A plasma instability named magnetic reconnection

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Outlook

We can look at magnetic reconnection from different angles which stress different features:

- as a process whereby magnetic energy inside highly inhomogeneous regions is converted into plasma particle energy, or as a process whereby the magnetic topology, more precisely the connection of the magnetic field lines, is altered;
- as a process forced by large scale plasma motions with time scales essentially determined by these motions, but made possible by the presence of non ideal effects, or as plasma instability where the non ideal effects play a fundamental role, at least within linear theory, in determining the reconnection rate.

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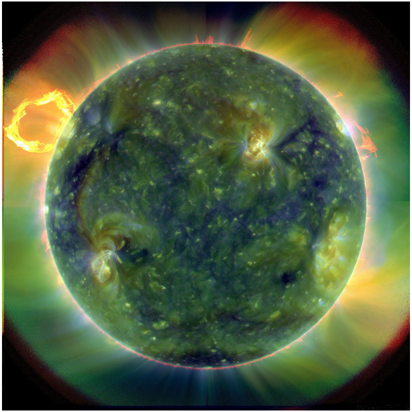
Solar landscape



A full-disk multiwavelength

extreme ultraviolet image

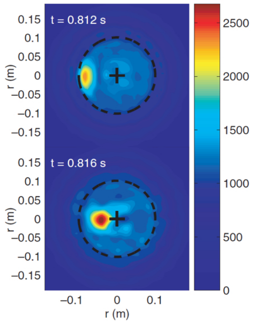
of the sun taken by SDO

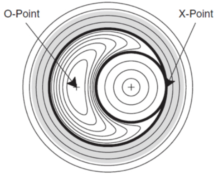
on March 30, 2010.

Credit: NASA/Goddard/SDO

AIA Team

Laboratory Reconnection





Giovannozzi Nucl. Fus. 2004

soft x-ray tomography of a

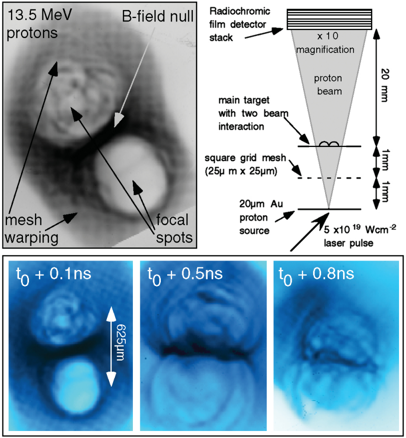
magnetic island in a sawtooth

(Molibdenum impurities)

The dashed line marks the

sawtooth inversion radius

Relativistic Reconnection



Nilson et al, PRL 97, 255001

2006 - Proton deflectometry

images of aluminum target

taken 100 , 500 and 800 ps

after the beams arrive

at the target surface

Outlook

After some soul searching on the very nature of reconnection in the first lecture, in the second lecture some well established results will be presented within the MHD framework.
In my presentation I will adopt the point of view of an instability that modifies the magnetic topology of an equilibrium configuration, and I will briefly refer to different magnetic field configurations and plasma regimes.

- Taking an initial value problem may be a restrictive point of view. For example in such an initial value formulation there is no explicit external forcing (physically it is substituted for by the “magnetic energy” that has been stored in the current inhomogeneities).

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Reconnection in high energy plasmas

Taking a wider perspective in a relativistic plasma we need to change the MHD equation as the displacement current and charge separation cannot be neglected.

- In addition the meaning of magnetic connection or of magnetic topology is not evident in the case of a relativistic dynamics where the distinction between electric and magnetic fields is frame dependent.
  Note that a covariant formulation of the topological MHD theorems can be derived (see Appendix and W.A. Newcomb, *Ann. Phys,*, **3** 347 (1958)).

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Magnetic connections

In an ideal MHD plasma if two plasma elements at $\vec{x}_1$ and $\vec{x}_2$ are initially connected by a magnetic line, then for every following time there will be a magnetic line connecting them[1].

- Consider the quantity $\delta\vec{l} \times \vec{B}$
- if at $t = 0$ we have $\delta\vec{l} \times \vec{B} = 0$ then

$$\frac{d}{dt}\left(\delta\vec{l} \times \vec{B}\right) = 0, \quad \forall t$$

and so $\delta\vec{l} \times \vec{B} = 0$ at every time.

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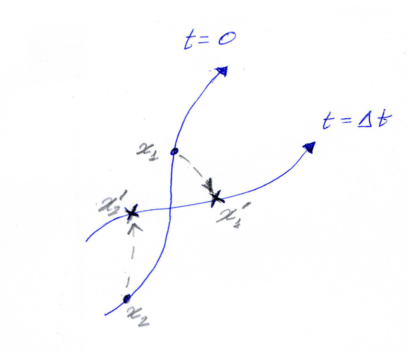
Connections at different times



Connection theorem

provides a meaning

to the expression

“Motion of a field line”

$$\frac{d}{dt}\left(\delta\vec{l}\times\vec{B}\right)=\frac{d}{dt}\delta\vec{l}\times\vec{B}+\delta\vec{l}\times\frac{d}{dt}\vec{B}$$

$$\frac{d}{dt}\delta\vec{l}=\left(\delta\vec{l}\cdot\vec{\nabla}\right)\vec{u}(\vec{l})$$

$$\frac{d}{dt}\vec{B}=\vec{\nabla}\times\left(\vec{u}\times\vec{B}\right)+\left(\vec{u}\cdot\vec{\nabla}\right)\vec{B}$$

$$\frac{d}{dt}\left(\delta\vec{l}\times\vec{B}\right)=-\left(\delta\vec{l}\times\vec{B}\right)\left(\vec{\nabla}\cdot\vec{u}\right)-\left[\left(\delta\vec{l}\times\vec{B}\right)\times\vec{\nabla}\right]\times\vec{u}$$

Breaking of Magnetic connections

Not all terms that violate the ideal Ohm's law break magnetic connections

$$\vec{E} + \frac{\vec{u}}{c} \times \vec{B} = \eta \vec{J} + \frac{m_e}{ne^2} \frac{d\vec{J}}{dt} + \frac{1}{nec} \vec{J} \times \vec{B} - \frac{1}{ne} \vec{\nabla} \cdot \Pi_e$$

- Resistivity breaks magnetic connections
- Electron inertia breaks magnetic connections but maintains generalized connections[2] related to the generalized magnetic field $\vec{B}_e$ defined as

$$\vec{B}_e = \nabla \times [\vec{A} - \vec{u}_e (m_e c/e)]$$

[2] Important in the case of dissipationless magnetic reconnection also called Hamiltonian reconnection.

