The Vlasov equation: Theory and Simulation

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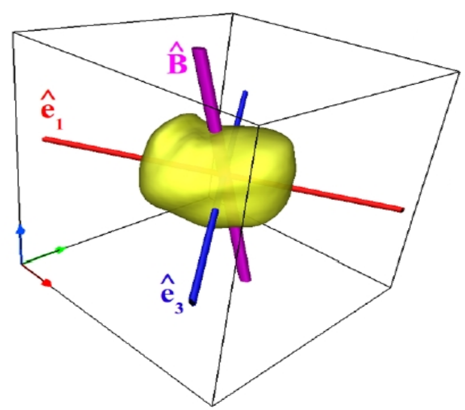

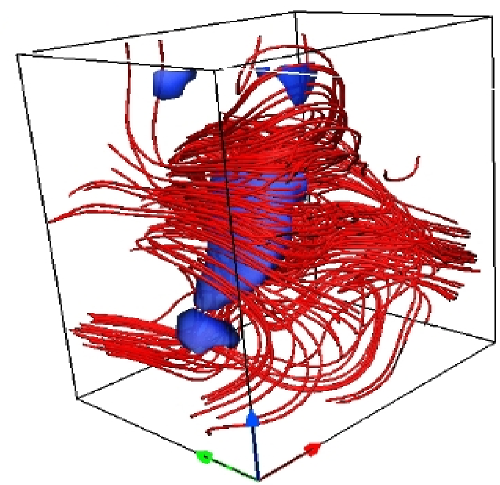

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EU School on Space Weather fundamental plasma processes, Spineto 4-9 June 2012

SPACE-WEATHER INTEGRATED FORECASTING FRAMEWORK SWIFF

OUTLINE

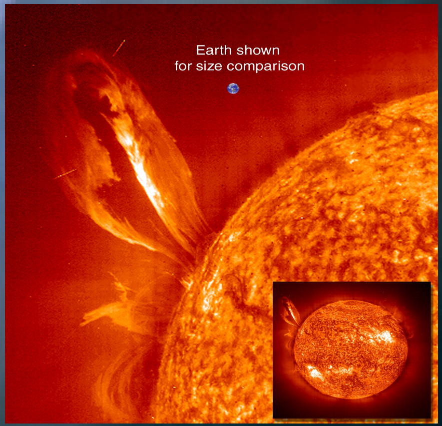
FIRST PART

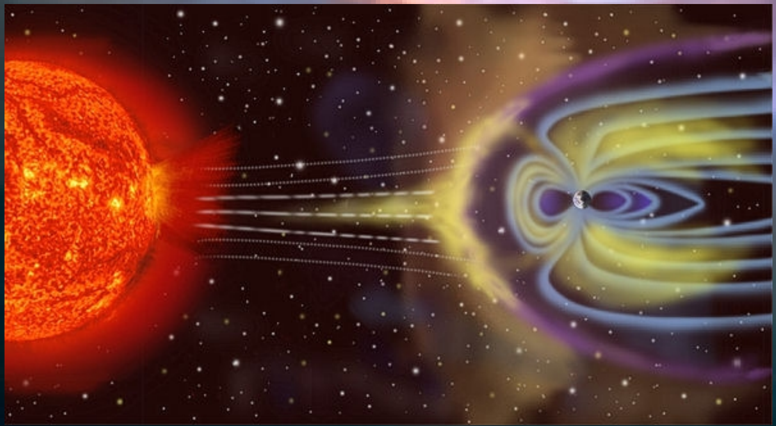
-The physics of the interplanetary medium: the solar wind plasma
- How to model a plasma? a statistical approach
- The kinetic dynamics of a collision-free plasma: the Vlasov equation
- The physics of wave-particle resonant interaction (linear and nonlinear regime)
- Plasma wave echoes

SECOND PART

- The numerical approach for the solution of the Vlasov equation
- Few words about finite difference schemes
- Eulerian algorithms for Vlasov simulations
- Distribution of particle velocities in space plasmas
- The tail at short spatial scales of turbulence in the solar wind: Hybrid Vlasov-Maxwell simulations
- State of the art and work in progress

Everywhere in the Universe, from near-Earth environments, like for example planetary magnetospheres and solar atmosphere, to further systems, like radio galaxies and supernovae, the dynamics is driven by plasma processes





A plasma is a collection of discrete charged particles globally neutral, behaving as a collective system, dominated by long range (electromagnetic) interactions

Plasma Physics

1. Astrophysics; space plasmas
2. Laboratory plasmas (fusion, ...)
3. Laser plasmas interaction

Plasma as an ideal, classical and collisionless gas

Plasma parameter

$$g = \left(\frac{e^2 n^{1/3}}{kT}\right)^{3/2}$$

Low density and high temperatures

$$g \ll 1 \Leftrightarrow \frac{3}{2}kT \gg \frac{e^2}{r}$$

Ideal gas with rare collisions

The mean free path of a particle in the solar wind is about 1AU

Debye length and Debye potential

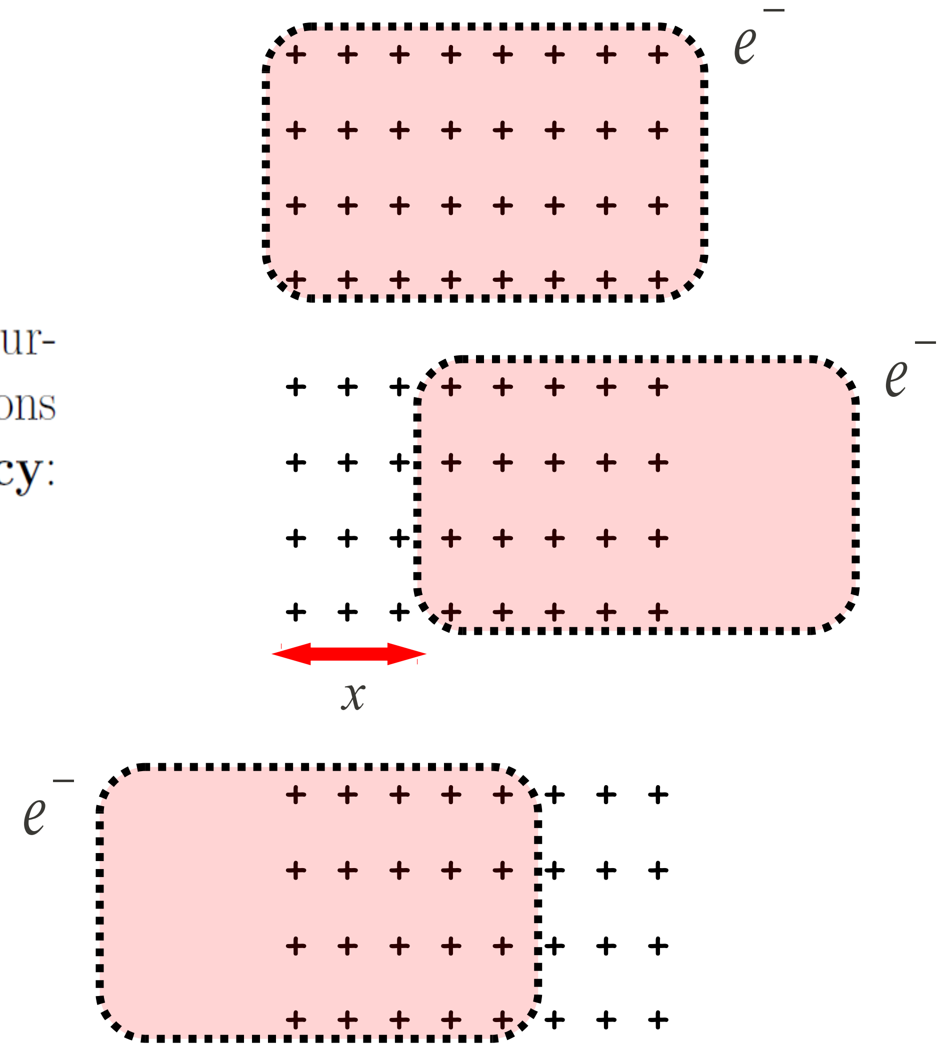
$$\lambda_D^2 = \frac{kT}{4\pi n e^2}; \quad \frac{1}{g} \simeq n\lambda_D^3 \qquad \Phi_D = \frac{q}{4\pi\epsilon_0 r}\, e^{-r/\lambda_D}$$

