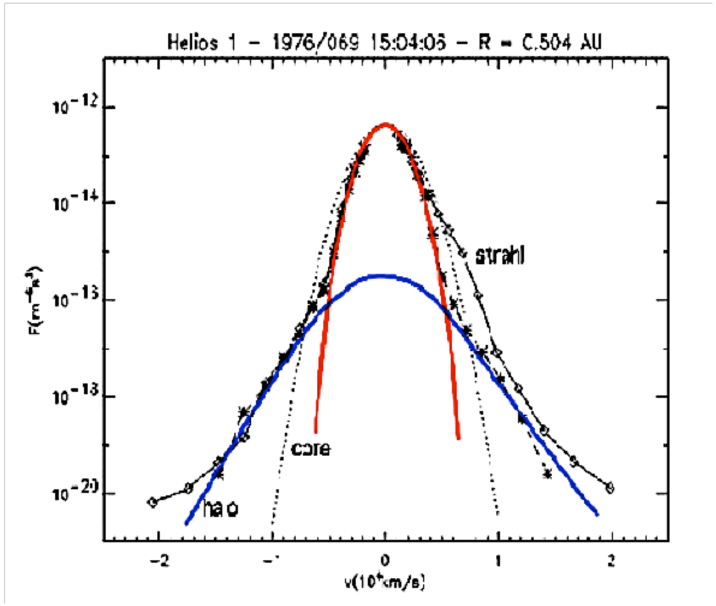
Typically, the diffusive time scale is many orders of magnitude larger than any dynamical or kinetic time scale:

Magnetic Reynolds

$$10^6 \leq R = \frac{\tau_{diff}}{\tau_{dyn}} \leq 10^{12}$$

non-Maxwellian distribution functions often observed as in the solar wind

An example, the ***Solar Wind****: no time to reach thermodynamical equilibrium: Temperature means "average energy"*



Non-Maxwellian particle distribution function (electrons)

Neutral Gaz: $\nu_{coll} \gg \omega$

Plasma: $\omega \gg \nu_{coll}$

Problems for plasma thermodynamics!

But we can do something....

Free charges with kinetic (thermal) energy much larger than the typical potential energy due to its nearest neighbor (ex. e^- and p^+)

$$E_k >> \Phi \quad or \quad n_0^{1/3} e^2 << m v_{th}^2$$

where $n_0^{1/3}$ is the mean particle distance

We need:

$$\Lambda_D = n\lambda_D^3 >> 1$$

$$\lambda_D = \sqrt{T / 4\pi n e^2}$$

Very large number of particles in a Debye sphere

where

λ_D = Debye length,

Λ_D = number of particles in a Debye sphere

In other words, particles must be ***"quasi non-correlated"***

free charges with kinetic (thermal) energy much larger than the potential energy due to its nearest neighbor: $E_k >> V$

$g = 1/n\,\lambda_D^3$ plasma parameter [$g \sim \nu_c / \omega_{pe}$]

$g << 1$ plasma approximations

$r_0 = e^2 / T$ [distance of min. approach ($E_k \sim \Phi_C$)]

$r_n = n^{-1/3}$ [mean particle distance]

$\lambda_D = \sqrt{T / 4\pi n e^2}$ [Debye length]

$d_{e/i} = c / \omega_{e/i}$ [e/i inertial length]

L_{HD} [hydro scale]

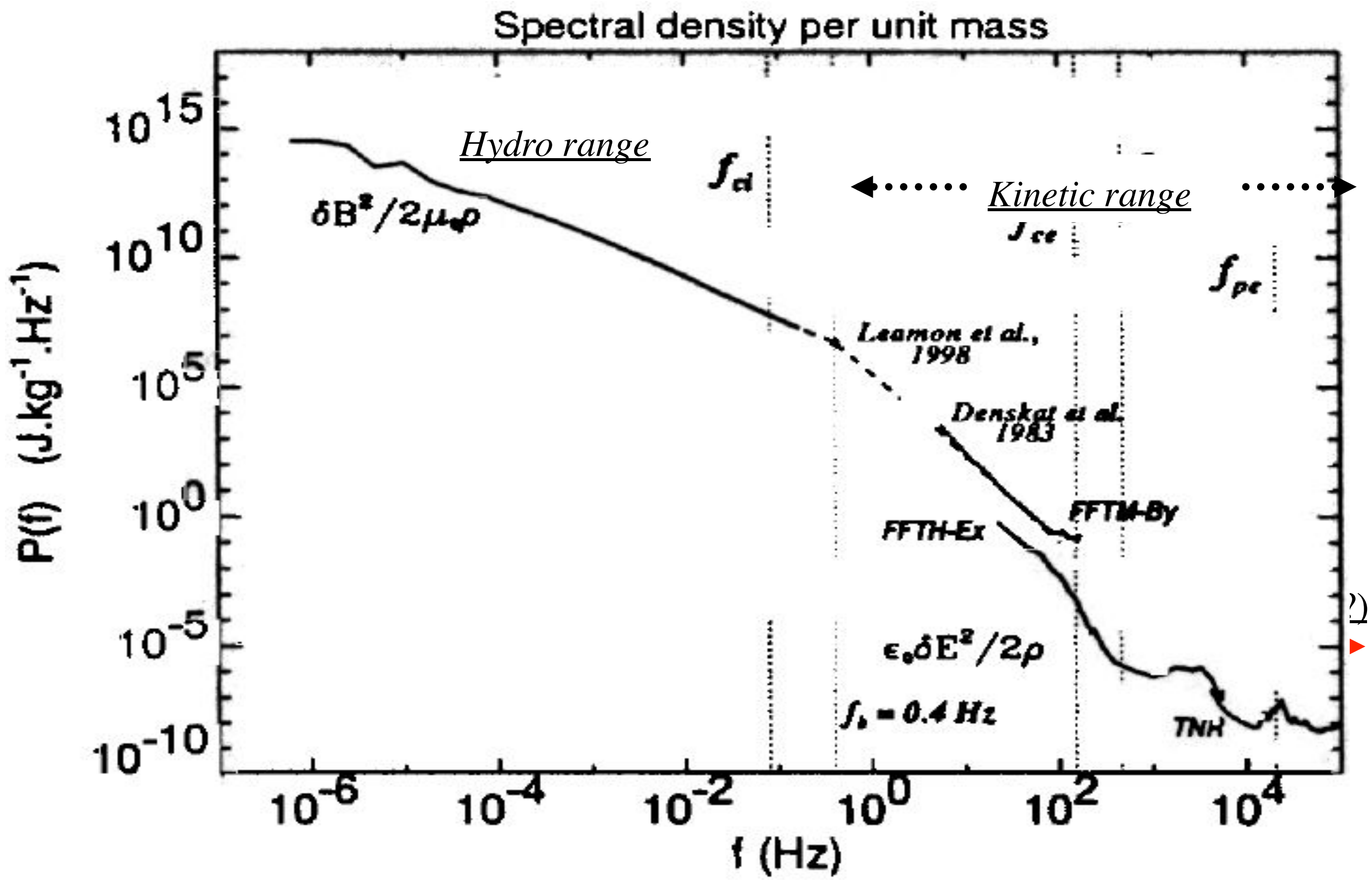
l_{mfp} [mean free path]

$L > \lambda_D$

A plasma must be larger than the Debye lebgth

(screen distance)

Many frequencies and scale lengths at play self-consistently coupled



*The statistical description of a **N particles plasma** is based on the **probability densities** **F** giving the probability of finding simultaneously the particles at locations (x_1, x_2, ..., x_N) in phase space. Too much complicate!*

The probability $f_1(x_1,v_1)$ of finding particle 1 at location x_1 is given by integrating the d.f. allover the particles except 1:

$$f_1(x_{1,}v_{1,}) = \int F_n(x_{2,}...,x_{n,}v_{2,}...,v_{n,})dx_{2,}...,x_{n,}v_{2,}...,v_{n,}$$

The probability density contains the effects of the interactions among particles

When the interaction potential can be neglected, the particles can be considered as ***statistically independent****:*

$$F_2(x_1 , x_2) = F_1(x_1) F_1 (x_2)$$

When instead the interaction potential among particles is present, the probability densities can be written trough a cluster expansion:

$$F_2(x_1 , x_2) = F_1(x_1) F_1 (x_2) [1 + P_{12} (x_1 , x_2)]$$

and so on for F_i , $i > 2$

P_{12} : *two particle correlation function*

In general, *<u>single particle interactions are assumed as negligible</u>*

$P_{12} \ll$ *1 (and so on)*

Neglecting single particle interactions, one make use of the ***probability*** $f^{(1)}$ *of finding particle 1 at location* $\mathbf{x}_1$ *in phase space* *under the action of the electromagnetic fields generated by the full system* - ***mean field theory***

In summary, a plasma is described in a reduced way in terms of the

$$f^{(1)}(\mathbf{x}_1, \mathbf{v}_1)\, d\mathbf{x}_1 d\mathbf{v}_1$$

CONTINUUM approach

One particle distribution function

Moments of the distribution function

Inter-particle forces can be dived into:

1. mean force ("many" distant particles)
2. Force due to nearest neighbor particles

Forces that do not depend on the exact location of all particles have the appearance of external forces.

PLASMA COLLISIONLESS DYNAMICS IS DESCRIBED BY THE

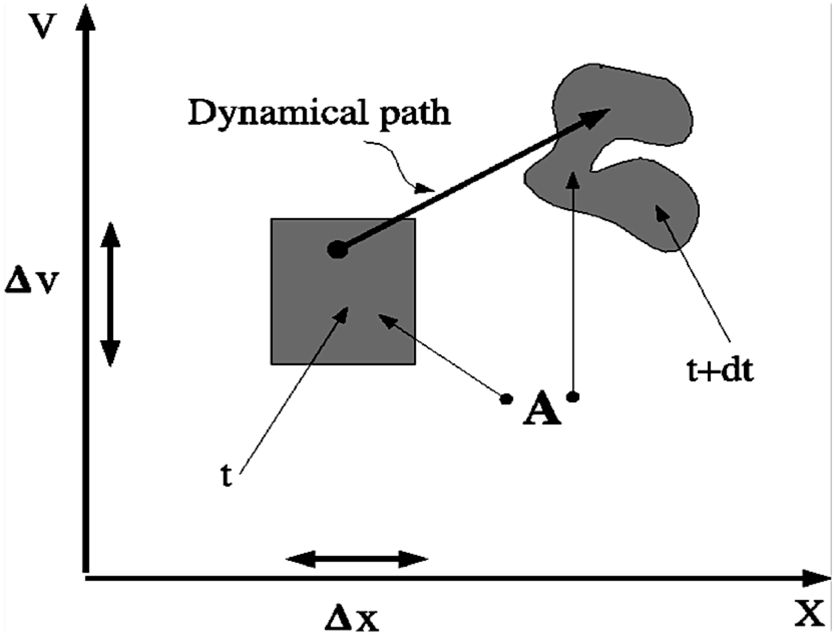
In a low density system the mean force due to many distant particles far exceeds the inter-particles forces: $\mathbf{a} = \mathbf{a}_{ext} + < \mathbf{a}_{int} > \simeq \mathbf{a}_{ext}$

Vlasov equation

$$\frac{\partial f_a}{\partial t} + \underline{v}\frac{\partial f_a}{\partial \underline{x}} + \frac{q_a}{m}\left(\underline{E} + \frac{\underline{v} \times \underline{B}}{c}\right)\frac{\partial f_a}{\partial \underline{v}} = 0$$

(neglecting collisions.)

The Vlasov equation is basically an advection equation in phase space:



Liouville's theorem:
The phase space volume can be deformed but its density is not changed during the dynamical evolution of the plasma.

It can be considered as a "transport" equation in phase space.