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- The pressure term does not break magnetic connections when the electron pressure is a scalar (Isotropic pressure $\Pi_e = p_e I$) and a polytropic equation of state of the form $p_e = p_e(n)$ holds[4].

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Related theorems

Closely related theorems are[5]

- magnetic flux conservation (Alfvèn theorem)
- Magnetic helicity density $(\vec{A} \cdot \vec{B})$ conservation
- Linking number conservation[6] (related to magnetic braids of field lines.)

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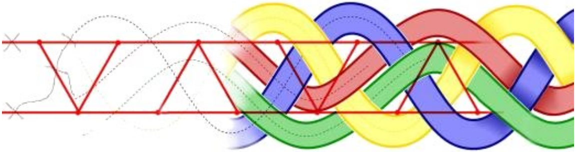
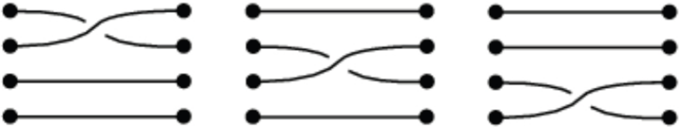
Braids





σ_1 σ_2 σ_3

Self Destroying theorems

The above theorems represent strong constraints on the (low frequency macroscopic) plasma dynamics that limit the accessible configuration space.

The net result of the **nonlinear** plasma dynamics is the *local* build up of small scale structures (such as current layers) that violate the very assumptions under which the ideal MHD description is formulated.
Note that the terms that violate the ideal condition depend on higher order spatial derivatives.

Reconnection and topology

- Although a clear cut definition of magnetic reconnection is not easy to formulate, *its traditional definition refers to the local violation of these topological theorems and in particular to the local breaking of the structure of magnetic connections*
- It is not yet clear to me how to implement this definition in high energy fast developing relativistic plasma event (e.g. in the "flaring" Crab Nebula)[7] with large velocity gradients.

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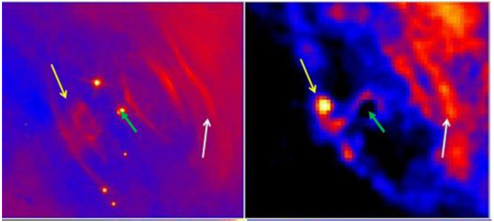
Detecting magnetic reconnection



Left: optical image of the inner nebula region on October 2, 2010. The pulsar position is marked with a green arrow. Arrows mark interesting features compared to archival data. Right: the same region on September 28, 2010. The pulsar does not show in this map and below because of pileup.[8]

[8]Taken from Tavani, M., *et al.*, *Science*, **331**, 736 (2011).

Magnetic field annihilation events

- My understanding is that, at the moment, in such systems by magnetic reconnection one simply means a fast release of magnetic energy and its conversion into particle energy.
- I think the same applies to the "reconnection events" in laser produced plasmas that should be probably more correctly called "magnetic field annihilation events". No obvious role appears to be played here by magnetic topology.

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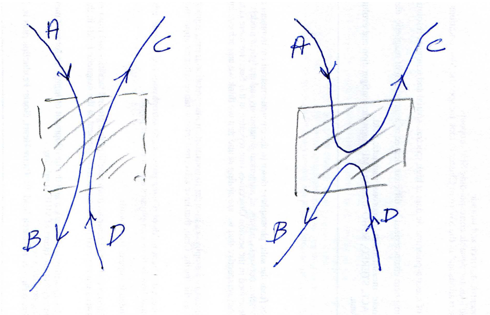

Example of reconnection

due to a violation of the

connection theorem in the

localized shaded area with

larger spatial derivatives

Reconnection and geometry

There is also a strong interplay between the geometry of the magnetic configuration and magnetic reconnection.

- Seen as an instability magnetic reconnection is driven by strong nonuniformities of the current density distribution[9].
- However the configuration may have additional sources of instabilities (such as in a toroidal configuration magnetic field curvature in the presence of pressure gradient).
- Other sources of instability, not directly related to geometry, come also into play in the onset and development of magnetic reconnection.

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Mixing of instabilities

Thus is general magnetic reconnection is not a “pure” instability driven by current inhomogeneities.

- It couples (linearly and nonlinearly) to different instabilities such as Rayleigh-Taylor type modes, Kelvin-Helmholtz instabilities, drift waves and, in the presence of strong current layers, to current instabilities (these latter contributing nonlinearly to the violation of ideal Ohm’s law).
- In the presence of anisotropy in velocity space[10] a mixing occurs between the Weibel instability and magnetic reconnection with important effects on the instability threshold and nonlinear development of reconnection depending on the sign of the anisotropy $(T_{||}/T_{\perp} - 1)$.

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Reconnection and geometry: initial configuration

The interplay between the geometry of the magnetic configuration and magnetic reconnection is evident when we compare two dimensional and three dimensional reconnection events.
There are different sides to this distinction: initial configuration.

- Very often, at least in analytical studies and as initial conditions in numerical simulations, we consider magnetic configurations where, in the absence of reconnection, magnetic surfaces $\psi(x,y,x)$ exist (globally)

$$\vec{B} \cdot \nabla \psi = 0.$$

  This is not a generic case.[11]

[11] It corresponds to the integrability of the magnetic Hamiltonian, i.e. to a zero measure set, see Appendix

Reconnection and geometry - chaotic field line configuration

- If magnetic surfaces exist, they are preserved in time by the ideal MHD evolution of the system[12]. Magnetic reconnection may destroy such surfaces although it may occur within field lines lying on a given surface.
- If such surfaces do not exist and the magnetic field lines are chaotic, the very concept of (global) magnetic topology becomes unclear as magnetically connected points are distributed in space in a disordered fashion[13].
- Magnetic topology may remain meaningful at scales shorter than that of exponential divergence of field lines.