Collective behavior of a plasma

A simple view: background of fixed ions

Collective response to external charge perturbations by high frequency electron oscillations around "fixed ions" at the **plasma frequency**:

$$\omega_{pe} = \left(4\pi n e^{2}/m_e\right)^{1/2}$$

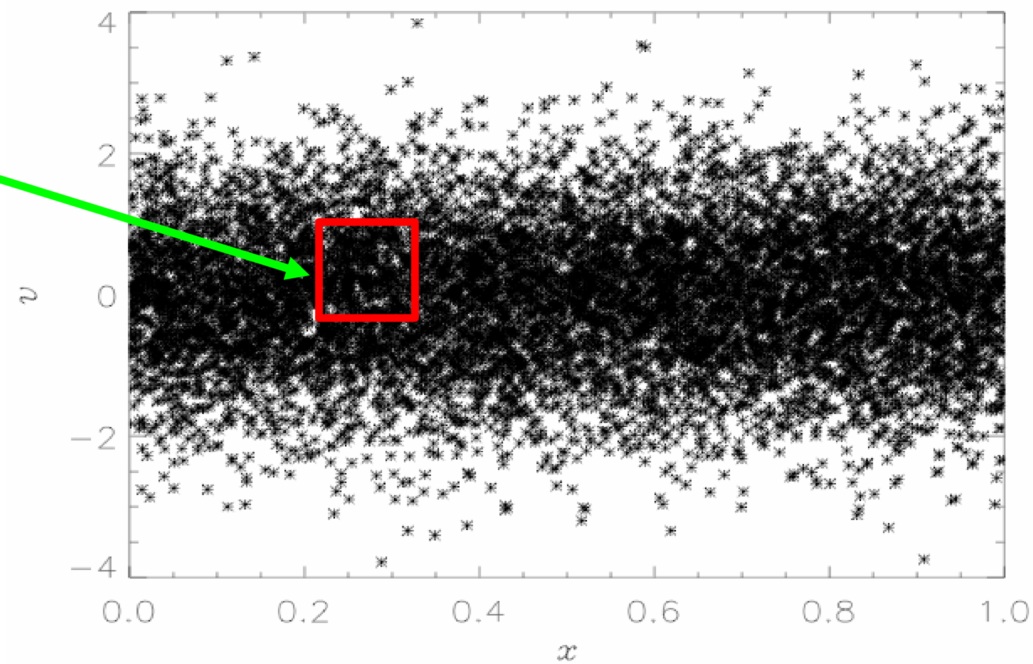


A statistical description of a plasma

Many instead of one particle in phase space (x,v):

$$f\left(x,v,t\right)dxdv$$

Average number of particles in $dxdv$



$$f=f\left(x,v,t\right)$$

Distribution function

Macroscopic variables of a plasma

$$f_\alpha = f_\alpha(\mathbf{x}, \mathbf{v}, t)$$

$$\int f_\alpha(\mathbf{x}, \mathbf{v}, t) d\mathbf{x} d\mathbf{v} = N_\alpha$$

The macroscopically observable quantities are found from the velocity moments of the *d.f.* :

Number of particles, Current density:

$$n_a(\mathbf{x}, t) = \int f_a(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v}$$

$$\mathbf{J}_a(\mathbf{x}, t) = q_a n_a(\mathbf{x}, t) \mathbf{V}_a(\mathbf{x}, t) = q_a \int \mathbf{v} f_a(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v}$$

Pressure tensor, Scalar pressure:

$$P_a(\mathbf{x}, t) = m_a \int (\mathbf{v} - \mathbf{V}_a)(\mathbf{v} - \mathbf{V}_a) f_a(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v}$$

$$p_a = \frac{1}{3}(p_{xx} + p_{yy} + p_{zz}) = n_a T_a$$

The Vlasov Equation

In the absence of collisions the Vlasov equations describes the evolution of the distribution function under the effects of self-consistent and external electromagnetic fields

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla f_\alpha + \frac{q_\alpha}{m_\alpha} \left[\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right] \cdot \nabla_v f_\alpha = 0$$

$$f_\alpha = f_\alpha(\mathbf{x}, \mathbf{v}, t)$$

$$\frac{df_\alpha}{dt} = 0$$

when calculated on the characteristics

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \frac{q_\alpha}{m_\alpha} \left[\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right]$$

Properties of the Vlasov equation

Invariants of the Vlasov equation:

$$I_a^n(t) = \int f_a^n(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{x}\, d\mathbf{v} \;=\; cste$$

Any power of the *d.f.* is a constant of motion.

More generally, any function $G(f)$ of f is a constant of motion:

$$\frac{dG}{dt} = \frac{dG}{df}\frac{df}{dt} = 0$$

For example, the entropy of the system is a constant:

$$\mathcal{S} = -\int f\, ln(f) d\mathbf{x}\, d\mathbf{v}$$

This is consistent with the fact that the Vlasov equation neglects the process (binary collisions) which causes statystical systems to increase their entropy and evolve toward a Maxwell-Boltzmann distribution

Vlasov-Maxwell equations (mean-field theory)

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla f_\alpha + \frac{q_\alpha}{m_\alpha}\left[\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}\right] \cdot \nabla_v f_\alpha = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = 4\pi \sum_\alpha q_\alpha \int f_\alpha(\mathbf{x}, \mathbf{v}, t) d\mathbf{v}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \sum_\alpha q_\alpha \int \mathbf{v} f_\alpha(\mathbf{x}, \mathbf{v}, t) d\mathbf{v} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

NO COLLISIONS!!!