From Vlasov to a fluid approach

Kinetic
3D-3V

Fluid
3D

Macroscopic variables of a plasma

The macroscopically observable quantities are found from the velocity moments of the *d.f.* :

Number of particles, Current density:

$$n_a(\mathbf{x},t) = \int f_a(\mathbf{x},\mathbf{v},t)\, d\mathbf{v}$$

$$\mathbf{J}_a(\mathbf{x},t) = q_a n_a(\mathbf{x},t)\mathbf{V}_a(\mathbf{x},t) = q_a \int \mathbf{v} f_a(\mathbf{x},\mathbf{v},t)\, d\mathbf{v}$$

Pressure tensor, Scalar pressure:

$$P_a(\mathbf{x},t) = m_a \int (\mathbf{v}-\mathbf{V}_a)(\mathbf{v}-\mathbf{V}_a) f_a(\mathbf{x},\mathbf{v},t)\, d\mathbf{v}$$

$$p_a = \frac{1}{3}(p_{xx}+p_{yy}+p_{zz}) = n_a T_a$$

ONE FLUID EQUATIONS -MHD

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \tag{1}$$

$$\rho \frac{\partial \mathbf{U}}{\partial t} + \rho(\mathbf{U} \cdot \nabla)\mathbf{U} = \frac{\mathbf{j} \times \mathbf{B}}{c} - \nabla P \tag{2}$$

$$\mathbf{E} + \frac{\mathbf{U} \times \mathbf{B}}{\mathbf{c}} = 0 \tag{3}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{4}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j}. \tag{5}$$

TWO FLUIDS EQUATIONS

$$\frac{\partial(n\mathbf{U})}{\partial t} + \nabla\left[n(\mathbf{u}_i\mathbf{u}_i + \varepsilon\mathbf{u}_e\mathbf{u}_e)\right] = -\frac{1}{m_i}\nabla\cdot\underline{(\mathbb{P}_i + \mathbb{P}_e)} + \frac{\mathbf{J}\times\mathbf{B}}{m_i c}$$

$$\frac{\partial n_a}{\partial t} + \frac{\partial}{\partial x_i}(n_a u_{a_i}) = 0$$

$\mathbf{U} = \mathbf{u}_i + \varepsilon\, \mathbf{u}_e\ ;\ \varepsilon = m_e / m_i$

$$\frac{d}{dt}\left(P_a \cdot n_a^{-\gamma_a}\right) = 0$$

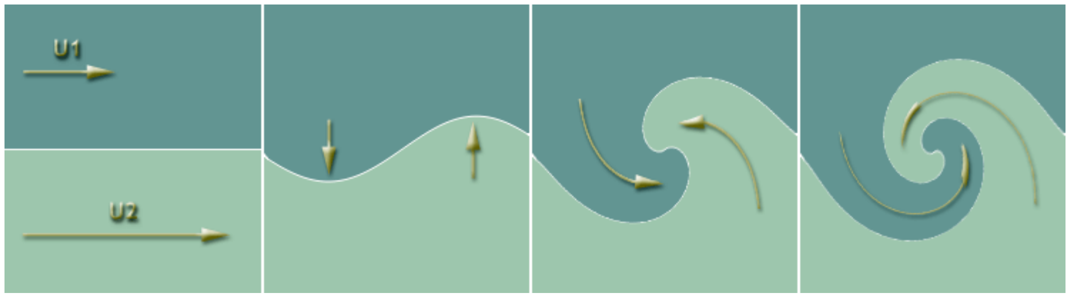
$$\left[1 + \varepsilon(1 - \underline{d_e^2\nabla^2})\right]\mathbf{E} = -\frac{(\mathbf{u}_e + \varepsilon\mathbf{u}_i)\times\mathbf{B}}{c} - \frac{1}{en}\nabla\left[\mathbb{P}_e - \cancel{\varepsilon\mathbb{P}_i} - \varepsilon m_i n(\mathbf{u}_i\mathbf{u}_i - \mathbf{u}_e\mathbf{u}_e)\right]$$

$$\mathbf{J} = \frac{c}{4\pi}(\nabla\times\mathbf{B}) + \cancel{\frac{1}{4\pi}\frac{\partial\mathbf{E}}{\partial t}} \simeq \frac{c}{4\pi}(\nabla\times\mathbf{B})$$

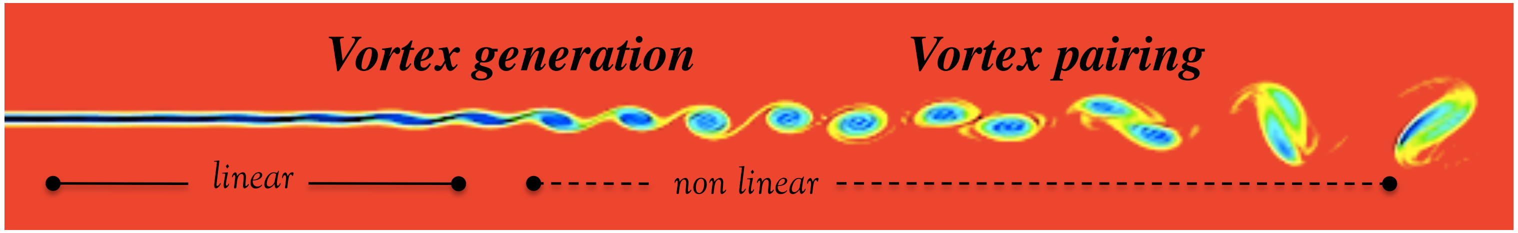
$$\frac{\partial\mathbf{B}}{\partial t} = -\,c\,\nabla\times\mathbf{E}$$

KELVIN-HELMHOLTZ INSTABILITY

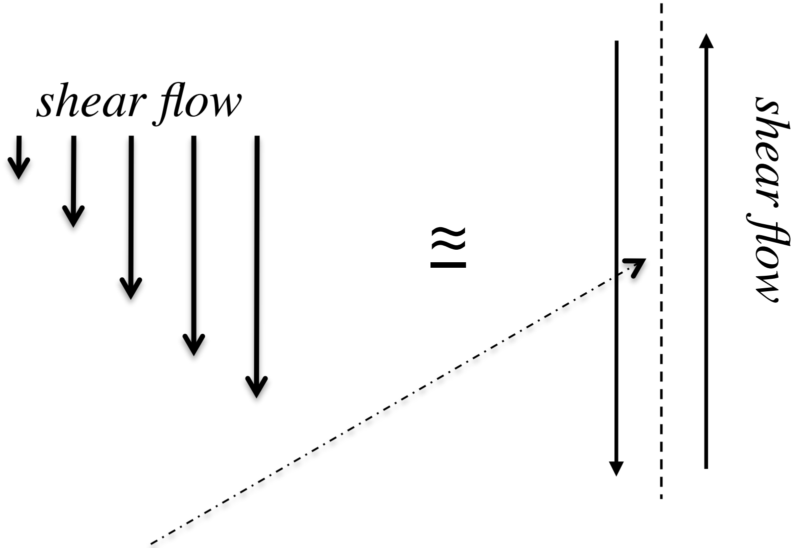
- It develops in the presence of a sheared flow
- It generates **vortices** in the flow plane



The K-H instability is of hydrodynamic nature



It is driven by a velocity shear perpendicular to the flow direction

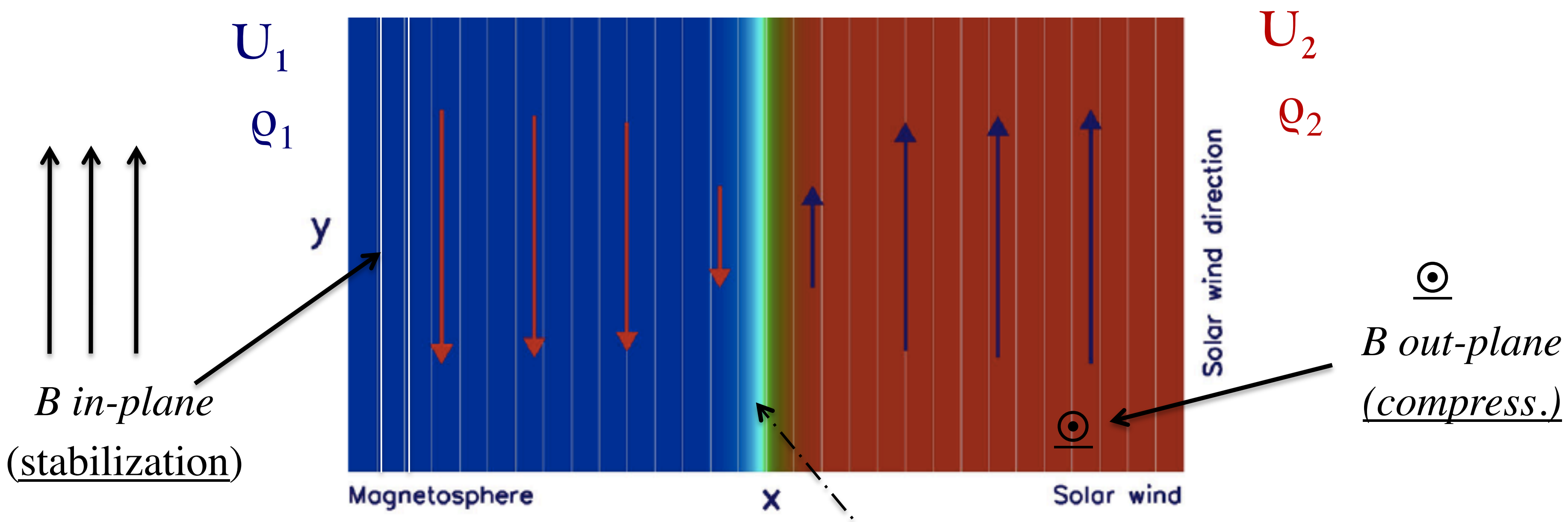


We assume an
<u>incompressible plasma</u>

plasma compressibility tends to stabilize

U = 0 ! Galilean transformation
(not equivalent in reconnection with a sheared B!)

Magnetized Kelvin – Helmholtz instability



Important Parameters:

Mach sonic,	$M_s = \Delta U / c_s$
Mach Alfvenic,	$M_A = \Delta U / c_A$
Mach magnetosonic	$M_f = \Delta U / \sqrt{c_s^2 + c_A^2}$

Linear regime

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0$$

$$\rho \frac{\partial \mathbf{U}}{\partial t} + \rho(\mathbf{U} \cdot \nabla)\mathbf{U} = \frac{\mathbf{j} \times \mathbf{B}}{c} - \nabla P$$

$$\nabla \cdot \overline{u} = 0$$

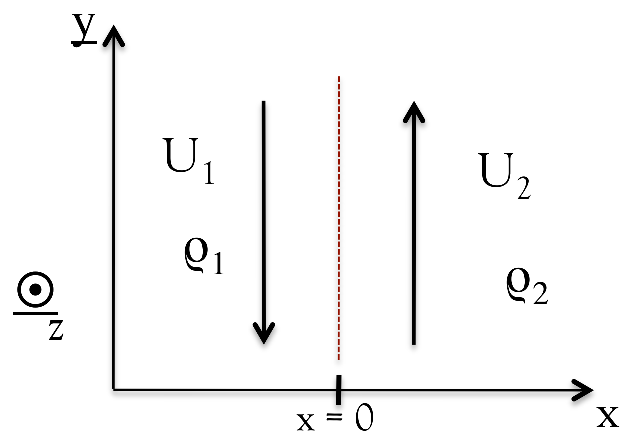
valid when $M_s = U_0 / C_s \ll 1$

Vel. Shear flow

Discontinuous simple model

$U_1 , \varrho_1 \; ; \quad x > 0$

$- U_2 \;\; \varrho_2 \; ; \quad x < 0$



$$\text{Eigen functions} \approx f(x)\, exp[i(k_y y + k_z z - \gamma t)]$$

By linearizing the HD equations:

$$\left(\frac{d^2}{dx^2} - k_y^2 - k_z^2\right)\tilde{u}_x = 0 \qquad (1)$$

By imposing the continuity of the displacement vector at the interface, we get

$$\left[\frac{\tilde{u}_x}{(\gamma + k_y U_1)}\right]_{-\epsilon} = \left[\frac{\tilde{u}_x}{(\gamma + k_y U_2)}\right]_{\epsilon}$$

By integrating (1) *between –ε and ε we get the second condition*

$$\left[\rho(\gamma + k_y U)\frac{d\tilde{u}_x}{dx} - \rho k_y \left(\frac{dU}{dx}\right)\tilde{u}_x\right]_{-\epsilon} = \left[\rho(\gamma + k_y U)\frac{d\tilde{u}_x}{dx} - \rho k_y \left(\frac{dU}{dx}\right)\tilde{u}_x\right]_{\epsilon}$$

Finally, by imposing $u_x \longrightarrow 0$ *for* $x \longrightarrow \pm\infty$*, we get*

$$\gamma = -\frac{k_y}{\rho_1 + \rho_2}(\rho_1 U_1 + \rho_2 U_2) \pm \left[-\frac{k_y^2}{(\rho_1 + \rho_2)^2}\rho_1 \rho_2 (U_1 - U_2)^2\right]^{\frac{1}{2}}$$

HD DISPERSION RELATION

An out-of-plane influences the KH by affecting the plasma compressibility

An in-plane directly stabilizes the KH due to magnetic tension

$$\gamma = -\frac{k_y}{\rho_1+\rho_2}(\rho_1 U_1 + \rho_2 U_2) \pm \left[\frac{k_y^2 B^2}{2\pi(\rho_1+\rho_2)} - k_y^2 \frac{\rho_1\rho_2}{(\rho_1+\rho_2)^2}(U_1-U_2)^2\right]^{\frac{1}{2}}$$