[12] If a Clebsch representation exists, $\vec{B} = \nabla\alpha \times \nabla\beta$, then $d\alpha/dt = d\beta/dt = 0$

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Reconnection and geometry - Closed field lines

- Assume that magnetic surfaces exist. If we have toroidal surfaces, as is the case in most fusion experiments in magnetized plasmas, closed field lines occur on the so called rational surfaces[14] .
- In this case there is an additional constraint on Ohm's law because the requirement that the instability electric field component vanishes along field lines may be in contrast with the induction equation unless the scalar electric potential is taken to be multivalued[15].
- Rational surfaces with a field "winding ratio" corresponding to the ratio of small integers are the most prone to magnetic reconnection in magnetic fusion experiments.

[14] Rational surfaces are dense in the configuration

[15] In a linearized description the line integral of the perturbed electric field along the unperturbed magnetic field must vanish.

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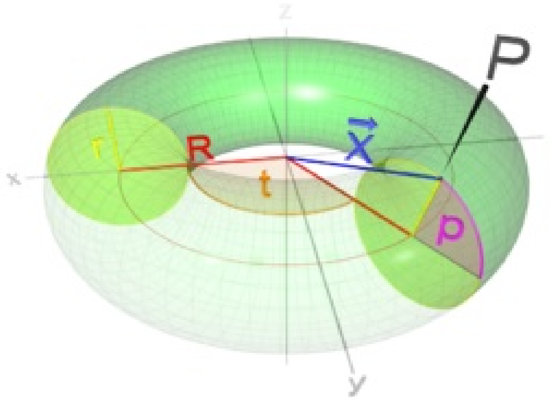
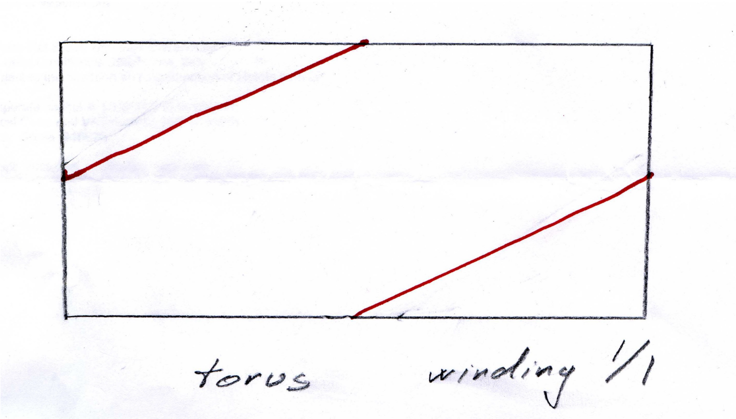
Magnetic toroidal surface



Closed field line on a toroidal surface. Winding number = 1



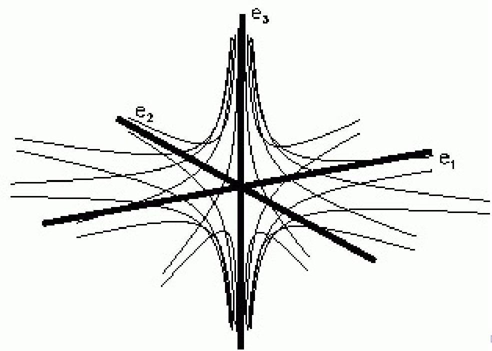
Reconnection and geometry, null surfaces

Ideal Ohm's law ($\vec{E} + \vec{u} \times \vec{B}/c$) requires that the electric field be balanced by the magnetic Lorentz force acting on the fluid plasma. This is obviously not possible at locations where the magnetic field vanishes (unless $\vec{E}$ vanishes too or the fluid velocity becomes arbitrarily large).
The same problem arises if the velocity field $\vec{u}$ is such that $\nabla \times (\vec{u} \times \vec{B})$ vanishes in the direction of $\nabla \times \vec{E} \neq 0$.
This may simply require that a given component of $\vec{B}$ vanishes, not the total field and selects a subset within the possible plasma motions (for linearized perturbations $\tilde{\vec{u}}(\vec{x}, t)$ it requires $(\vec{B}_0 \cdot \nabla)\tilde{\vec{u}} = 0$, i.e. $\vec{k} \cdot \vec{B}_0 = 0$).

Reconnection and geometry, null surfaces

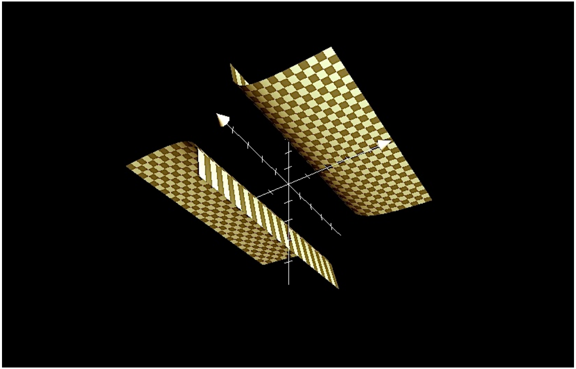
Consider the first case (vanishing of $\vec{B}$) and assume that magnetic surfaces exist. The magnetic field $\vec{B}$ can vanish at separate points (3D configurations), or along extended curves (2D configurations), or on surfaces (1D configurations).

3D: $B = (x, y, -2z)$, null point $x = y = z = 0$



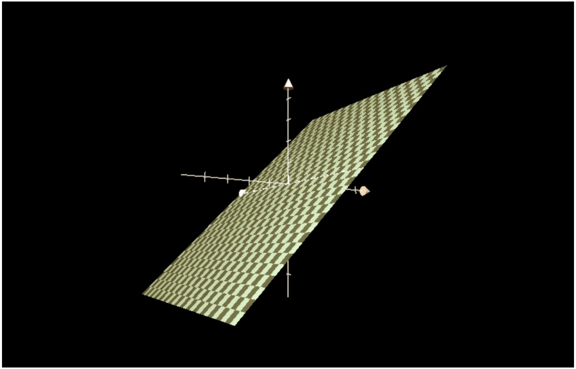
Reconnection and geometry, null surfaces

2D: $B = (y, x, 1)$, null line $x = y = 0$



Reconnection and geometry, null surfaces

1D: $B = (0, x, 1)$, null surface $x = 0$



Reconnection and geometry - Perturbations

Geometry comes into play also in the consequences of reconnection.

- If magnetic surfaces exist before the onset of a magnetic reconnection instability, they are generally torn apart by its development. This may lead to large scale loss of confinement e.g., in fusion experiments.
- However it is possible that new magnetic surfaces exist even after the reconnection event (“single helicity perturbations” in the fusion community slang).
- In general after the onset of reconnection (e.g. in the presence of multiple helicity perturbations) the new magnetic Hamiltonian is not integrable and the field lines are locally or even globally chaotic. This leads to enhanced particle and energy diffusion.