Vlasov equation coupled to Maxwell equations is a nonlinear problem, whose analytical solution is availaible only in few simplified cases

Linear regime of wave-particle interaction

Electron distribution function; fixed ions with constant density

Vlasov-Poisson (1D-1V)

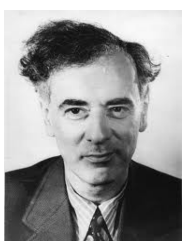
$$\frac{\partial f_e}{\partial t} + v\frac{\partial f_e}{\partial x} - \frac{eE}{m}\frac{\partial f_e}{\partial v} = 0$$

$$\frac{\partial E}{\partial x} = 4\pi e\left[n_0 - \int f_e(x,v,t)dv\right]$$

Equilibrium configuration

$$f_e(v,t=0) = \frac{m_e^{1/2} n_0}{(2\pi kT)^{1/2}} \exp\left[-\frac{m_e}{2kT}v^2\right]$$

$$n_0 = const.; \quad E_0 = 0; \quad B_0 = 0$$



ON THE VIBRATIONS OF THE ELECTRONIC PLASMA

By L. LANDAU

Institute for Physical Problems, Academy of Sciences of the USSR

(Received June 2, 1945)

The vibrations of the electronic plasma are considered, which arose as a result of an arbitrary initial non-equilibrium distribution in it. It is shown that the vibrations of the field in plasma are always damped, and the dependence of the frequency and of the damping decrement on the wave vector is determined for small and for large values of the latter.

The penetration of a periodical external electric field into the plasma is considered. The case of the frequency of the external field being almost at resonance with the proper frequency of plasma is considered separately.

Small amplitude perturbations

$$f_e(x,v,t) = f_e(v,t=0) + \delta f_e(x,v,t)$$

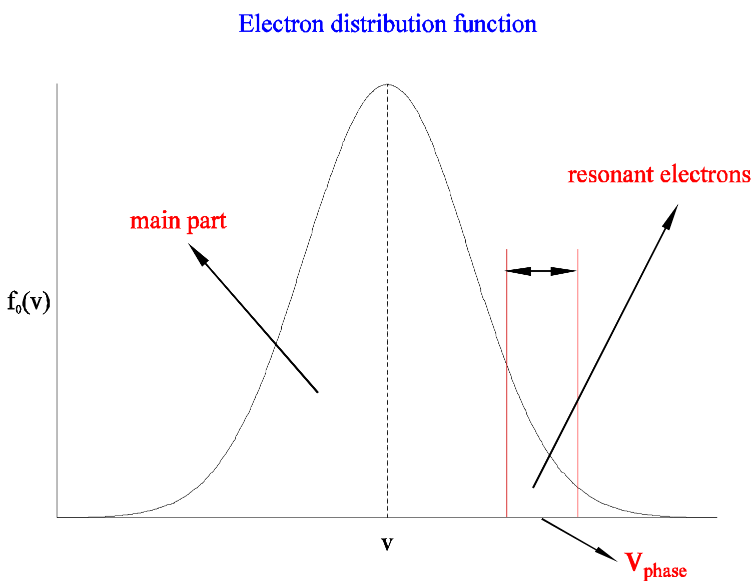
$$E(x,t) = \delta E(x,t)$$

$$\delta E(x,t) \simeq \exp\left[ikx - i\omega_R t + \omega_I t\right]$$

$$\omega_R \simeq \omega_{pe}$$

$$\omega_I \propto \left[\frac{df_e(v,t=0)}{dv}\right]_{v=v_\phi}$$

A simple case of wave damping



Electric sinusoidal perturbation
(in the lab frame)

$$\delta E(x',t) = \hat{E} sin(kx' - \omega t)$$

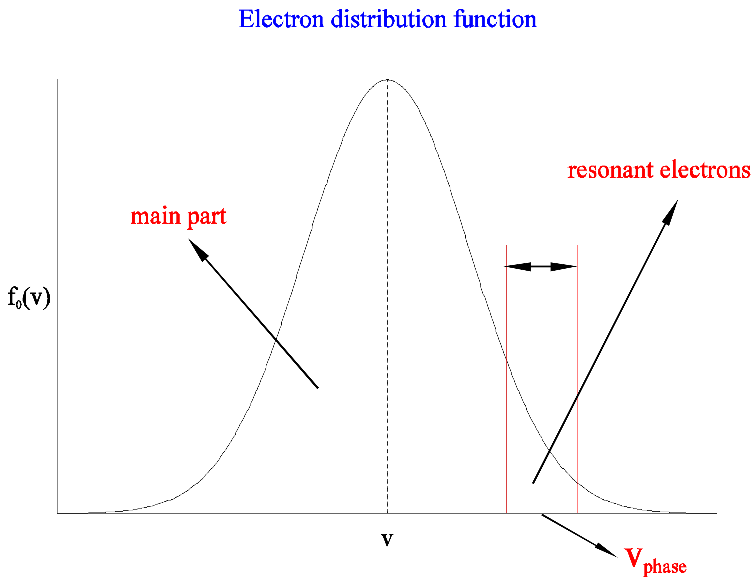
$$\delta E(x,t) = \hat{E} sin(kx)$$ In the wave frame

One can calculate the wave damping coefficient by setting the rate of increase of kinetic energy of resonant particles equal to the rate of decrease of wave energy

The solution for the perturbed distribution function

$$\delta f(x,v,t) = \delta f(v,t=0)\cos(kx - kvt) -$$
$$\frac{e}{m}\hat{E}\frac{\partial}{\partial v}f_e(v,t=0)\left[\frac{\cos(kx) - \cos(kx - kvt)}{kv}\right]$$

A simple case of wave damping



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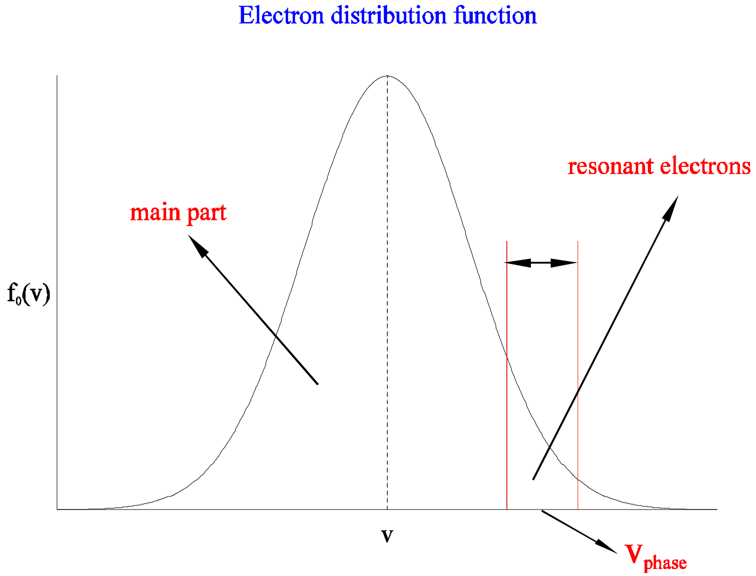
$$\delta f(x,v,t) = \delta f(v,t=0)\cos(kx - kvt) - \frac{e}{m}\hat{E}\frac{\partial}{\partial v}f_e(v,t=0)\left[\frac{\cos(kx) - \cos(kx - kvt)}{kv}\right]$$

$$\frac{dE_k}{dt} = \frac{nm}{2}\int_{-\lambda/2}^{\lambda/2}\frac{dx}{\lambda}\int_{-\infty}^{+\infty} dv\left(v + \frac{\omega}{k}\right)^2\frac{\partial\delta f}{\partial t}$$

$$\frac{d\hat{E}^2}{dt} = 2\omega_I \hat{E}^2$$

A simple case of wave damping



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$$\delta f(x,v,t) = \delta f(v,t=0)\cos(kx - kvt) - \frac{e}{m}\hat{E}\frac{\partial}{\partial v}f_e(v,t=0)\left[\frac{\cos(kx) - \cos(kx-kvt)}{kv}\right]$$

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$$\frac{d\hat{E}^2}{dt} = 2\omega_I \hat{E}^2$$

$$\frac{dE_k}{dt} = -\frac{d}{dt}\left(\frac{\hat{E}^2}{8\pi}\right) \Rightarrow \omega_I = \frac{\pi}{2}\omega_{pe}^2\frac{\omega_R}{k^2 n_0}\left[\frac{df_e(v,t=0)}{dv}\right]_{v=0}$$

The Landau damping rate

What happens to the wave energy?