DISPERSION RELATION with B

As a result, the magnetic tension inhibits the KHI if

$$\varrho_1\varrho_2\,(U_1 - U_2)^2 / (\varrho_1 + \varrho_2) \leq \overline{V}_A^2 \qquad \overline{V}_A = B / \sqrt{2\pi(\varrho_1 + \varrho_2)}$$

$$(U_1 - U_2)^2 / 4 \leq \overline{V}_A^2 \qquad \text{if } \varrho_1 = \varrho_2 = \varrho$$

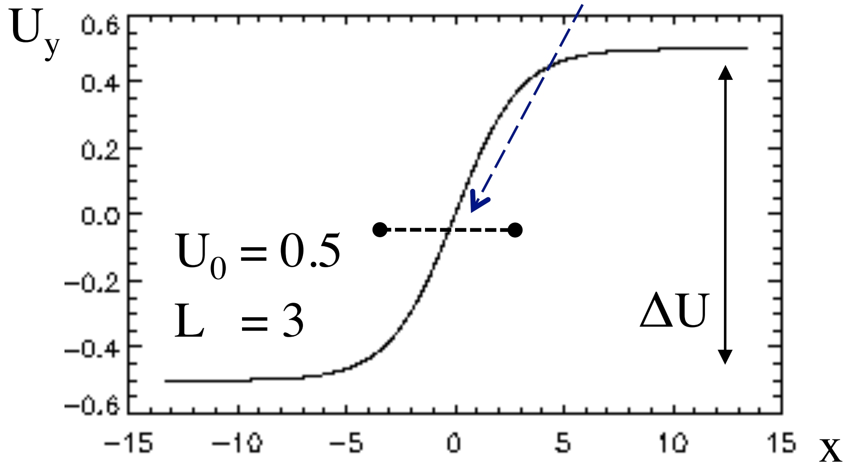
Note that the analysis is 3D ($k_z \neq 0$), *but the most unstable modes are 2D;*
The vortices will be generated in the (x,y) *plane.*

S. Chandrasekhar. Hydrodynamic and Hydromagnetic Stability, Oxford University Press, 1961.

Finite velocity shear layer

$$U_y(x) = (U_0 / 2) \tanh(x/L)$$

γ_{max} at $2k_yL \sim O(1)$



$k_yL << 1$: homogeneous flow!

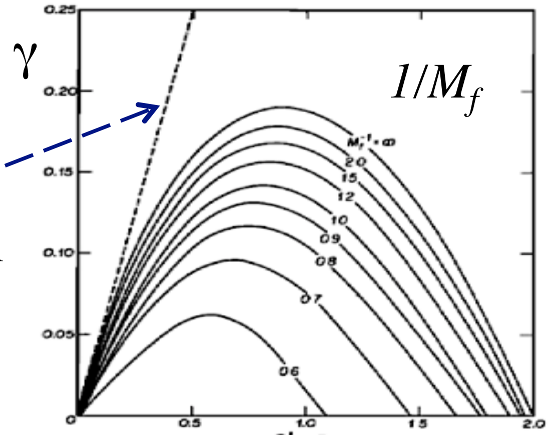
finite velocity shear layer

Unstable only for $M_f < 2$

γ decreases for M_f increasing

Unstable only for $M_A > 2$, $M_s < 2$

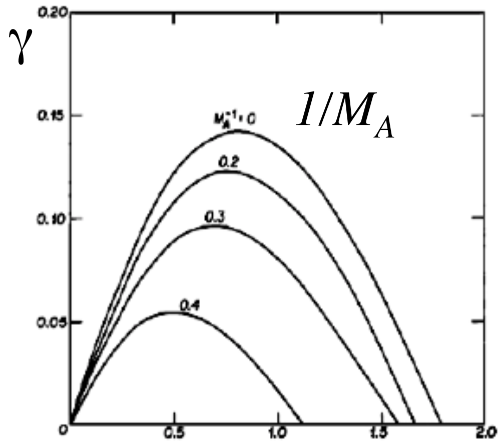
B_{out} , compr



k_yL

discont. incompr. model

B_{in} , compr *M_s fixed*



k_yL

$M_s = \Delta U / c_s$

$M_{A,||} = \Delta U / c_{A,||}$

main parameter controlling

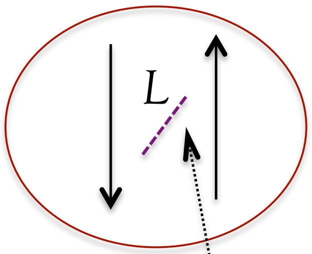
compress.: $M_f = \Delta U / \sqrt{c_s^2 + c_A^2}$)

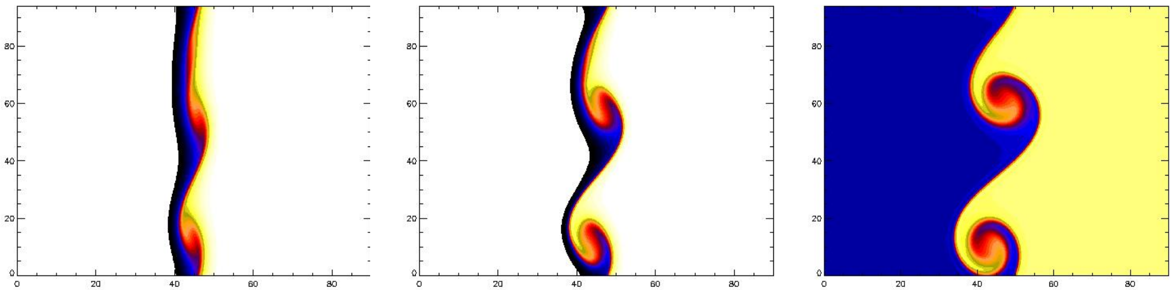
Magnetic tension stabilizes

A. Miura e P. L. Pritchett. J. Geophys. Res., 87(A9) 7431, 1982

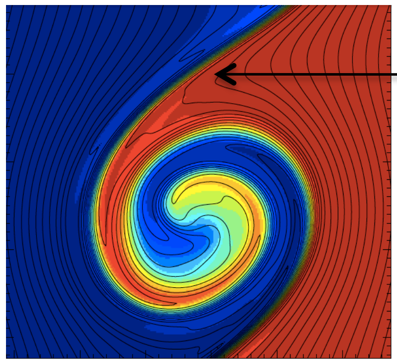
Example of KH vortices
Numerical integration of MHD equations

End of linear regime *Non linear regime: fully rolled-up vortices*





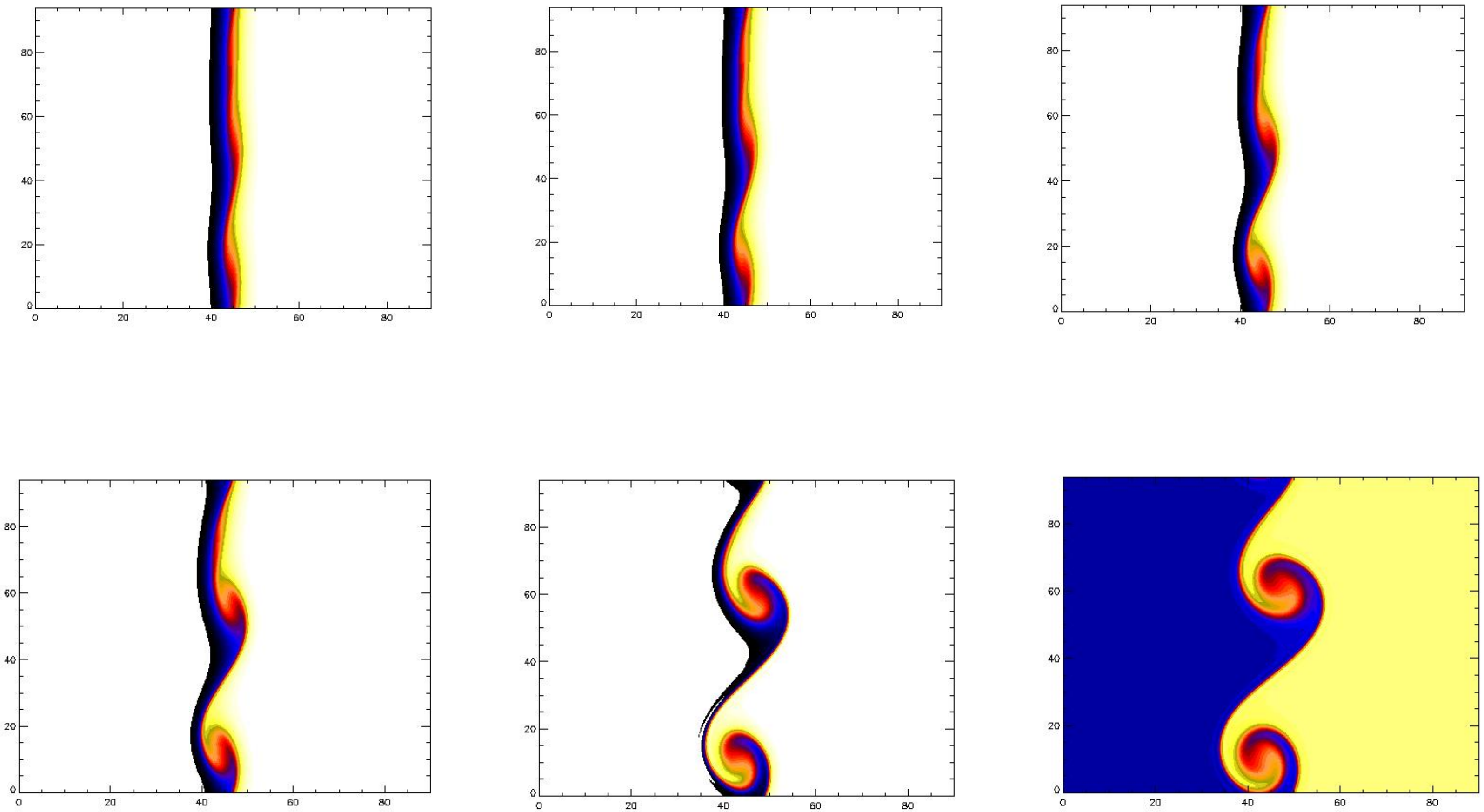
The only characteristic length in MHD



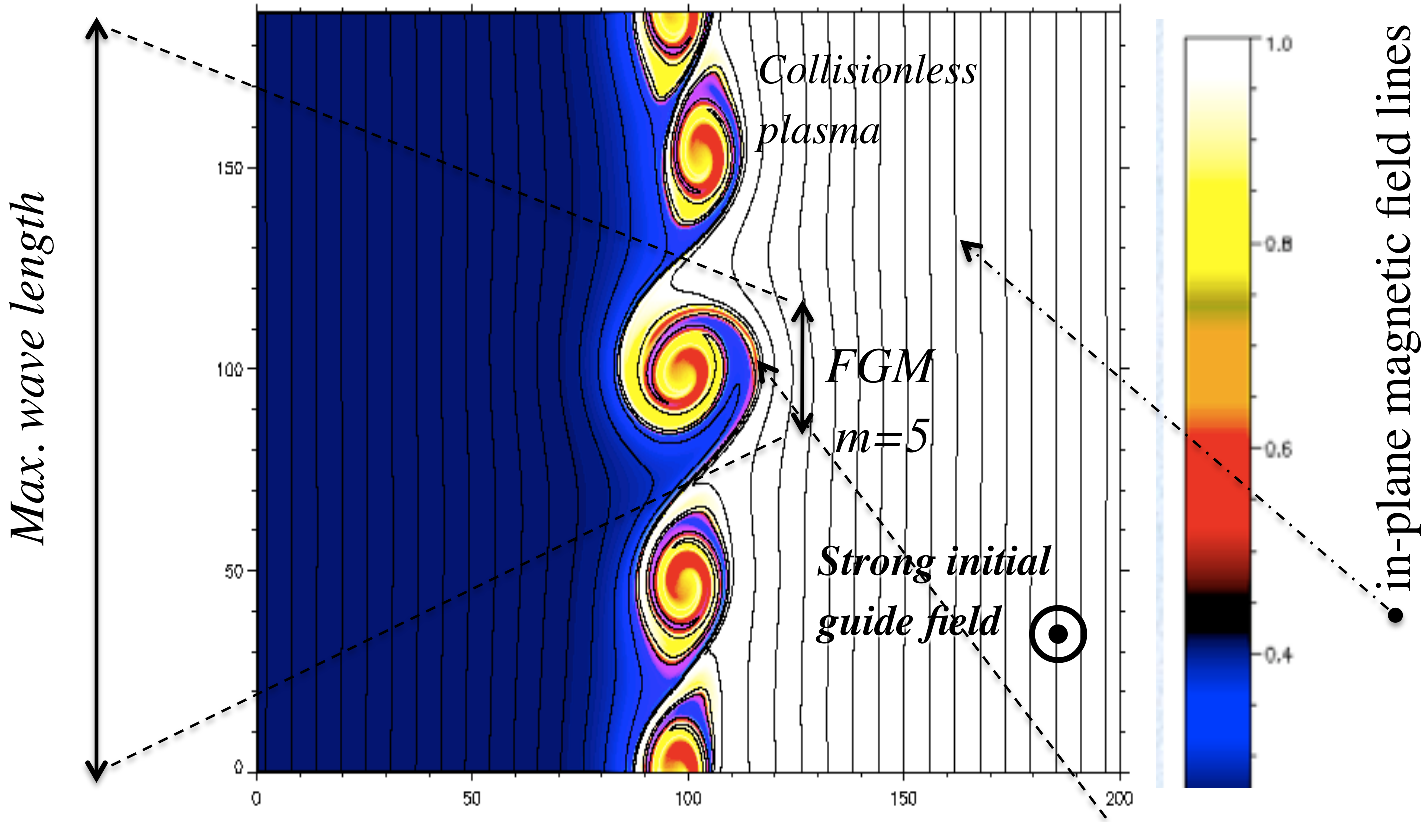
advection, stretching and compression of B field lines