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A simplified case of reconnection instability mode threshold, structure and growth rate scaling

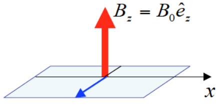
I will describe some basic features of magnetic reconnection in a simple configuration in a regime that goes under the name of “guide field reconnection” and within the framework of resistive MHD[16] limiting myself to linearized results.

- Clearly such a linearized analysis has a limited range of validity but is shows that within its framework the instability must grow at a rate that is slower than the Alfvèn time and that this growth rate must depend on the specific physical mechanism that violates the ideal Ohm’s law.

[16] I will also mention how to substitute electron inertia for resistivity within this analysis

Model geometry

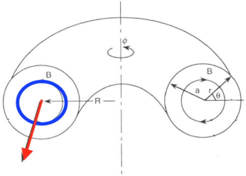
Reconnection in cartesian and toroidal geometry

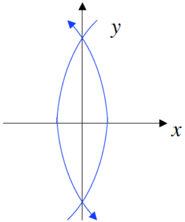


$$B_z = B_0 \hat{e}_z$$

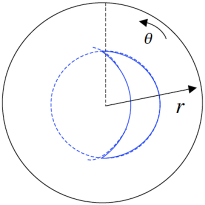
$$B_y(x) = -B_0 \vec{\nabla} \times \hat{e}_z \psi$$

$$x \longleftrightarrow r$$
$$y \longleftrightarrow \theta$$
$$z \longleftrightarrow \varphi$$





$$B_x \longleftrightarrow B_r$$
$$B_y \longleftrightarrow B_\theta$$
$$B_z \longleftrightarrow B_\varphi$$



Mode threshold, structure and growth rate scaling

I will consider a configuration with a large magnetic field component B_0 along the z direction and a smaller “shear field " $\vec{B}_{sh}$ in the x-y plane. In the initial configuration $\vec{B}_{sh}$ has only a y component and is odd in x (thus vanishes at $x = 0$).

- In the limit of very large B_0 this field component can be assumed to remain unaffected by the instability, the plasma motion can be taken incompressible and the Hall term in Ohm’s law does not play a relevant role.
- In the limit there is no decoupling between electrons and ions and the scale $d_i = c/\omega_{pi}$ does not appear[17].

[17] If we introduce electron inertia effects with $d_e = c/\omega_{pe}$ the validity of the description requires that $k_\perp d_e \gg k_{||} d_i$.

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Mode threshold, structure and growth rate scaling

Note in passing that while in this limit the uniform component $B_0\vec{e}_z$ of the magnetic field does not enter the solution of the MHD equations, it takes an important role in a kinetic modeling: in a sheared magnetic field $B_y = B_y(x)$ with $B_0\vec{e}_z = 0$, near the null point (where $B_y = 0$), particles move along “Parker's orbits” (figure 8 orbits, meandering orbits)) with characteristic transverse length $l \sim \sqrt{L\rho_L}$, where L is the scale length of the B_y magnetic field component, and ρ_L is the asymptotic value of the corresponding Larmor radius.

Reduced MHD

- This plasma description goes under the name of Reduced MHD (RMHD)[18] (not to be confused wit Relativistic MHD) and can be derived formally by a proper expansion procedure[19]involving the geometry and the value of the β parameter of the plasma.
- We parameterize the magnetic field in the form:

$$\vec{B} = B_z \hat{e}_z + \vec{\nabla}\psi(x,y,t) \times \hat{e}_z$$

where $\psi(x,y,t)$ is the magnetic surface function (equal in this geometry to the z component of the vector potential) and $\psi(x,y,t=0) = \psi_0(x)$ is an even function of x.

[18] H.R. Strauss, *Phys. Fluids*, **20**, (1977) 1354.

[19] See e.g. http://www.physics.wisc.edu/grads/courses/726-f07/files/Section_13_Reduced_MHD_02.pdf http://www.afs.enea.it/vlad/Papers/review_RNC_2_latex2html/node28.html

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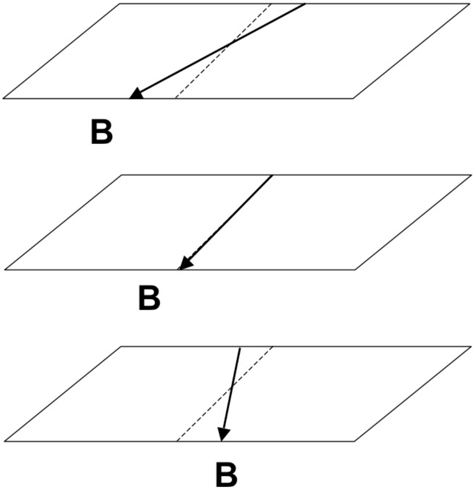
Shear magnetic field



Reduced MHD

- We take perturbations such that $\partial/\partial z = 0$
  (This is simply a consequence of taking a single mode and of *the choice of the geometry*)
- We take the plasma flow in the x-y plane incompressible:

$$\vec{\nabla} \cdot \vec{u} = 0 \qquad \text{and write}$$

$$\vec{u} = \vec{\nabla}\varphi \times \hat{e}_z$$

Note that the neglecting of the z component of the perturbed magnetic field is equivalent to the assumption $\partial A_\perp/\partial t \approx 0$, Thus the perpendicular electric field is purely electrostatic: $\vec{E}_\perp \approx -\vec{\nabla}\phi$.

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Reduced resistive Ohm's law

$$\vec{E} + \frac{\vec{u}}{c} \times \vec{B} = \eta \vec{J}$$

Using the magnetic field representation in terms of the magnetic surface function ψ, of the velocity $\vec{u}$ in terms of the the stream function φ, Ohm's law takes the form:

$$-\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \psi}{\partial t}\hat{e}_z + \frac{1}{c}\left(\vec{\nabla}\varphi \times \hat{e}_z\right) \times \left(B_z\hat{e}_z + \vec{\nabla}\psi \times \hat{e}_z\right) = \eta \vec{J}.$$

In the perpendicular direction we obtain $-\vec{\nabla}_\perp \phi \sim B_z \vec{\nabla}_\perp \varphi / c$: the perpendicular plasma motion is simply given by the $\vec{E} \times \vec{B}$ drift (no pressure terms and resistive effects do not count).