Suppose you have a perturbation of the form

$$\simeq \exp\left[ik_1 x - i\omega_R t + \omega_I t\right]$$

The energy of this waves can be dissipated through Landau damping

However, in absence of collisions, the memory of the initial perturbation cannot be lost, but it remains locked in the distribution function, as a perturbation in the form

$$\simeq \exp\left[ik_1 x - i\omega_R t + \omega_I t\right] + \exp\left[ik_1 x - ik_1 v t\right]$$

Ballistic contribution

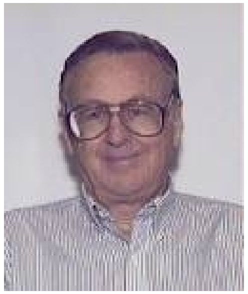
If a second wave is launched into the plasma after the first wave has been Landau damped

A third wave called plasma echo will apperar after a certain delay time

$$\delta E_2 \simeq \exp\left[ik_2 x - i\omega_R^{(2)} t + \omega_I^{(2)} t\right]$$

$$\delta f_2 \simeq \exp\left[ik_2 x - i\omega_R^{(2)} t + \omega_I^{(2)} t\right] + \exp\left[ik_2 x - ik_2 v t\right]$$

Plasma wave echoes



VOLUME 19, NUMBER 5 PHYSICAL REVIEW LETTERS 31 JULY 1967

PLASMA WAVE ECHO*

R. W. Gould
California Institute of Technology, Pasadena, California

and

T. M. O'Neil and J. H. Malmberg
General Atomic Division of General Dynamics Corporation,
John Jay Hopkins Laboratory for Pure and Applied Science, San Diego, California
(Received 29 May 1967)

It is shown that if a longitudinal wave is excited in a collision-free plasma and Landau-damps away, and a second wave is excited and also damps away, then a third wave (i.e., the echo) will spontaneously appear in the plasma.

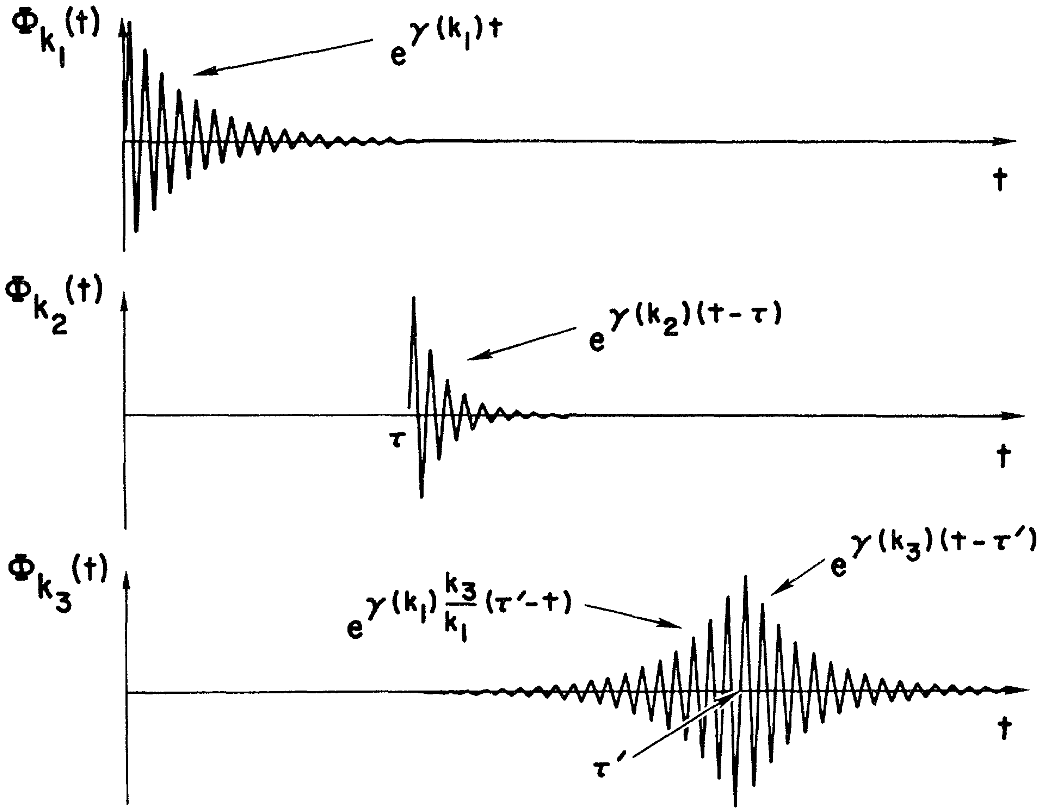
VOLUME 20, NUMBER 3 PHYSICAL REVIEW LETTERS 15 JANUARY 1968

PLASMA WAVE ECHO EXPERIMENT*

J. H. Malmberg, C. B. Wharton, R. W. Gould,† and T. M. O'Neil
General Dynamics, General atomic Division, John Jay Hopkins Laboratory
for Pure and Applied Science, San Diego, California
(Received 22 September 1967)

Experimental observation of a new nonlinear plasma phenomenon, the plasma wave echo, is reported.

Plasma wave echoes



$k_3 = k_2 - k_1$ $\qquad$ $\tau' = \tau\left[\frac{k_2}{k_2 - k_1}\right]$

Nonlinear regime of wave-particle interaction

THE PHYSICS OF FLUIDS VOLUME 8, NUMBER 12 DECEMBER 1965

Collisionless Damping of Nonlinear Plasma Oscillations

THOMAS O'NEIL*
Department of Physics, University of California, San Diego, La Jolla, California
(Received 12 July 1965)

It is well known that the linear theory of collisionless damping breaks down after a time $\tau = (m/e\mathcal{E}k)^{\frac{1}{2}}$, where k is the wavenumber and $\mathcal{E}$ is the amplitude of the electric field. Jacobi elliptic functions are now used to provide an exact solution of the Vlasov equation for the resonant electrons, and the damping coefficient is generalized to be valid for times greater than $t = \tau$. This generalized damping coefficient reduces to Landau's result when $t/\tau \ll 1$; it has an oscillatory behavior when t/τ is of order unity, and it phase mixes to zero as t/τ approaches infinity. The above results are all shown to have simple physical interpretations.