M_A > 5 *to have fully rolled-up vortices*

Fully rolled-up vortex formation



Vortex chain generated by the KH instability



In-plane magnetic field advected by the rolled-up vortices,
thus <u>*increasingly stretched and compressed*</u>

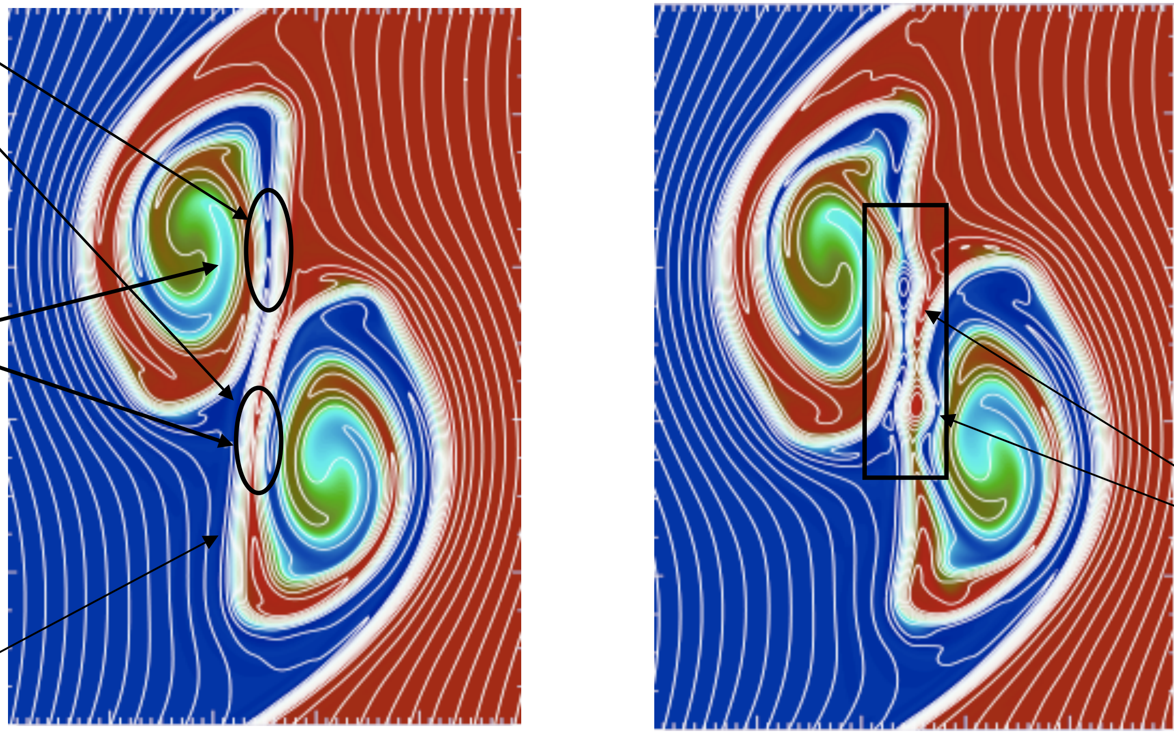
An example of KH fully rolled-up vortices

$M_{A,||} = 20$

Plasma passive tracer and magnetic field lines: formation of two vortices by the K-H instability

X-points

current layers



magnetic islands
typical size ~ d_i

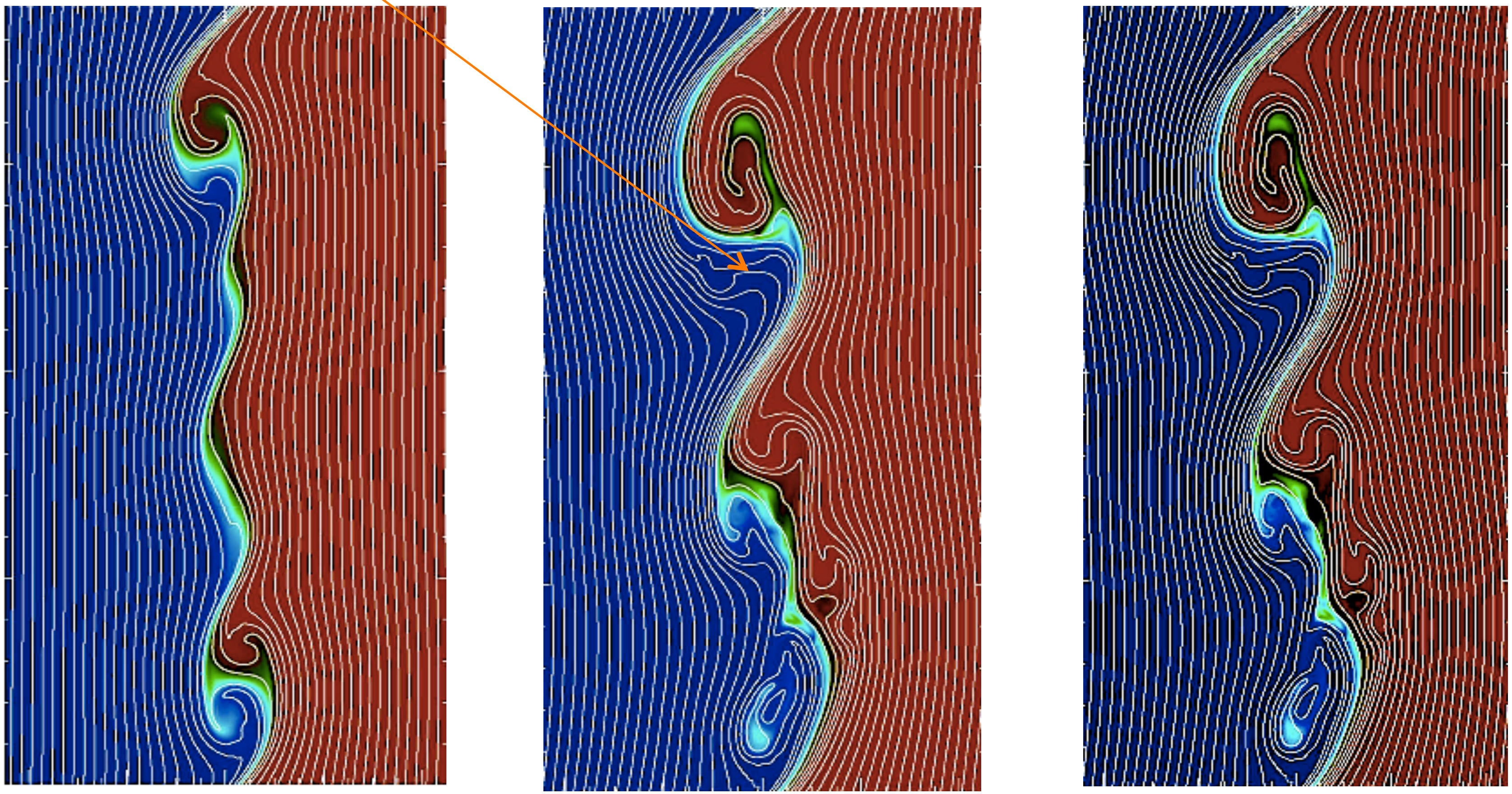
***ribbon** of nearly parallel, compressed magnetic lines.*

Magnetic reconnection develops at the X-points

Near the threshold

$M_{A,\parallel} = 5$

$\mathbf{E} + \mathbf{u} \times \mathbf{B}/c = 0$!



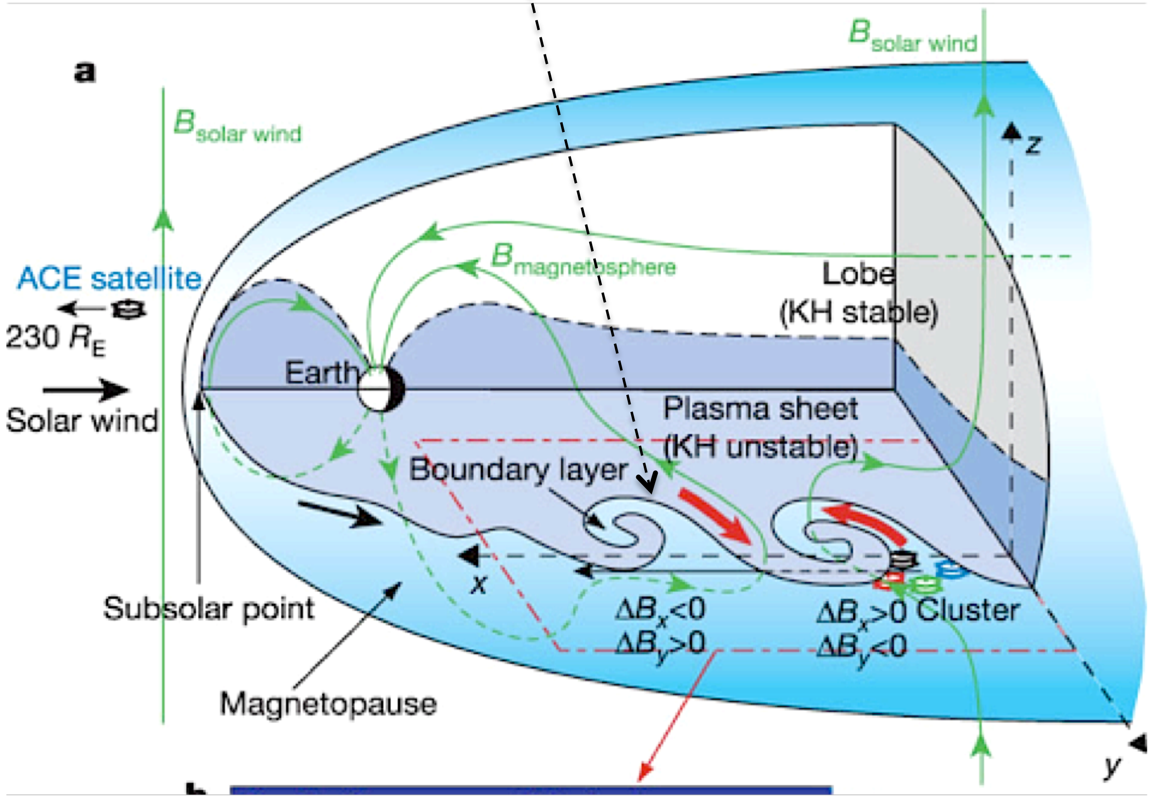
Magnetic tension is strong enough to provide nonlinear stability.