Reduced resistive Ohm's law

The z component of Ohm's law gives[20]

$$\frac{\partial}{\partial t}\psi + \vec{u}\cdot\vec{\nabla}\psi = c\eta J_z$$

i.e.,

$$\frac{\partial}{\partial t}\psi + \vec{\nabla}\varphi\times\hat{e}_z\cdot\vec{\nabla}\psi = c\eta J_z$$

where $\vec{\nabla}\varphi\times\hat{e}_z\cdot\vec{\nabla}\psi = \vec{\nabla}\psi\times\vec{\nabla}\varphi\cdot\hat{e}_z$

[20] Independently on the the condition of incompressibility, if we can neglect the electrical resistivity the magnetic flux function ψ is a Lagrangian invariant satisfying the equation: $d\psi/dt = 0$

Momentum equation

$$\frac{\partial}{\partial t}\vec{u} + \left(\vec{u}\cdot\vec{\nabla}\right)\vec{u} = -\frac{\vec{\nabla}P}{\rho} + \frac{1}{c}\frac{\vec{J}\times\vec{B}}{\rho}$$

Assume a barotropic closure $P = P(\rho)$

$$\frac{\partial}{\partial t}\vec{u} - \vec{u}\times\left(\vec{\nabla}\times\vec{u}\right) = -\vec{\nabla}\left(\frac{P}{\rho} - \frac{1}{2}u^2\right) + \frac{1}{c}\frac{\vec{J}\times\vec{B}}{\rho}$$

Taking the z-component of its curl (as for the incompressible Euler equation):

$$\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{u})\cdot\hat{e}_z - \hat{e}_z\cdot\vec{\nabla}\times\left[\vec{u}\times\left(\vec{\nabla}\times\vec{u}\right)\right] = \frac{1}{c}\hat{e}_z\cdot\vec{\nabla}\times\left(\vec{J}\times\vec{B}/\rho\right)$$

Two scalar equations

Expressing $\vec{u}$ in terms of the stream function φ and J_z in terms of ψ we have $\hat{e}_z \cdot \vec{\nabla} \times \vec{u} = -\nabla^2\varphi, \;\; J_z = -(c/4\pi)\nabla^2\psi.$
Normalizing spatial scales to the equilibrium length L and times to the Alfvén time $\tau_A = L/c_A$ with $c_A^2 = B_{y0}^2/(4\pi\rho)$, the parallel components of Ohm's law and of the vorticity equation become

$$\frac{\partial}{\partial t}\psi + [\varphi\,,\,\psi] = \varepsilon_\eta \nabla^2\psi$$

$$\frac{\partial}{\partial t}\nabla^2\varphi + \left[\varphi\,,\,\nabla^2\varphi\right] = \left[\psi\,,\,\nabla^2\psi\right]$$

with the Poisson brackets $[f\,,\,g] = \partial_x f \partial_y g - \partial_y f \partial_x g$
and $\varepsilon_\eta = \tau_A/\tau_R$,

Linearized reconnection equations

Consider perturbations of the form $\psi(x), \varphi(x) \exp(ik_y y - i\omega t)$ with $\psi(x), \varphi(x)$ the perturbation amplitudes:

$$-i\omega\psi - i\psi_0' k_y \varphi = \varepsilon_\eta^2 \left[-k_y^2 + \frac{\partial^2}{\partial x^2}\right]\psi$$

$$i\omega\left[-k_y^2 + \frac{\partial^2}{\partial x^2}\right]\varphi = -ik_y\psi_0'\left[-k_y^2 + \frac{\partial^2}{\partial x^2}\right]\psi - ik_y\psi\psi_0'''$$

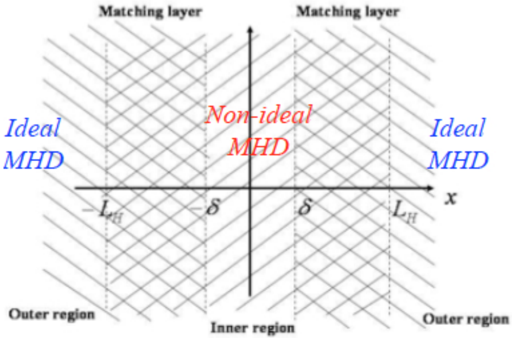
The term ψ_0''' gives the current density inhomogeneity.

Translation:

$\psi(x)$ is proportional to the x component of the perturbed magnetic field,
$\varphi(x) = -\omega\xi_x/k_y$, with ξ_x the x component of the displacement vector.

Boundary layer approach

We search for modes where ω scales with a positive power (smaller than one[21]) of ε_η and adopt a boundary layer approach.



[21] otherwise we would be looking for resistive diffusion

Boundary Layer and asymptotic matching

When the length scales of the equilibrium field are larger than those related not-ideal effects ($\delta \ll L$) we can solve (to the required order) the dynamical equations in two regions that partly overlap and then match the solutions in the overlapping region.

The condition $\delta \ll L$ implies that we can expand locally the various quantities; the not-ideal terms are not negligible only in the inner region ($x \sim \delta$) where the higher order derivatives become larger.
The ideal equations, which are valid in the outer region, impose the boundary conditions for the inner solution.

Outer solution

Leading order equations:

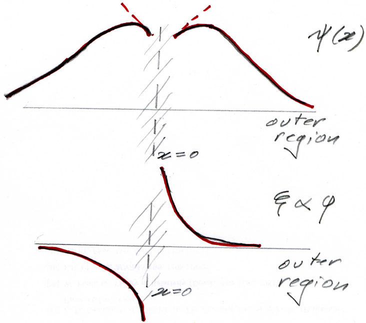
$$-i\omega\psi - i\psi_0' k_y \varphi = 0$$

$$0 = -ik_y\psi_0'\left[-k_y^2 + \frac{\partial^2}{\partial x^2}\right]\psi - ik_y\,\psi\,\psi_0'''$$

$k_y\psi_0'$ plays the role of $k_{||}$ and vanishes for $x \to 0$. Take $\psi_0''' \sim 0$ for $|x| \to \infty$ (local current inhom.) $\psi_{out}(|x| \gg 1) \sim \exp(-k_y|x|)$

The sign of ψ_0'''/ψ_0' determines how the flux function "enters" the inner region, i.e. the sign $\partial\psi_{out}/\partial x$ in the matching region. If $\psi_0'''/\psi_0' < 0$ so that $(k_y^2 + \psi_0'''/\psi_0') < 0$, the magnetic surface function can turn convex near the matching region.