Nonlinear effects: Particle trapping

$$\frac{\partial f_e}{\partial t} + v\frac{\partial f_e}{\partial x} - \frac{eE}{m}\frac{\partial f_e}{\partial v} = 0$$
$$\frac{\partial E}{\partial x} = 4\pi e\left[n_0 - \int f_e(x,v,t)dv\right]$$

L. Landau (1946) linearized the VP equations in the perturbation amplitudes

O'Neil (1965) showed that nonlinear effects are no longer negligible for times comparable to

$$\tau = \left(\frac{m}{eEk}\right)^{1/2}$$

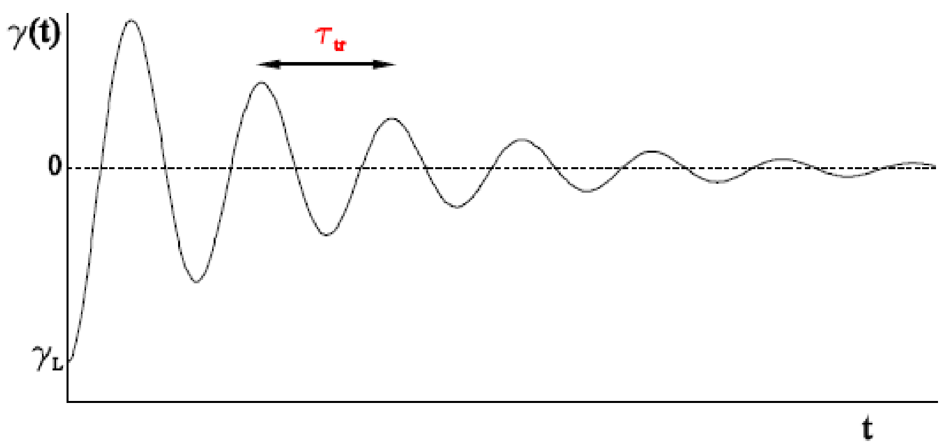
For an electron moving at velocity close to the wave phase speed:

$$m\frac{d^2x}{dt^2} = -eE_k \sin(kx) \simeq eE_k kx$$

Therefore, the charge oscillates with period

$$T_B = \frac{1}{2\pi}\sqrt{\frac{m}{ekE_k}} = \frac{2\pi}{\omega_B}$$

Damping rate time behavior

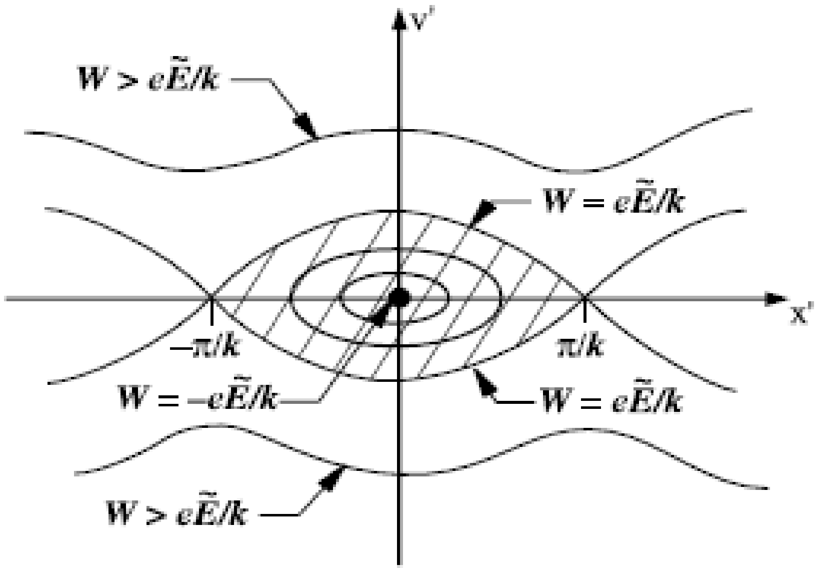


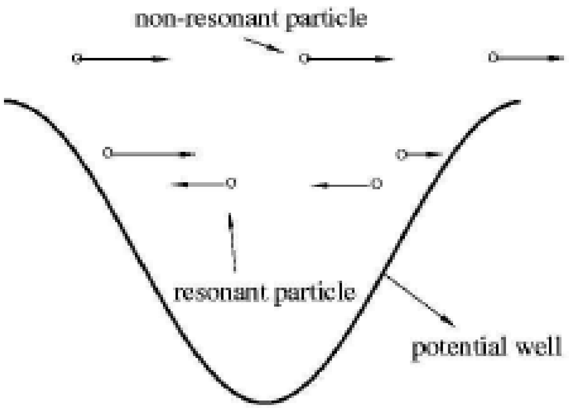
Nonlinear effects get important when resonant particles are trapped in the wave potential well

Phase space particle trajectories

$$E(x,t) = a sin(kx - \omega t); \quad \phi(x,t) = (a/k) cos(kx - \omega t)$$

$$Particle\ energy:\ W = \frac{1}{2}mv^2 - e\phi = cste$$





The simple harmonic motion in the trough of the plasma wave is a

major perturbation of the particle orbit

Linear approximation no longer valid in the resonant region

Eulerian algorithms for Vlasov simulations

The Vlasov equation is discretized on a grid in phase space and the field equations on a spatial grid.

Numerical integration of the Vlasov equation in phase space

Electrostatic case. fixed ions with constant density. 1D-1V phase space (x,v) for the sake of simplicity, but the generalization to multi-dimensional geometry is straightforward

$$\frac{\partial f_e}{\partial t}+v\frac{\partial f_e}{\partial x}+a\frac{\partial f_e}{\partial v}=0$$

Advection equation in phase space

$$f_e=f_e(\mathrm{x,v,t});\mathrm{a}=-\frac{\mathrm{eE}}{m}$$

VLASOV-POISSON SYSTEM

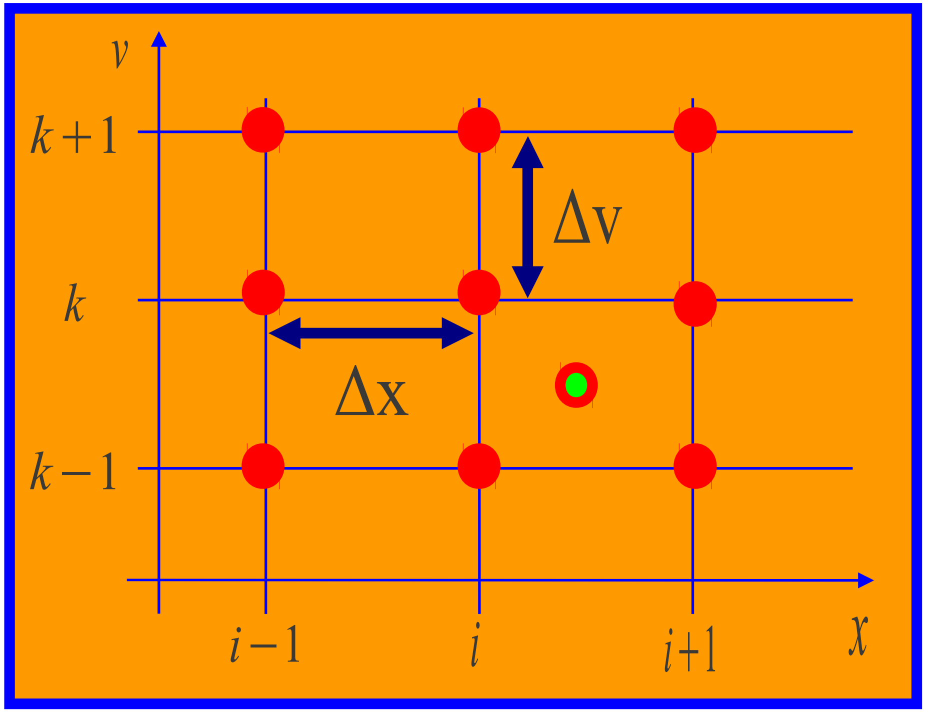
$$\frac{\partial E}{\partial x}=4\pi\mathrm{e}\left[n_i-\int_{-\infty}^{+\infty} f_e(\mathrm{x,v,t})\,\mathrm{dv}\right]$$