the vortices not as rolled-up as for larger $M_{A,\parallel}$

A research example

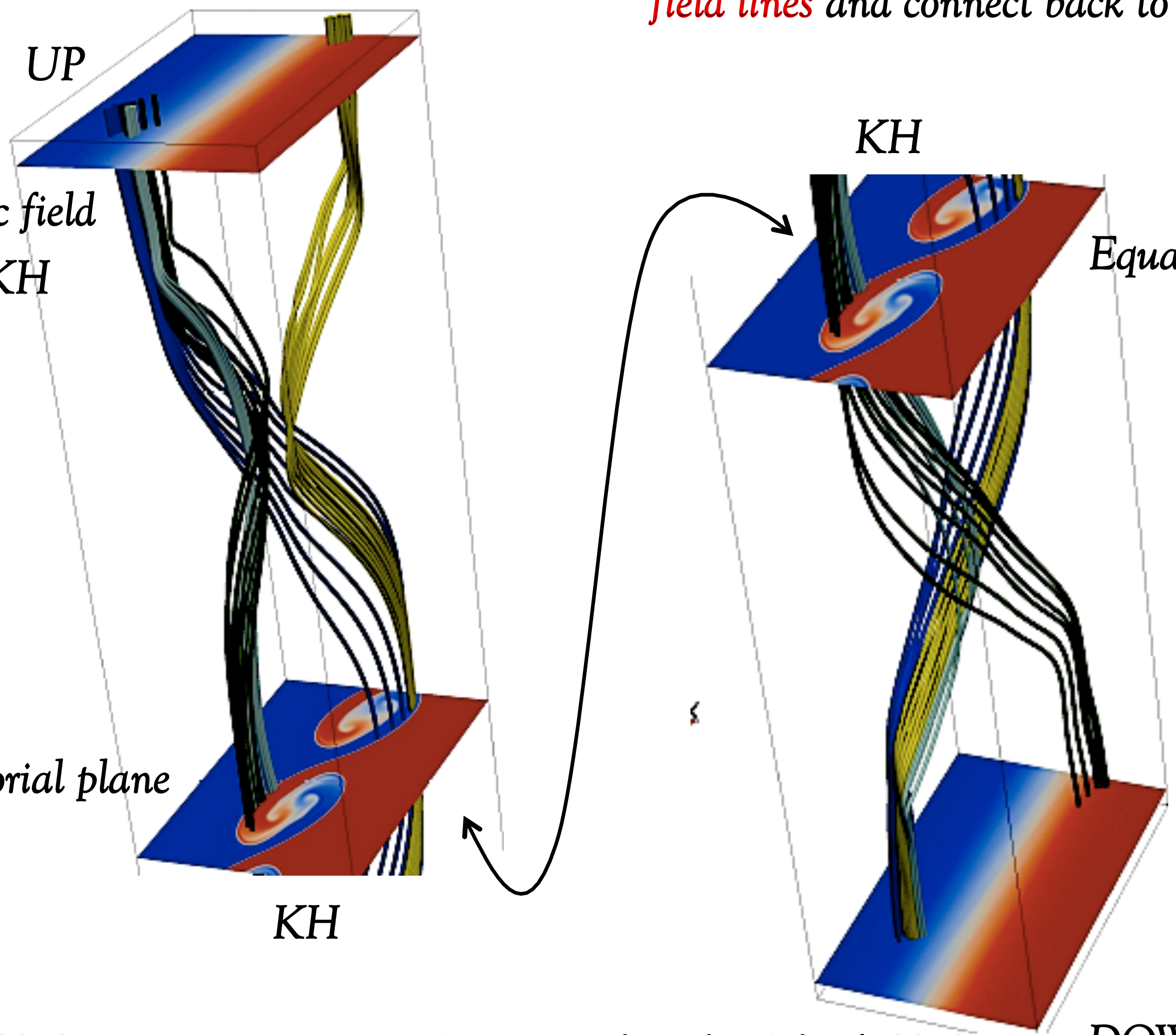
The ***Solar Wind - Earth's Magnetosphere* interaction** in regions where the velocity shear generates rolled-up vortices has been evidenced by *satellite in-situ*

The physical system:
solar wind - magnetosphere



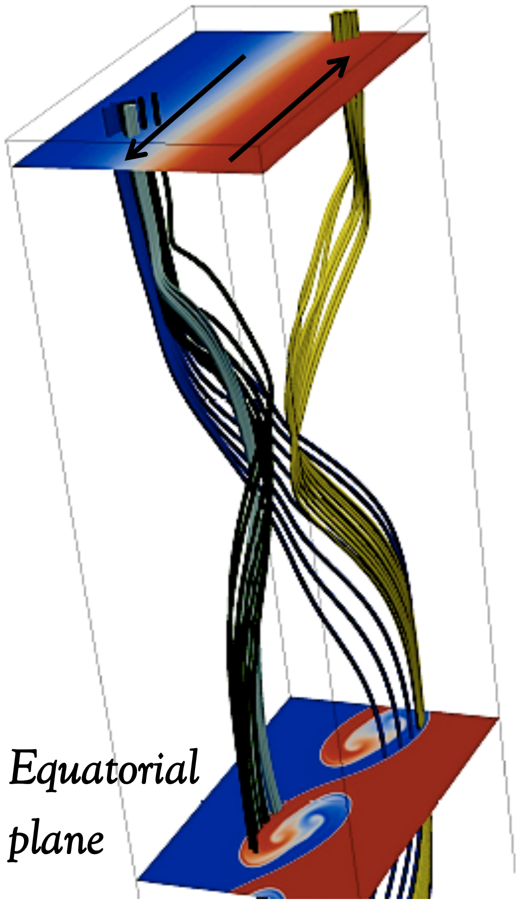
A 3D example: Double Mid-Latitude Dynamical Reconnection at the Magnetopause: an Efficient Mechanism Allowing Solar Wind to Enter the Earth's Magnetosphere

Magnetospheric field lines connect to solar wind field lines and connect back to the magnetosphere



A 3D example

Frozen-in lines!



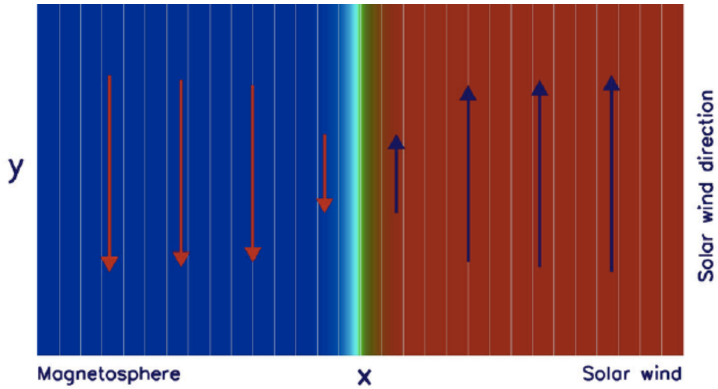
KH

Blue lines: Magnetosphere

Red lines: Solar Wind

The differential advection between field lines is driven not only by the rolling-up of the vortices but mainly by the motion of the field lines themselves, anchored in the moving plasma at high latitude, with respect to their anchoring in the vortices at rest in the equatorial region.

Role of density variation on the KH instability



U_1 ϱ_1 *low density plasma*

U_2 ϱ_2 *high density plasma*

$\varrho_1 << \varrho_2$

For $\Delta\varrho = 0$, at "large" values U (i.e. increasing M_f) the KH instability is more and more suppressed [**stable for $M_f \geq 2$**]

This condition is relaxed for $\Delta\varrho \neq 0$ as the magneto-sonic speed increases where ϱ decreases (and so M_f)

1) As a result, K-H vortices are generated even at relatively strong flows for which the plasma is super magneto-sonic.

2) The vortices propagate at speeds different from $\Delta U/2$ [or U(x=0) for a g.t.]

Super-magneto-sonic regimes

The physical properties of the solar wind change crossing the Earth's bow shock moving tail-ward inside the magnetosheath where, according to the ***Rankine-Hugoniot relations****, the plasma density and temperature increases, thus leading to subsonic velocities.*

However, at larger distances the shocked solar wind regains a fraction of its initial speed as it flows past the magnetosphere while the plasma temperature decreases more and more. Near the magnetopause flanks, the velocity of the magnetosheath plasma is increasingly accelerated as the distance from the Earth increases:

We expect a transition to supersonic regime for the KHI in the tail region of the magnetopause

Fairfield et al., J. Geophys. Res., 105, 21.159 (2000)

Sreiter et al., Planet. Space Sci., 14, 223 (1966)

Super-magneto-sonic regimes

We can study this regime by **increasing** U_0

Transition towards magneto-sonic Mach numbers ≥ 1: the **vortex** acts as an **obstacle** leading to the *formation of shocks structures*

In this regime, *rarefaction and compression effects* play a key role. In particular the **vortices** are now of **low density** thus *modifying the non linear dynamics* (pairing, secondary instabilities) observed in the "low" Mach number regime.

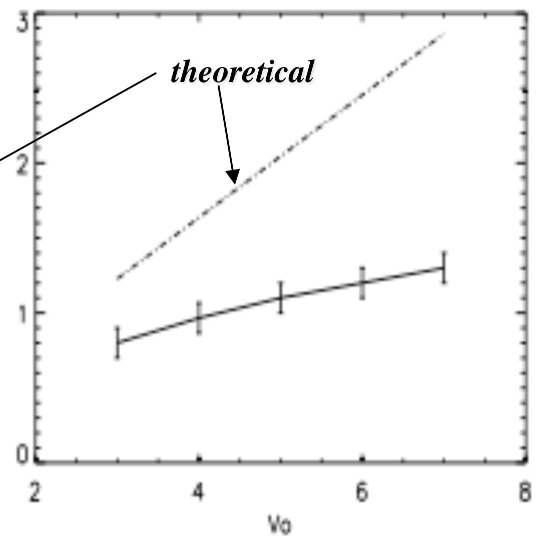
Vortex propagation due to density variations

The most important effect with respect to the uniform density regime is that ***the vortices propagate*** *in the same direction of the flow where the plasma density is larger (but less rapid than expected).*

incompressible plasma

with density discontinuity

$$V_{vort,theory} = \tfrac{1}{2}U(n_{0,L} - n_{0,R})/(n_{0,L} + n_{0,R})$$

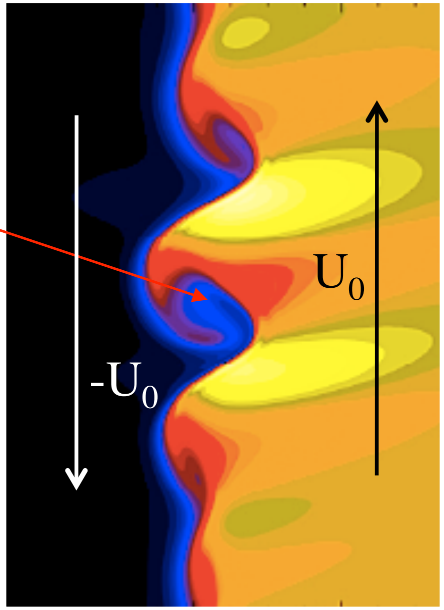


Vortex induced shock formation (U_0 = 5)

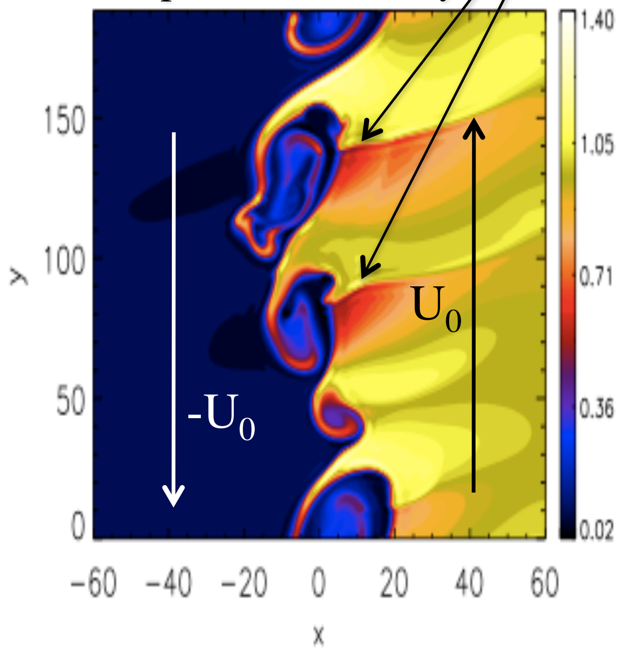
Shocks

plasma density

low density, nearly uniform vortices



plasma density



Relative speed of the vortex with respect to the flow $V \approx U/2 \pm V_{vort}$

Vortex (Convective) Mach number: V/c_f $\qquad$ $c_f = \left[c_s^2 + c_A^2 \right]^{1/2}$

Palermo et al., J. Geophys. Res. 116, A04223 (2011)
Palermo et al., Ann. Geophys., 29, 1169 (2011)

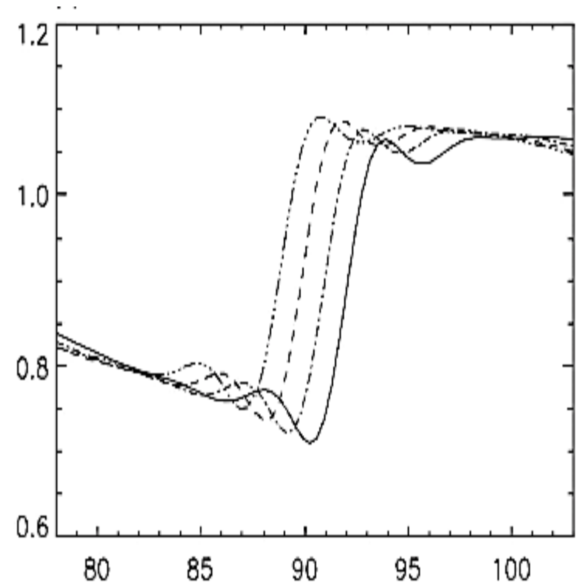
The shocks

The perpendicular magnetic fluctuations δb_z are superposed to the density fluctuations δn thus identifying the shock as a ***perpendicular magneto-sonic shock***

Formally "collisionless shocks"

In the shock frame of reference, the **Rankine-Hugoniot** conditions for a fast magneto-sonic shock are satisfied.