Outer solution



We have taken $\psi(x)$ an even function; the x component of the perturbed magnetic field must not vanish at $x = 0$.
With this choice the first derivative of $\psi(x)$ is discontinuous for $|x|_{out} \to 0$ while ξ_x is odd and diverges as $1/x$.

Instability parameter: Δ'

Take the limit for $|x|_{out} \to 0$ of the logarithmic derivative of $\psi(x)$

$$\Delta' = \lim_{|x_{out}|\to 0} \frac{\psi'_{out}(|x_{out}|) - \psi'_{out}(-|x_{out}|)}{\psi_{out}(|x_{out}|)}$$

$\Delta' = \Delta'(\psi_0, k_y)$ depends on the chosen equilibrium and on the wavelength of the perturbation.

Matching condition $\psi_{out} \approx 1 + \Delta'|x|/2$

Reconnection is driven by the current inhomogeneity that provides the (magnetic) energy from the outer domain, and is made possible by the not-ideal parameters (here by resistivity). Δ' measures the rate at which magnetic energy enters the inner domain[22].

[22] R White, *Rev. Mod. Phys.*, **58**, 183, (1986) and references therein

Inner equations

Order $\partial/\partial x \gg k_y$, expand $\psi_0(x)$ around $x = 0$ and use the stretched variable x/δ

$$-i\,\omega\psi_{in} - i\psi_0' k_y \varphi_{in} = \varepsilon_\eta\, \psi_{in}''$$

$$i\,\omega\varphi_{in}'' = -i\,k_y\,\psi_0'\,\psi_{in}''$$

Length are normalized to the width of the inner domain δ. The limit $|x| \to \infty$ taken in the inner domain defines spatial scales that can be “arbitrarily small” (i.e.: corresponds to the limit $|x| \to 0$) if seen from the outer region where scale lengths are normalized with respect to L ($\delta \ll L$).

Inner equations

Revert to more standard notation $\omega = -i\gamma$ and express φ in terms of the displacement ξ_x (along x).

$$\psi + x\xi_x = \frac{\varepsilon_\eta}{\gamma}\psi'' \qquad \gamma^2 \xi_x'' = -k_y^2 x\psi''$$

GENERAL RESISTIVE ORDERING

Assume $x \sim \delta$, $\partial/\partial x \sim \delta^{-1}$ (when acting on ψ and on ξ_x) with $\psi \sim \delta^0$ and $\xi_x \sim \delta^{-1}$. Set

$$\delta \sim \varepsilon_\eta^\alpha, \qquad \gamma \sim \varepsilon_\eta^\beta$$

$0 = 1 - 2\alpha - \beta,\ 2\beta - 3\alpha = -\alpha,$ $\rightarrow$ resistive ordering

$$\alpha = \beta = 1/3$$

Inner equations

The resistive ordering requires a large value of Δ' ($\Delta \sim 1/\delta$)
If this is not the case (as determined by the outer solution) we can refine the ordering[23] by assuming that
$\partial/\partial x \sim \delta^{-1}$ when acting on ξ_x while
$\partial\psi/\partial x \sim \delta^0$ and $\partial^2\psi/\partial x^2 \sim \delta^{-1}$

Constant ψ ordering (Tearing ordering)

Set again $\delta \sim \varepsilon_\eta^\alpha, \quad \gamma \sim \varepsilon_\eta^\beta$

$$0 = 1 - \alpha - \beta, \; 2\beta - 3\alpha = 0, \qquad \rightarrow \text{constant } \psi \text{ ordering}$$

$$\alpha = 2/5, \qquad \beta = 3/5$$

[23] H. P. Furth, J. Killeen, and M. N. Rosenbluth, *Phys. Fluids*, **6**, 459 (1963)

Full solution and dispersion relation

The dispersion relation can be obtained by matching the inner and the outer solutions in the matching region.

The asymptotic matching is also applicable in Fourier space[24] $\rightarrow$ a second order differential equation in the Fourier variable k.

Conditions in Fourier space: for a smooth $\psi(x)$ its Fourier transform must be at least exponentially decreasing as $k \rightarrow \infty$
The spatial matching in the matching layer is transformed into a boundary condition for $k < 1/\delta$.

[24] however, "distributions" are needed since the functions to transform are not L^2.

Dispersion relation

In k space the inner equations can be solved in terms on known functions[25] for all values of Δ' i.e., without the need of resort to to the "constant ψ subordering".
In this way we recover the FKR dispersion relation:

$$\hat{\gamma} = -\frac{1}{\hat{\Delta}'}\left[\frac{\hat{\gamma}^{\frac{9}{4}}}{8}\frac{\Gamma\left[(\hat{\gamma}^{3/2}-1)/4\right]}{\Gamma\left[(\hat{\gamma}^{3/2}+5)/4\right]}\right]$$

where $\hat{\gamma} = \gamma/(\varepsilon_\eta)^{\frac{1}{3}} \leq 1$ and $\hat{\Delta}' = \Delta'(\varepsilon_\eta)^{\frac{1}{3}}$.
$\Gamma(\hat{\gamma}^{3/2} - ..)$ are Γ functions.

Instability requires $\Delta' \geq 0$

[25] F. Pegoraro, T.J. Schep, *Plasma Phys. Contr. Fus.*, **28**, 647 (1986)

Dispersion relation in the resistive and in the tearing orderings

- In the general $\varepsilon_\eta^{1/3}$ ordering the dispersion relation reads:

$$\hat{\gamma}^{5/4} = -8\hat{\Delta}' \frac{\Gamma\left[(\hat{\gamma}^{3/2}+5)/4\right]}{\Gamma\left[(\hat{\gamma}^{3/2}-1)/4\right]}$$

- $\hat{\gamma} = 1$ for $\Delta' \to \infty$
- In the tearing ordering we have $\hat{\gamma} \ll 1$ and the dispersion equation becomes : $\hat{\gamma}^{5/4} = -8\hat{\Delta}'\Gamma(5/4)/\Gamma(-1/4)$ i.e.,[26]

$$\gamma^{5/4} = 2\,\varepsilon_\eta^{3/4}\Delta' \frac{\Gamma(5/4)}{\Gamma(3/4)}$$

recovering the $\gamma \sim \varepsilon_\eta^{3/5}$ ordering .