Phase space and time discretization

$$x_i = (i-1)\Delta x$$

$$i = 1, N_x$$

$$\Delta x = \frac{L_x}{N_x}$$



$$v_k = k\Delta v$$

$$k = -N_v, N_v$$

$$\Delta v = \frac{V_{max}}{N_v}$$

Time discretization:

$$t_n = n\Delta t$$

$$n = 0, n_{step}$$

$$\Delta t \leq \min\left\{\frac{\Delta x}{v_{max}}, \frac{\Delta v}{a_{max}}\right\}$$

CFL stability condition

Splitting method for the Vlasov equation

Distribution function in physical space

$$\frac{\partial f_x}{\partial t} + v\frac{\partial f_x}{\partial x} = 0$$

Advection in physical space (describes translation in physical space)

$$\frac{\partial f_v}{\partial t} + a\frac{\partial f_v}{\partial v} = 0$$

Advection in velocity space (describes translation in velocity space)

Distribution function in velocity space

The time evolution of the distribution function is divided into two steps:

1) EVOLUTION in physical space
2) EVOLUTION in velocity space

The time splitting algorithm

$$\frac{\partial f_x}{\partial t} + v\frac{\partial f_x}{\partial x} = 0 \Rightarrow f_x(t+\Delta t) = \Lambda_x(\Delta t)\, f_x(t)$$

$$\frac{\partial f_v}{\partial t} + a\frac{\partial f_v}{\partial v} = 0 \Rightarrow f_v(t+\Delta t) = \Lambda_v(\Delta t)\, f_v(t)$$

Translation operators

Finite differences for the translation operators

$$f(x_{i+1}) = f(x_i) + \Delta x \left(\frac{df}{dx}\right)_{x_i} + \frac{1}{2}\Delta x^2 \left(\frac{d^2 f}{dx^2}\right)_{x_i} + o(\Delta x^3) \qquad (1)$$

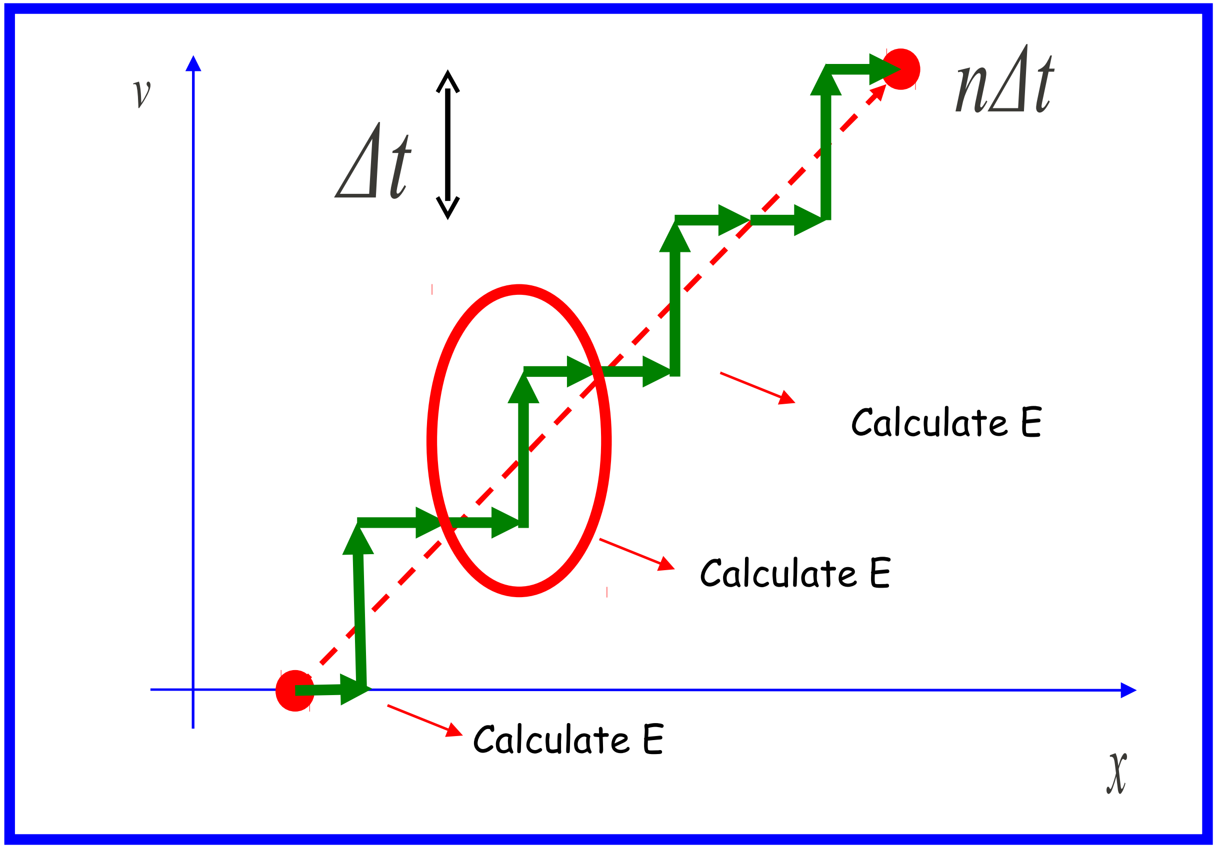
$$f(x_{i-1}) = f(x_i) - \Delta x \left(\frac{df}{dx}\right)_{x_i} + \frac{1}{2}\Delta x^2 \left(\frac{d^2 f}{dx^2}\right)_{x_i} + o(\Delta x^3) \qquad (2)$$

$$(1) - (2) \Rightarrow \left(\frac{df}{dx}\right)_{x_i} = \frac{f(x_{i+1}) - f(x_{i-1})}{2\Delta x} + o(\Delta x^2)$$