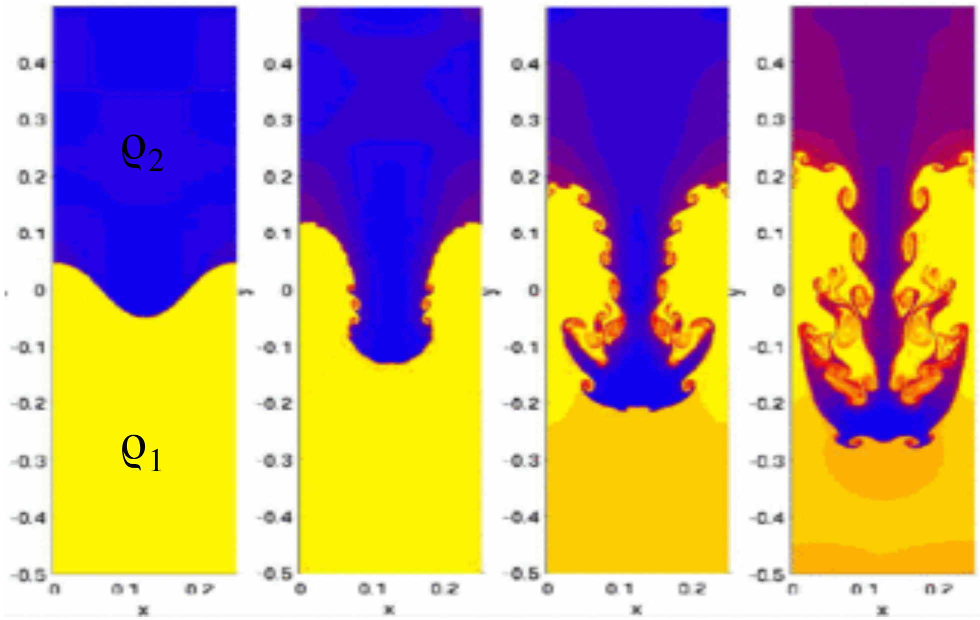
$$n_2 / n_1 \approx B_2 / B_1 \approx u_{y,1} / u_{y,2}$$



The upstream (downstream) plasma velocity is > (<) than the magneto-sonic velocity

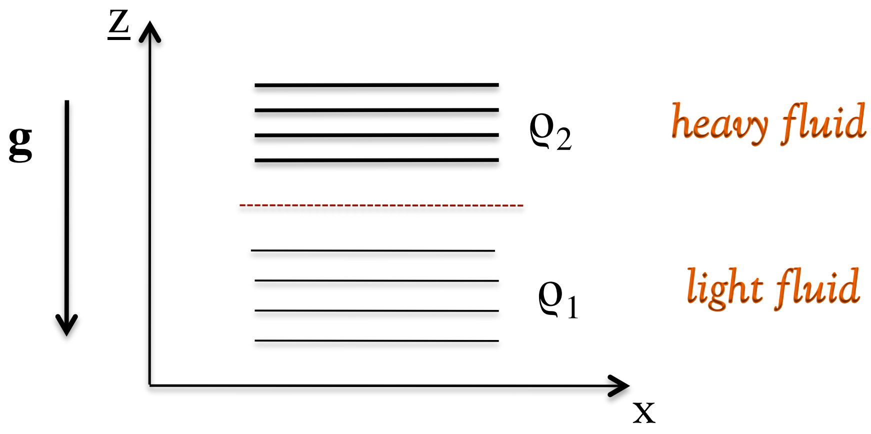
RAYLEIGH-TAYLOR

Hydrodynamic instability of an **heavy fluid** *accelerated over a* **light fluid**



$$\rho \frac{\partial \underline{U}}{\partial t} + \rho \underline{U} \cdot \nabla \underline{U} = -g\rho \underline{z} + \frac{\underline{J} \times \underline{B}}{c} - \nabla P$$

Rayleigh – Taylor instability



incompressible plasma discontinuity

Very important in laser plasma interaction

As for the K-I instability

linearization $$\left(\frac{d^2}{dz^2} - k^2\right)u_z = 0 \qquad (1)$$

Eigen functions f(z) exp$[i(k_x x - \gamma t)]$

Incompr. plasma $\nabla \cdot \underline{u} = 0$

displacement vector continuity at the interface gives

$$u_z(-\varepsilon) = u_z(+\varepsilon)$$

By integrating (1) between – ε *and* ε *we get the condition*

$$\left[\rho \frac{du_z}{dz}\right]_{+\varepsilon} - \left[\rho \frac{du_z}{dz}\right]_{-\varepsilon} = \frac{k^2}{\gamma^2} g(\rho_2 - \rho_1) u_z$$

finally: $u_z \longrightarrow 0$ *for* $z \longrightarrow \pm\infty$*, we get*

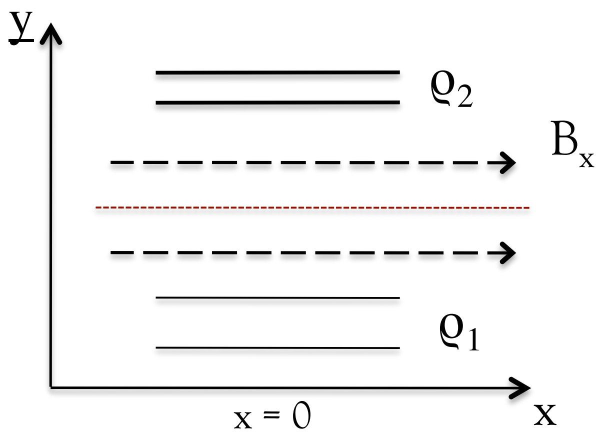
$$\gamma = \pm \sqrt{gk \frac{(\rho_1 - \rho_2)}{(\rho_1 + \rho_2)}}$$

Rayleigh – Taylor instability

Stability condition ($V_g = \sqrt{g/k}$)

$$\overline{V_A^2} > V_g^2(\rho_2 - \rho_1)/(\rho_1 + \rho_2)$$

incompressible plasma, discontinuity



$$\gamma = \pm\sqrt{gk\left(\frac{B^2 k}{2\pi g(\rho_1 + \rho_2)} + \frac{(\rho_1 - \rho_2)}{(\rho_1 + \rho_2)}\right)}$$

Magnetic stabilization

S. Chandrasekhar: Hydrodynamic and Hydro-magnetic Stability, Oxford University Press, 1961

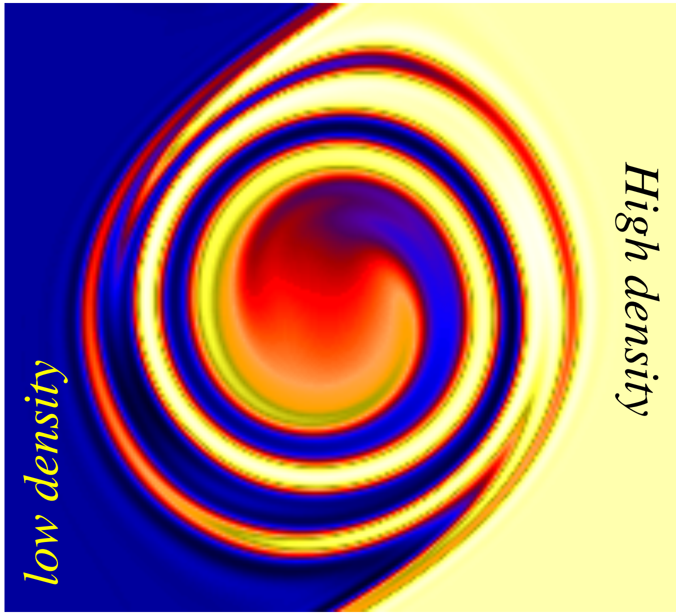
Density jump between solar wind and magnetosphere:

the rolled-up K-H vortices are characterized by

alternating density layers in the vortex arms

The centrifugal acceleration of the rotating K-H vortices acts as an "effective" gravity force on the plasma.

K-H VORTEX



W.D. Smyth, J. Fluid Mech. 497, 67 (2003)
Y. Matsumoto et al., Geophys. Res. Lett. 31, 2807 (2004)
Faganello *et al*., Phys. Rev. Lett., 100, 015001, 2008

Theory on the onset of the secondary instabilities

We consider each vortex separately and to be stationary

We model the vortex as an "equilibrium”. Inside two nearby vortex arms:

n_1, u_1 more dense ; n_2, u_2 less dense => density and velocity values of *two superposed fluid plasmas* in slab geometry.

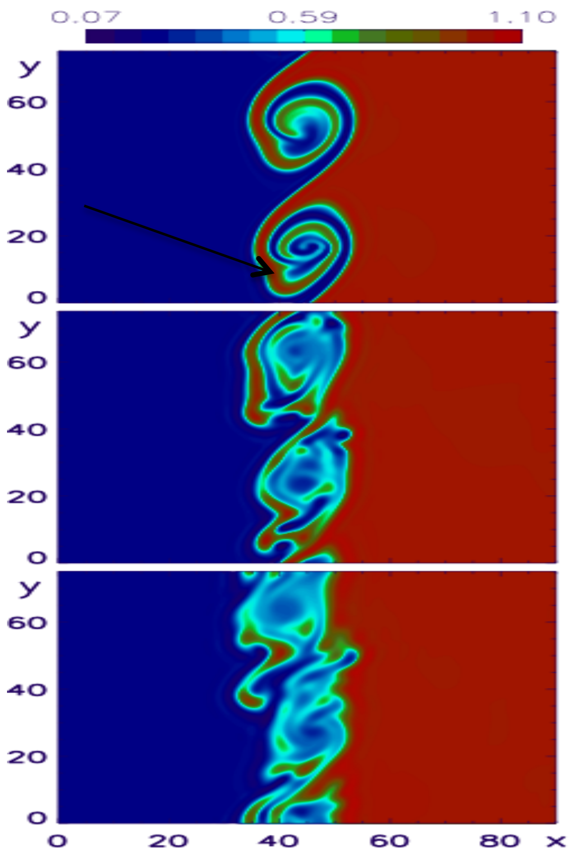
*The plasma slabs are subjected to an "**effective gravity**" which corresponds to the centripetal acceleration arising from the arms curvature*

ℓ_u ℓ_n scale length of the velocity and density gradient between the two arms; λ wave length along the vortex arm associated to the observed R-T.

Typical values: $\ell_u \sim \ell_n \sim 1$; $1 < \lambda < 10$ (dimensionless)

Onset of R-T. Linear analysis

If the density variation is large enough, the Rayleigh-Taylor instability can grow along the vortex arms.



The R-T growth rate is compatible with simulations

We model the system by a step-like configuration since the R-T instability is not affected by the finite value of the length l_n , at least when $\lambda \geq l_n$

$$\gamma_{RT} \approx [\, g_{eff}\, k\, (\alpha_1 - \alpha_2)]^{1/2}$$

where $\alpha_1 = \rho_1/(\rho_1+\rho_2)$, $\alpha_2 = \rho_2/(\rho_1+\rho_2)$

$g_{eff} \approx 0.1$ estimated using the ω_{vortex} and r_{arms}

For λ= 10, 4, 1 we get $\gamma_{RT} = 0.2, 0.3, 0.6$

Why so interesting this study ?

non linear, multi-scale collisionless dynamics

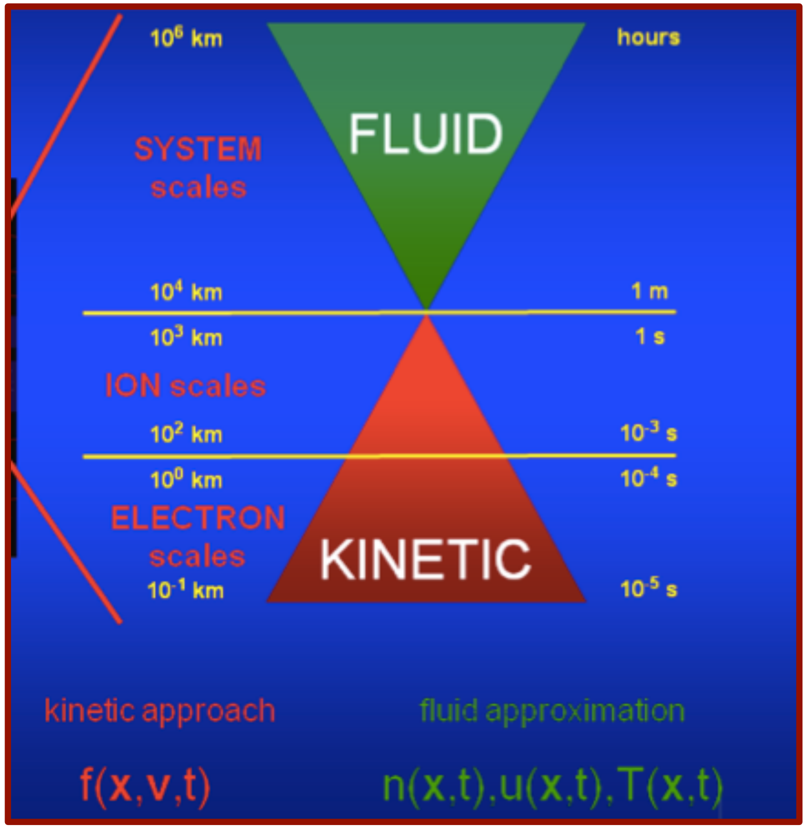
Typically: $L \gg d_i \, , \varrho_i, \lambda, d_e, \varrho_e \gg \lambda_{coll}$