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Some conclusions

- In all cases in which a boundary layer theory applies, (i.e., the equilibrium and the reconnection scales are separated) the linear growth rate width depends on fractional powers of ε_η
- Similar dependences on the microscopic physics that makes reconnection possible are obtained (with the appropriate scalings) when electron inertia effect are considered ($\eta \leftrightarrow \gamma/\omega_{pe}^2$) and in general on the microscopic physics that occurs in the reconnection layer (including ion electron decoupling, "neoclassical effects", anisotropic pressure ...).
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- Geometric effects change these scalings: e.g., along a neutral line (in a hyperbolic configurations) one finds dependence of the type $\ln \varepsilon_\eta$[27]
- The specific dependences of the growth rate on the microscopic effect predicted by linear theory appear to be “non-robust” (can be easily modified by a change of regime)
- How to apply these considerations to nonlinear regimes is not straightforward

[27] I.J. Craig, P.G. Watson, *Ap J* **393**, 385 (1992), or $\varepsilon_\eta \ln \varepsilon_\eta$ in the presence of a guide field: M. De Benedetti, F. Pegoraro, *Plasma Phys. Control. Fus.*, **37** 103 (1995)

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- How to apply these considerations to nonlinear regimes is not straightforward

[27] I.J. Craig, P.G. Watson, *Ap J* **393**, 385 (1992), or $\varepsilon_\eta \ln \varepsilon_\eta$ in the presence of a guide field: M. De Benedetti, F. Pegoraro, *Plasma Phys. Control. Fus.*, **37** 103 (1995)

Some conclusions

- What determines the nonlinear time scales? Are they Alfvènic? (linear resistive theory would predict $\varepsilon_\eta^{1/3}$ scaling even for infinitely strong "drive" ($\Delta' \to \infty$).
- Scale separation can be huge $\varepsilon_\eta \sim 10^{-8}, 10^{-10}$ (different scalings are really different !) but the equilibrium scale may shrink to the reconnection scale
- In fully nonlinear regimes simulations cannot be run in such extreme asymptotic regimes.
- Open problems, trigger, time and space scale, energy redistribution, particle acceleration, signatures .... kinetic effects, anisotropy, instability mixing, relativistic effects....

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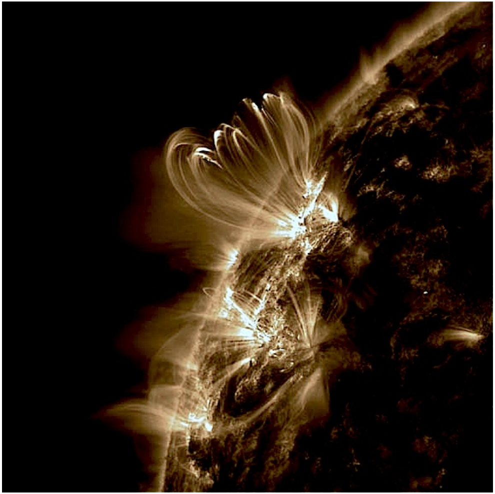
THANKS FOR YOUR ATTENTION



Appendix - Relativistic "dynamo" equation

- The dynamo equation for the magnetic field can be written in general in three-dimensional notation although it is not covariant

$$\partial\vec{B}/\partial t - \nabla \times (\vec{v} \times \vec{B}) = 0$$

- Ohm's law $F_{\mu\nu}u_\mu = 0$ can be rewritten as a covariant equation for the variation of the four vector potential along the fluid motion in the (Lagrangian) form

$$\partial A_\nu/\partial\tau = u_\mu(\partial_\nu A_\mu),$$

where $\partial/\partial\tau = u_\mu\partial_\mu$.

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Appendix - Relativistic "dynamo" equation

From the equation for the vector potential we obtain a covariant form for relativistic dynamo equation in terms of the field tensor[28]

$$\partial_\tau F_{\mu\nu} = (\partial_\mu u_\alpha) F_{\nu\alpha} - (\partial_\nu u_\alpha) F_{\mu\alpha}$$

- In a given frame, by projecting this equation onto its space-space and space-time components[29] we recover the non covariant equation for the magnetic field $\partial\vec{B}/\partial t - \nabla \times (\vec{v} \times \vec{B}) = 0$ and an associated equation for the time derivative of the electric field $\vec{E}$.

---

[28] $\partial_\mu \partial_\tau \neq \partial_\tau \partial \mu$

[29] Here we use $F_{\mu\nu} u_\mu = 0$ when writing $(\partial_\mu u_\alpha) F_{\nu\alpha} = \gamma F_{\nu\alpha} \partial_\mu (u_\alpha / \gamma)$.

Appendix - Relativistic Connection theorem

Define the infinitesimal (spacelike) dl_μ (distance between two close events) such that

$$dl_\mu F_{\mu\nu} = 0.$$

In the frame where $dl_o = 0$ this is equivalent to $d\vec{l} \times \vec{B} = 0$ and includes $d\vec{l} \cdot \vec{E} = 0$. Note that for such a four vector to satisfy the above condition, the Lorentz invariant $F_{\mu\nu}F^*_{\mu\nu}$ must vanish (i.e. $\vec{E} \cdot \vec{B} = 0$).
If this condition is satisfied dl_μ belongs to a 2-dimensional hyperplane. If the Lorenz invariant $E^2 - B^2 < 0$ (this is the case we must consider since u_μ is timelike), we can choose one timelike (e.g., along u_μ) and one (dl_μ) spacelike direction.

Appendix - Relativistic Connection theorem

Using $\Delta x_\mu = u_\mu \Delta\tau,\ \Delta dl_\mu = [dl_\alpha(\partial_\alpha u_\mu)]\Delta\tau + u_\mu[dl_\alpha(\partial_\alpha \Delta\tau)]$ and $\partial_\tau F_{\mu\nu} = (\partial_\mu u_\alpha) F_{\nu\alpha} - (\partial_\nu u_\alpha) F_{\mu\alpha}$ together with $F_{\mu\nu} u_\mu = 0$, we obtain

$$\partial_\tau (dl_\mu F_{\mu\nu}) = -(\partial_\nu u_\beta)(dl_\alpha F_{\alpha\beta}).$$

which is the covariant form of the 3D connection theorem. Note that in a reference frame where $dl_o \neq 0$ we can "project" dl_μ onto 3D space by defining $dl'_\mu = dl_\mu - u_\mu\, dt$ without changing the connection theorem since $F_{\mu\nu} u_\mu = 0$

Thus we simply have to reset the time by moving the endpoints of the line connecting the two close events along their trajectories in order to recover the standard form of the connection theorem.