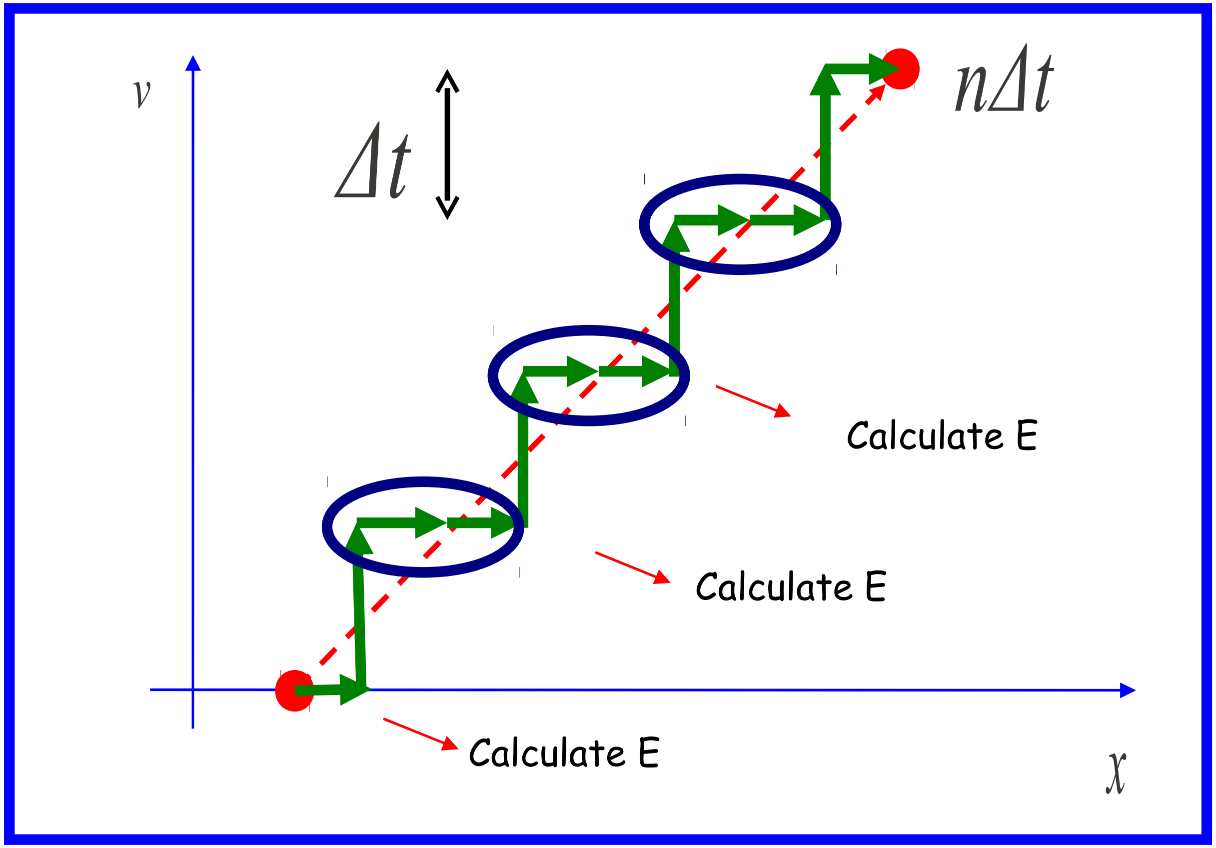
Centred difference

The time splitting algorithm

$$f(n\Delta t) = \{\Lambda_x(\Delta t/2)\Lambda_v(\Delta t)\Lambda_x(\Delta t/2)\}^n f_0 + o(\Delta t^3)$$



The time splitting algorithm



$$f(n\Delta t) = \{\Lambda_x(\Delta t/2)\Lambda_v(\Delta t)[\Lambda_x(\Delta t)\Lambda_v(\Delta t)]^{n-1}\Lambda_x(\Delta t/2)\}f_0$$

Chen and Knorr, JCP (1977)
Mangeney et al. JCP (2002)