(same for frequencies…)

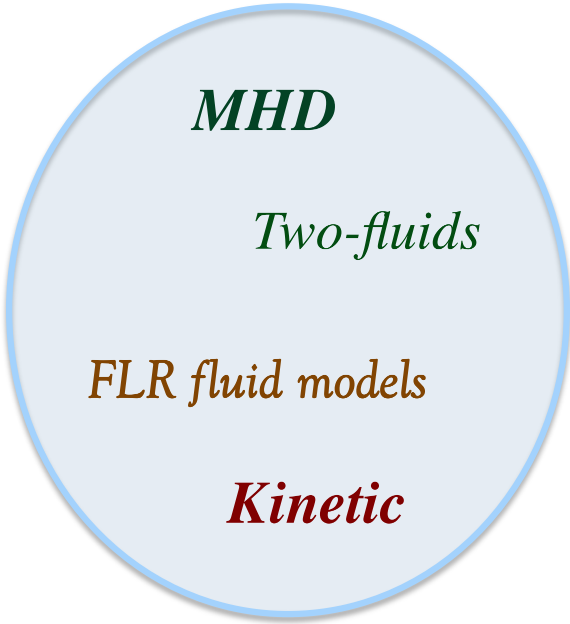
MHD - two-fluids - kinetic

Very rich Physics: Hydrodynamics vortices, Fluid instabilities, magnetic reconnection, shocks structures, turbulence

How to model such a multi-scale system ?



Shear flows: Pressure tensor ?



Pressure Tensor

$$\frac{\partial \Pi_{a_{ij}}}{\partial t} + \frac{\partial}{\partial x_k}\left(\Pi_{a_{ij}} u_{a_k} + Q_{a_{ijk}}\right) + \left(\Pi_{a_{ik}} \frac{\partial u_{a_j}}{\partial x_k} + \Pi_{a_{jk}} \frac{\partial u_{a_i}}{\partial x_k}\right) =$$

$$= \frac{e_a}{m_a c}\left(\epsilon_{ilm} \Pi_{a_{lj}} + \epsilon_{jlm} \Pi_{a_{li}}\right) B_m$$

Gyrotropic pressure equations, ϱ/L << *1*

$$\frac{\partial p_{a_\perp}}{\partial t} + \frac{\partial}{\partial x_i}(p_{a_\perp} u_{a_i}) = -\, p_{a_\perp} \frac{\partial u_{a_i}}{\partial x_i} + p_{a_\perp} b_i b_j \frac{\partial u_{a_i}}{\partial x_j} - \frac{\partial}{\partial x_i}(q_{a_\perp} b_i) - q_{a_\perp} \frac{\partial b_i}{\partial x_i}$$

$$\frac{\partial p_{a_\parallel}}{\partial t} + \frac{\partial}{\partial x_i}(p_{a_\parallel} u_{a_i}) = -\, 2\, p_{a_\perp} b_i b_j \frac{\partial u_{a_i}}{\partial x_j} - \frac{\partial}{\partial x_i}(q_{a_\parallel} b_i) + 2\, q_{a_\perp} \frac{\partial b_i}{\partial x_i}$$

+ FLR in motion equation (first order development)

Chew-Goldberger-Low equations, q=0, (par and perp. energy transport along **B***)*

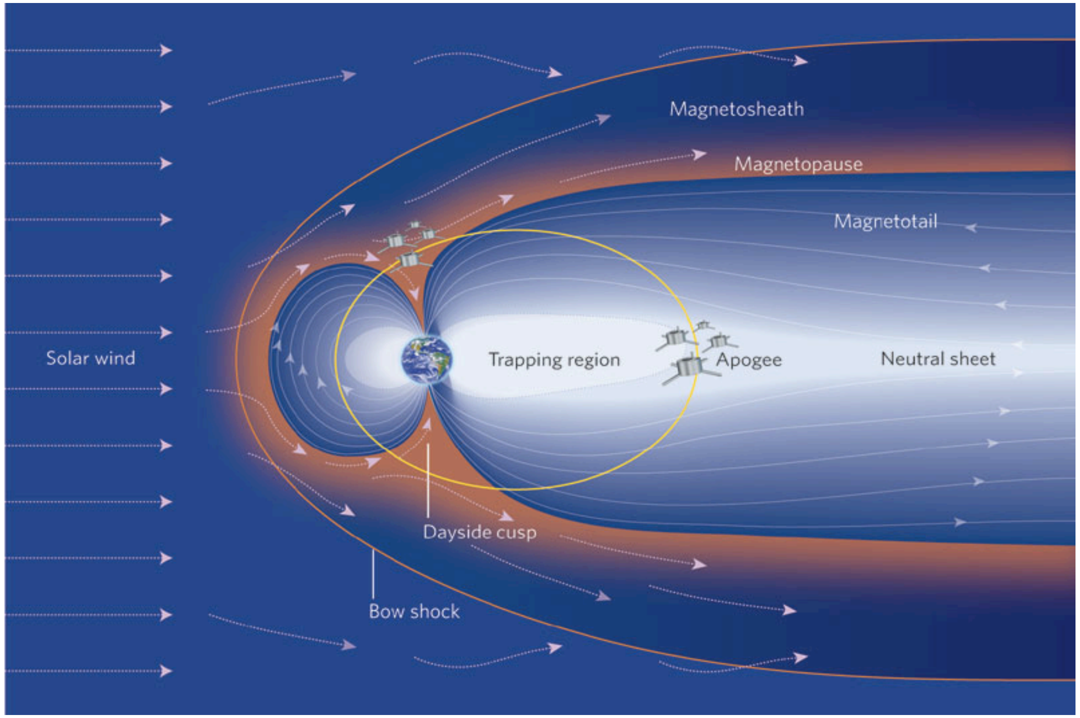
$$\frac{d}{dt}\left(\frac{p_{a_\perp}}{nB}\right) = 0 \quad \text{and} \quad \frac{d}{dt}\left(\frac{p_{a_\parallel} B^2}{n^3}\right) = 0$$

A posteriori equivalent to $\Upsilon_{par} = 1$ (i.e. isoth. Along B), $\Upsilon_{perp} = 2$

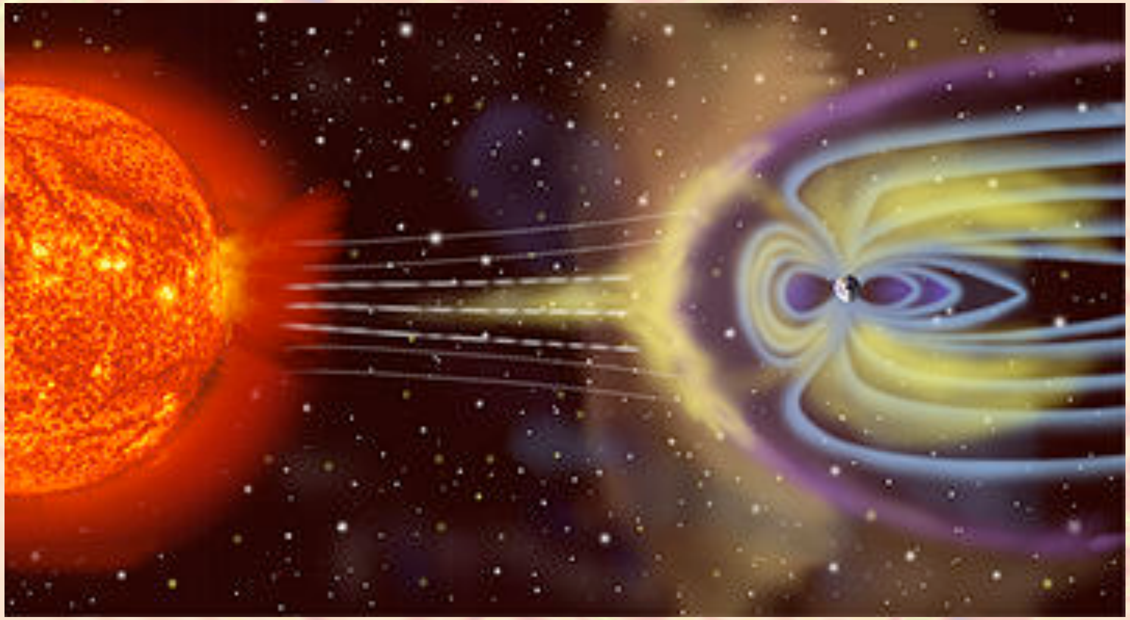

THANK YOU FOR
YOUR ATTENTION !

The physical system: Solar Wind - Magnetosphere

The connection between the solar wind and the Earth's magnetosphere is mediated through the magnetosheath and magnetopause boundaries.



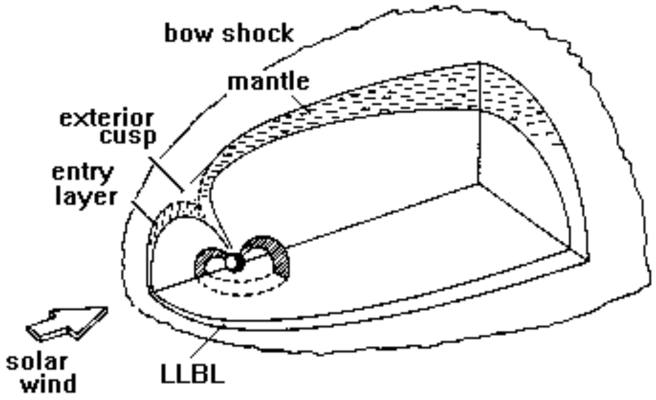
The solar wind-magnetosphere coupling strongly depends on solar wind properties and their variability, as the density and velocity value or the ***Interplanetary Magnetic Field orientation*** *with respect to the Earth's dipole*.



The **great interest in the analysis of the processes** at play is:

i) Importance in the shaping and dynamics of the system

ii) A wealth of in-situ diagnostics of improving quality (electromagnetic profiles and particle distribution functions)



Questions:

- *how can we represent the 3D large scale field ?*
- *Is it a true equilibrium in the MHD sense?*
- *Can satellite data help us ?*

At low latitude, when the IMF is mostly southward: *magnetic reconnection dominates the transport*

If reconnection at low latitude would be the only relevant phenomena for mixing, the northward periods (IMF and geomagnetic field parallel) should be relatively quiet and the flank regions should be dominated by the tenuous and hot plasma of the Earth plasma sheet

On the contrary, during **northward periods** the near-Earth plasma sheet becomes denser and colder near the flanks suggesting an **enhancement of the plasma transport** across the magnetopause

[Terasawa et al., Geophys. Res. Lett. 24, 935, 1997]

Two *main processes* have been proposed in order to explain this efficient transport

*1) High-latitude **magnetic reconnection** in both hemispheres converts northward magnetosheath field lines into closed geomagnetic field lines allowing for the entry of the magnetosheath plasma into the magnetosphere*

McFadden et al., 2008 and refs. therein

2) Development of ***Kelvin-Helmholtz instability*** at low-latitude magnetopause. Several nonlinear processes efficient for the formation of a mixing layer :

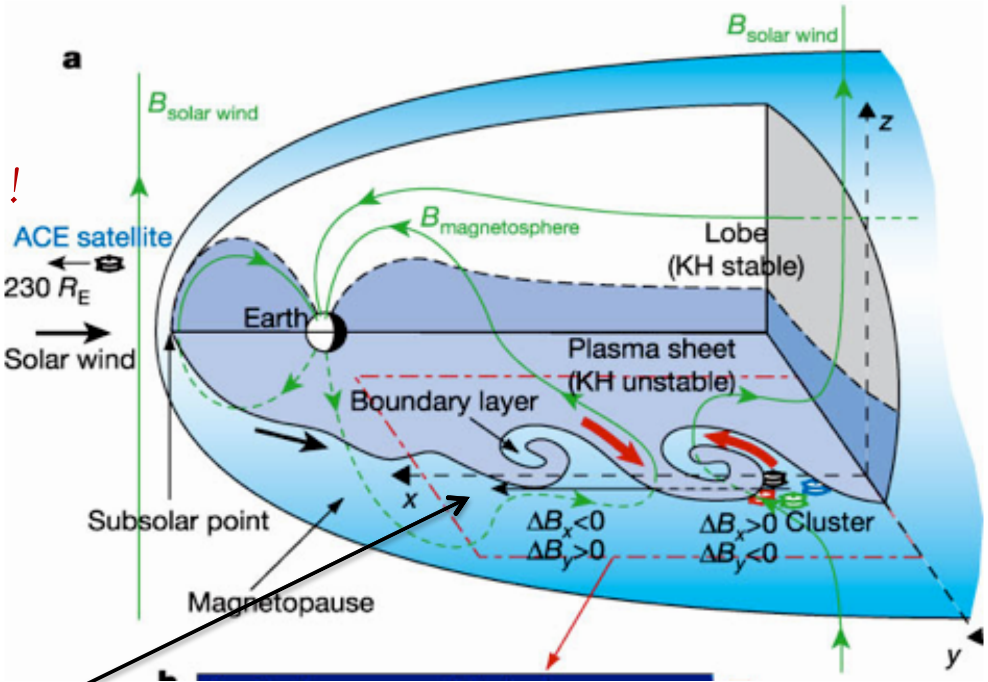
vortex pairing (standard HD non linear process)
twists up magnetic field lines leading to magnetic reconnection
secondary fluid and magnetic instabilities (on the shoulder of the vortices)

Belmont and Chanteur, 1989; Fairfield et al. , 2000; Nakamura and Fujimoto, 2005; Miura, 1997

Miura, Matsumoto, Hoshino, Hashimoto, Otto, Faganello, …

The solar wind flow provides an important source of “free energy” generating large-scale vortices driven by the development of **shear-flow** instability

Mixing efficiency strongly increased !