Appendix - Relativistic Magnetic helicity

3D Magnetic helicity is the time component of the four vector

$$K_\mu = F^*_{\mu\nu} A_\nu$$

which obeys the standard gauge transformations (for $A_\nu \rightarrow A_\nu + \partial_\nu \psi$ a four divergence $\partial_\nu (F^*_{\mu\nu}\psi)$ is added since Maxwell's equations give $\partial_\nu F^*_{\mu\nu} = 0$) and satisfies the conservation equation

$$\partial_\mu K_\mu = F^*_{\mu\nu}(\partial_\mu A_\nu) = F^*_{\mu\nu} F_{\mu\nu}/2 = 0$$

which is the covariant form of the three-dimensional continuity equation for the helicity density.

Appendix No quasineutrality

- Note that the relativistic Ohm's law $F_{\mu\nu}u_\mu = 0$ is obtained independently of any quasineutrality assumption.
- Taking the four divergence of the above equation we obtain an Eulerian relationship involving the fluid four velocity and vorticity tensor and the electromagnetic field tensor and four current density

$$-u_\nu \partial_\mu F_{\mu\nu} = (4\pi/c)\, u_\nu j_\nu = F_{\mu\nu}(\partial_\mu u_\nu - \partial_\nu u_\mu)/2.$$

  This equation cannot be used in standard quasineutral nonrelativistic MHD.

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This equation cannot be used in standard quasineutral nonrelativistic MHD.

Introduce three (curvilinear) coordinates $\chi_i,\ i = 1,2,3$. Write the electromagnetic vector-potential $\vec{A}$ as:

$$\vec{A}' = A'_1 \vec{\nabla}\chi_1 + A'_2 \vec{\nabla}\chi_2 + A'_3 \vec{\nabla}\chi_3$$

Use gauge invariance and choose a scalar function K satisfying $\vec{\nabla}K = K_1 \vec{\nabla}\chi_1 + K_2 \vec{\nabla}\chi_2 + K_3 \vec{\nabla}\chi_3$ where $K_3 = -A'_3$.
This means $K = -\int_0^{\chi_3} A'_3\,(\chi_1, \chi_2, \xi_3)\, d\xi_3$.
Then

$$\vec{A} = \vec{A}' + \vec{\nabla}K = A_1 \vec{\nabla}\chi_1 + A_2 \vec{\nabla}\chi_2$$

and

$$\vec{B} = \vec{\nabla} \times \vec{A} = \vec{\nabla}A_1 \times \vec{\nabla}\chi_1 + \vec{\nabla}A_2 \times \vec{\nabla}\chi_2$$

Assume $\vec{B} \neq 0$, and take

$$\vec{B} \cdot \vec{\nabla}\chi_1 \neq 0 \qquad \Leftrightarrow \qquad \left(\vec{\nabla}A_2 \times \vec{\nabla}\chi_2\right) \cdot \vec{\nabla}\chi_1 \neq 0$$

If $\vec{B}\cdot\vec{\nabla}\chi_1 \neq 0$ the Jacobian determinant of the coordinate transformation $(\chi_1,\chi_2,\chi_3) \to (\chi_1,\chi_2,A_2)$ is non-zero, and the transformation is *locally invertible*. Using

$$\vec{B} = \vec{\nabla}A_1(\chi_1,\chi_2,A_2)\times\vec{\nabla}\chi_1 + \vec{\nabla}A_2\times\vec{\nabla}\chi_2$$

and $d\vec{l}\times\vec{B} = 0$ we obtain

$$(d\vec{l}\cdot\vec{\nabla}A_1)\vec{\nabla}\chi_1 - (d\vec{l}\cdot\vec{\nabla}\chi_1)\vec{\nabla}A_1 + (d\vec{l}\cdot\vec{\nabla}A_2)\vec{\nabla}\chi_2 - (d\vec{l}\cdot\vec{\nabla}\chi_2)\vec{\nabla}A_2 = 0$$

This leads to three scalar equations:

$$(d\vec{l}\cdot\vec{\nabla}A_2) - (d\vec{l}\cdot\vec{\nabla}\chi_1)\frac{\partial A_1}{\partial\chi_2} = 0 \qquad \Rightarrow \qquad \frac{dA_2}{d\chi_1} = \frac{\partial A_1}{\partial\chi_2}$$

$$(d\vec{l}\cdot\vec{\nabla}\chi_1)\frac{\partial A_1}{\partial A_2} + (d\vec{l}\cdot\vec{\nabla}\chi_2) = 0 \qquad \Rightarrow \qquad \frac{d\chi_2}{d\chi_1} = -\frac{\partial A_1}{\partial A_2}$$

$$(d\vec{l}\cdot\vec{\nabla}A_1) - (d\vec{l}\cdot\vec{\nabla}\chi_1)\frac{\partial A_1}{\partial\chi_1} = 0 \qquad \Rightarrow \qquad \frac{dA_1}{d\chi_1} = \frac{\partial A_1}{\partial\chi_1}$$

We can interpret these results in terms of Hamiltonian mechanics:

- χ_1 can be considered as the “time” variable.
- χ_2 and A_2 play the role of the two canonical coordinates: $d\chi_2 \leftrightarrow dq$ and $dA_2 \leftrightarrow dp$.
- $A_1(\chi_2, A_2, \chi_1)$ becomes then the “Hamiltonian” $\mathscr{H}$.

Integrability requires the existence of a constant of the motion: e.g., the energy if the Hamiltonian does not depend on χ_1.
In this latter case we have that $A_1 = const$ corresponds to a magnetic surface: $\vec{B} \cdot \vec{\nabla} A_1 = \vec{\nabla} A_2 \times \vec{\nabla} \chi_2 \cdot \vec{\nabla} A_1(\chi_2, A_2) = 0$

Magnetic Hamiltonian

In this case it can be shown that the ideal MHD evolution corresponds to a canonical transformation of the the Hamiltonian A_1 and of the coordinates A_2 and χ_2 depending parametrically on time t and that Ohm's law plays the role of the equation for the generating function (related to the scalar electric potential).