Complex non linear phenomenology induced by the K-H vortices

Quasi periodic perturbations often observed near the flank magnetopause

Hasegawa et al., Nature, 2004

KHI: Fast Growing Mode and vortex pairing

Net transport of momentum across the initial velocity shear occurs both when the Fast Growing Mode and its sub-harmonics (paired vortices) grow, and when the vortex pairing process takes place.

In a homogeneous density system, the momentum transport caused by vortex pairing process is much larger than that due to the growth of the FGM[1] thus leading to a faster relaxation of the velocity shear.

Vortex pairing is therefore expected to be an efficient process in the nearly two-dimensional external region of the magnetopause at low latitude[1].

[1] A. Otto et al., J. Geophys. Res. 105, 21175 (2000)

The equation: from MHD to EMHD regime

We adopt a "**simple**" **fluid approach**

$$B_0(x) = \left[B_{0,R}^2 + 2\left(P_{0,R} - P_0(x)\right)\right]^{1/2}$$

P_o *total thermal pressure*

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{U}) = 0$$ *Quasi neutrality*

$$\frac{\partial (nS_{e,i})}{\partial t} + \nabla \cdot (nS_{e,i}\mathbf{u}_{e,i}) = 0 \qquad S_{e,i} = P_{e,i}n^{-\gamma}$$

Isothermal or Adiabatic closure

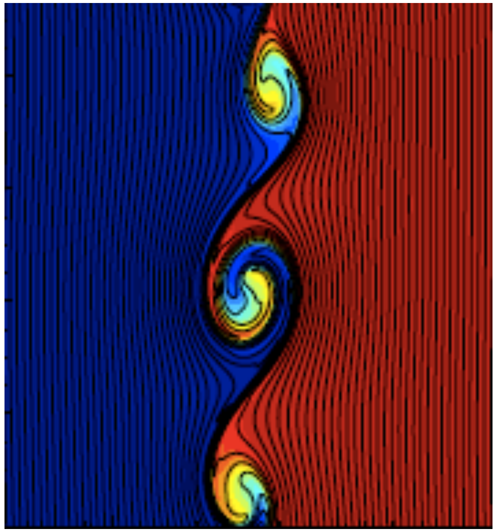
$$\frac{\partial (n\mathbf{U})}{\partial t} + \nabla \cdot \left[n(\mathbf{u}_i\mathbf{u}_i + d_e^2\mathbf{u}_e\mathbf{u}_e) + P\bar{\bar{\mathbf{I}}} - \mathbf{B}\mathbf{B}\right] = 0$$

$$(1 - d_e^2\nabla^2)\mathbf{E} = -\mathbf{u}_e \times \mathbf{B} - \frac{1}{n}\nabla P_e \qquad \partial B/\partial t = -\nabla \times E$$

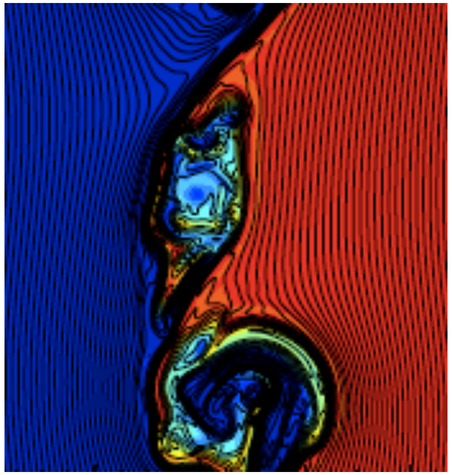
Mixing layer

***Strong density jump**, $\Delta n = 0.8$*

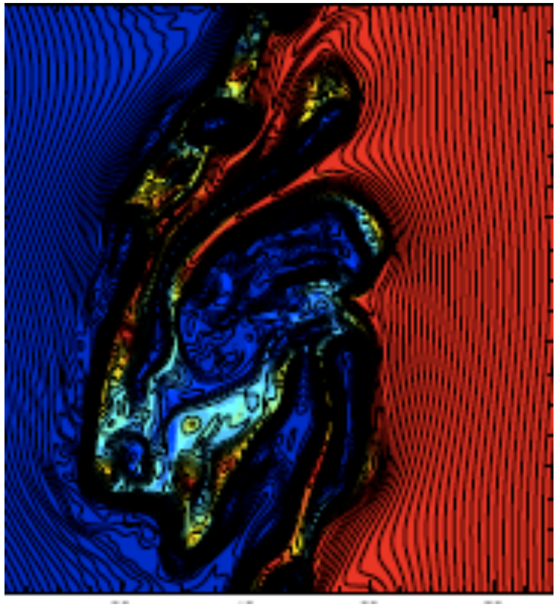
KHI



Development of** **fluid instabilities** **in the vortex arms



formation of a turbulent layer



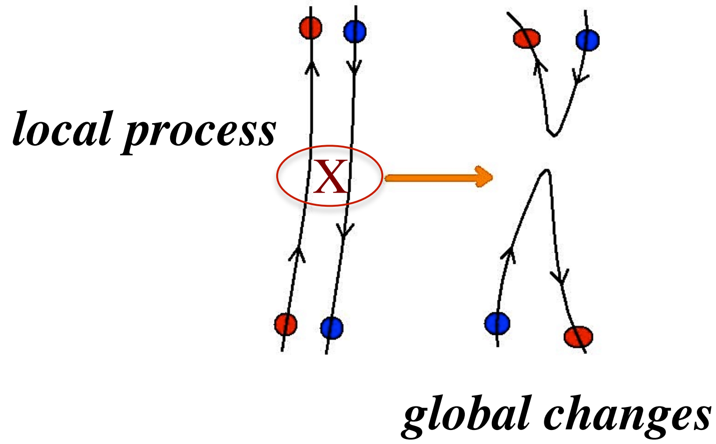
M. Faganello, F. Califano, F. Pegoraro, Phys. Rev. Lett. 100, 015001 2008

Importance of magnetic field

In a plasma the process of ***Magnetic Reconnection*** play a fundamental role in the dynamics by violating (locally) the "ideal" Ohm law thus allowing the system to access ideally forbidden energetic states.

The only process capable of violating the linking condition is known as

Magnetic Reconnection



Magnetic Reconnection:

affects the *global energy balance of the system* (astrophysics)

reorganizes the *large scale magnetic topology* (laboratory)

Conclusions

The Solar wind - Magnetosphere low latitude boundary layer:

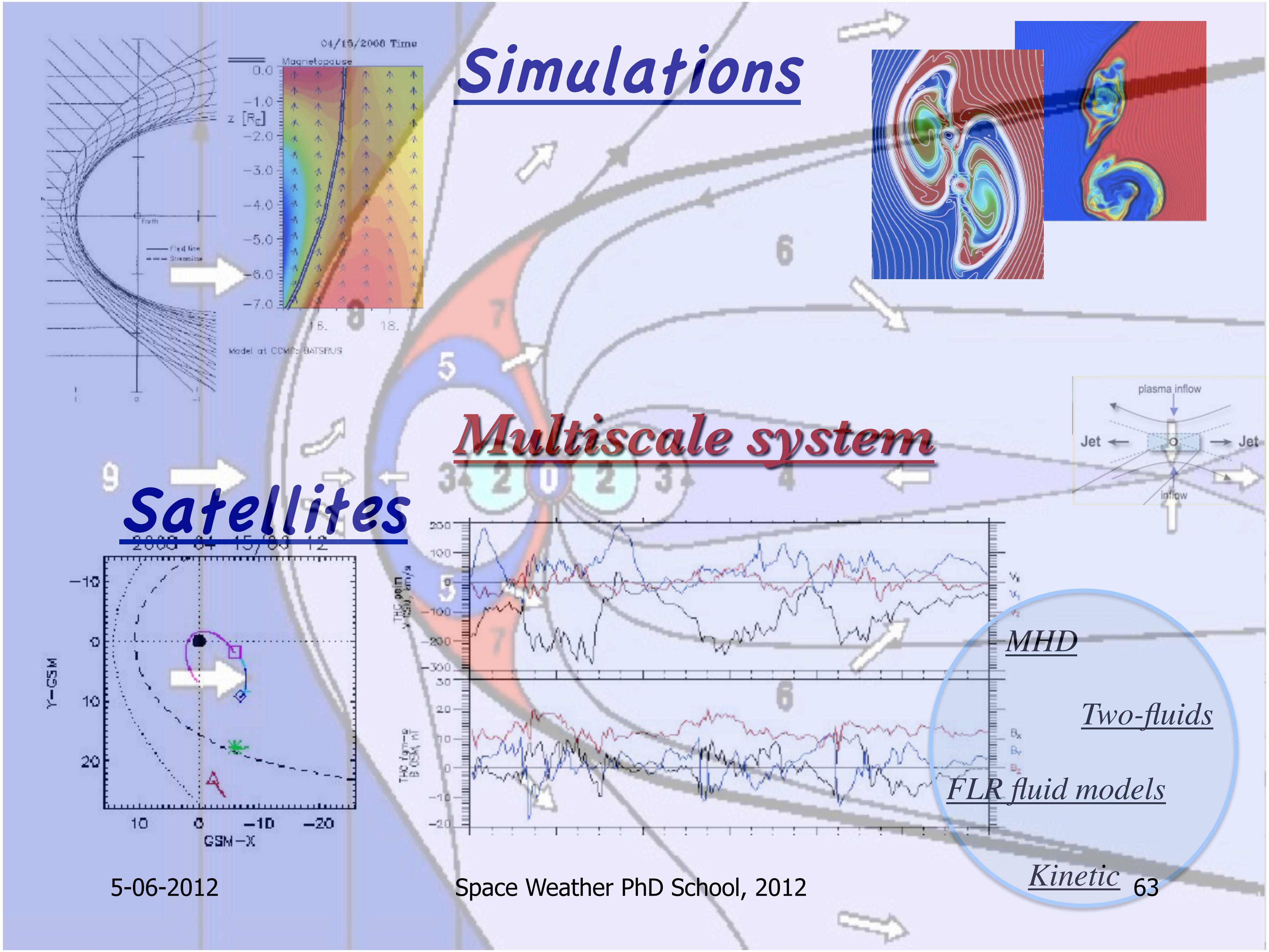
i) Play a key role for the entry of solar wind plasma in the Magnetosphere
ii) It is a laboratory of excellence for basic processes in plasmas
iii) It is one of the best example of multi-scale plasma dynamics

Results

We have understood many key processes at play in the dynamics

Problems and future work

i) Need for satellite data analysis in the transition region: large scale fields
ii) Need of a 3D initial configuration (MHD equilibrium ?)
iii) Need of kinetic simulations


Simulations
Multiscale system
Satellites
MHD
Two-fluids
FLR fluid models
Kinetic
plasma inflow
Jet
Jet
inflow
Magnetopause
Model at CCMC: BATSRUS
GSM-X
Y-GSM

First European School on: Fundamental processes in Space Weather: a challenge in numerical modeling

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Addressed to PhD and researchers in
Space plasmas, Plasma Physics, Computational Astrophysics

Basic processes for Space Weather
Modeling of space weather
Magnetic Reconnection
Instabilities in space
Fluid and kinetic simulations

Coupling at the solar surface
Coupling in the Earth environment
Multi-scale and multi-physics modeling
parallel, high performance computing