Vlasov simulations of Landau damping

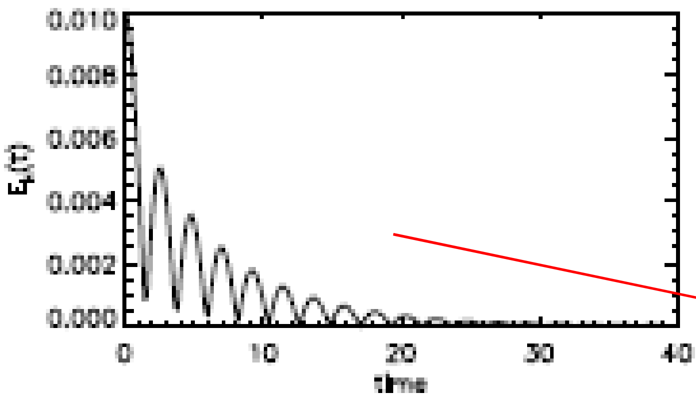
Initial electric perturbation

$$E(x,t=0)=E_0 \sin(kx)$$

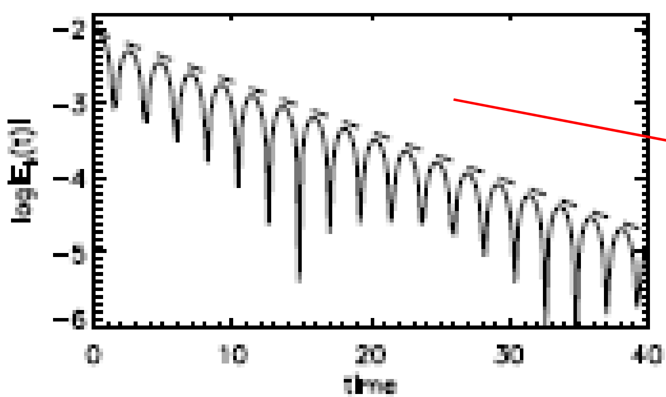
$$E_0=0.02; k=\frac{2\pi}{L_x}=0.5$$

Periodic boundary conditions in the physical domain

Electric field spectral component

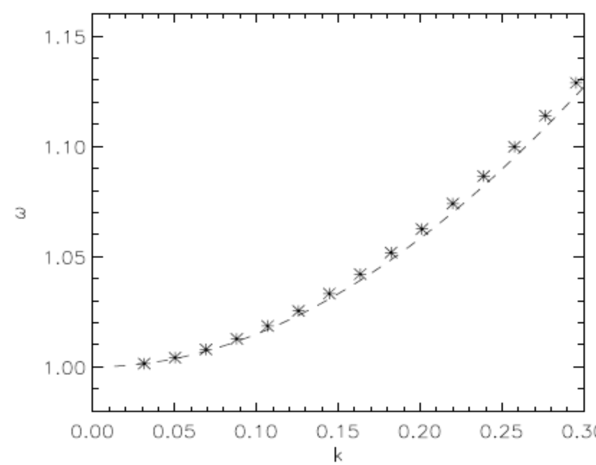


Exponential damping in time



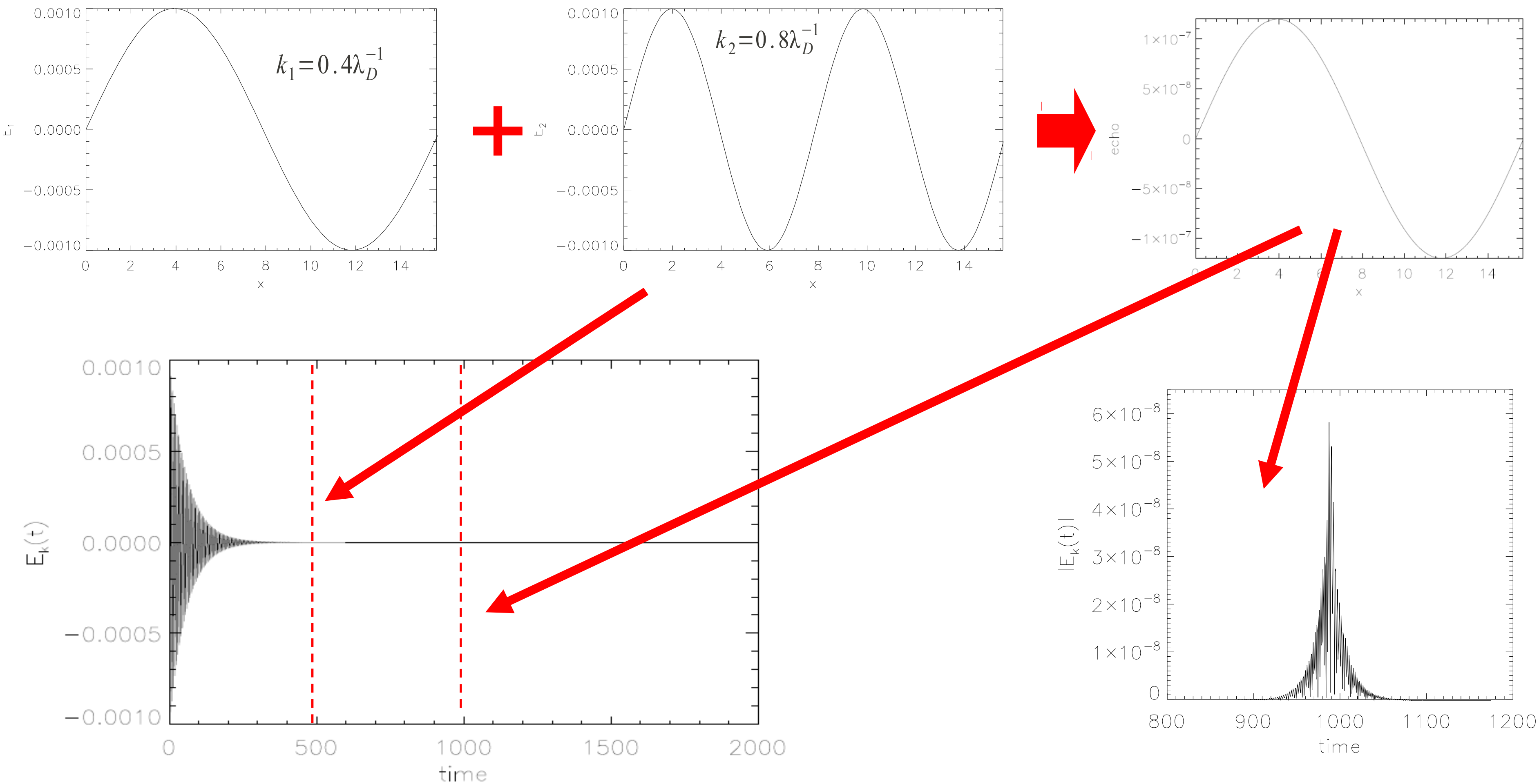
Comparison with linear theory of Landau damping

$$\omega_R^2=\omega_p^2(1+3k^2\lambda_D^2)$$



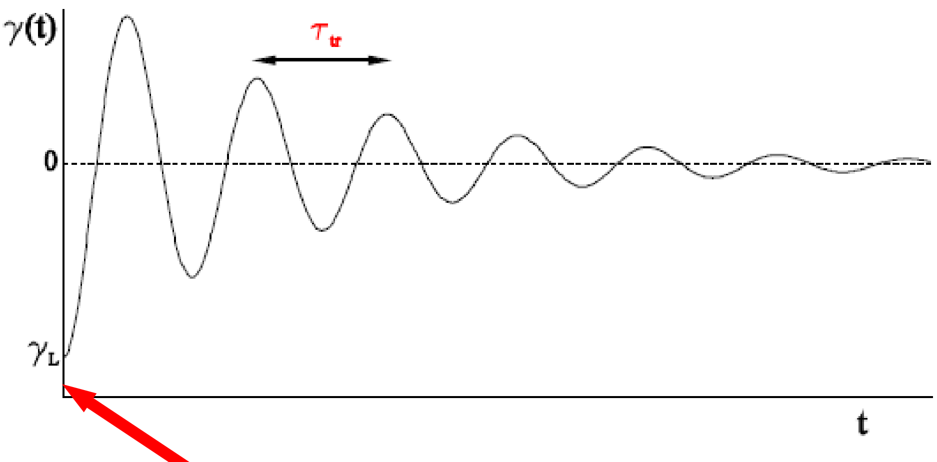
Plasma wave echoes (Vlasov simulation)

$$k_{echo} = k_2 - k_1 = 0.4\lambda_D^{-1}$$



Nonlinear saturation of Landau damping

Damping rate time behavior



Landau linear prediction

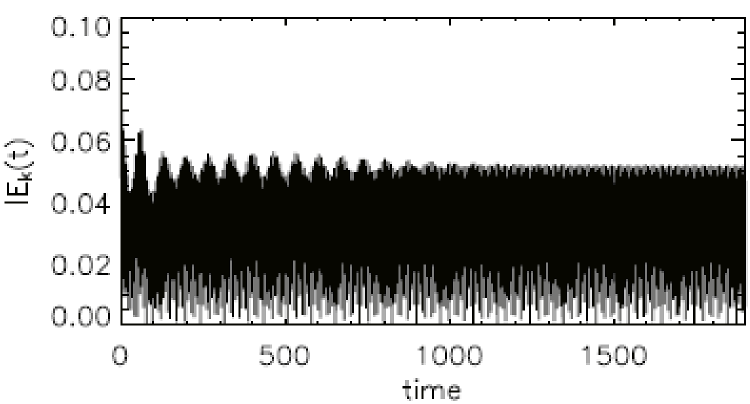
O'Neil found that, due to nonlinear effects, the Landau damping rate of the wave starts oscillating around zero on the trapping time scale with decreasing amplitude.

SATURATION OF DAMPING

Vlasov simulations

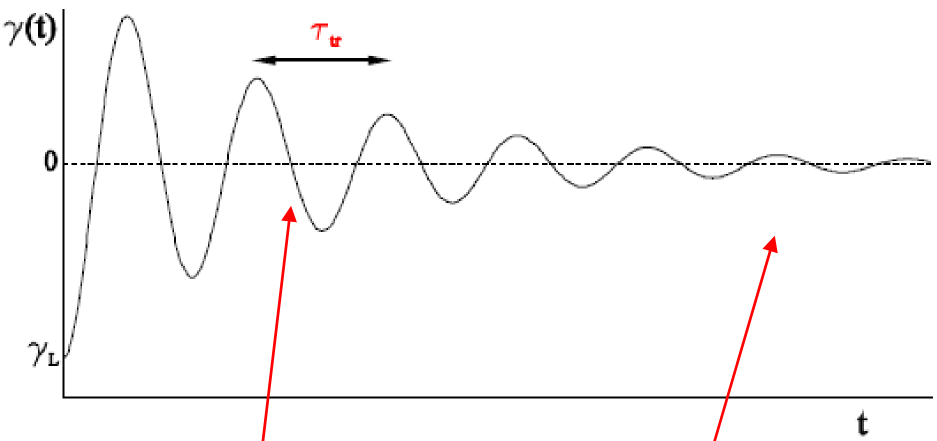
$$E(x,t=0)=E_0\sin(kx)$$

$$E_0=0.2; k=\frac{2\pi}{L_x}=0.5$$



Nonlinear saturation of Landau damping

Damping rate time behavior



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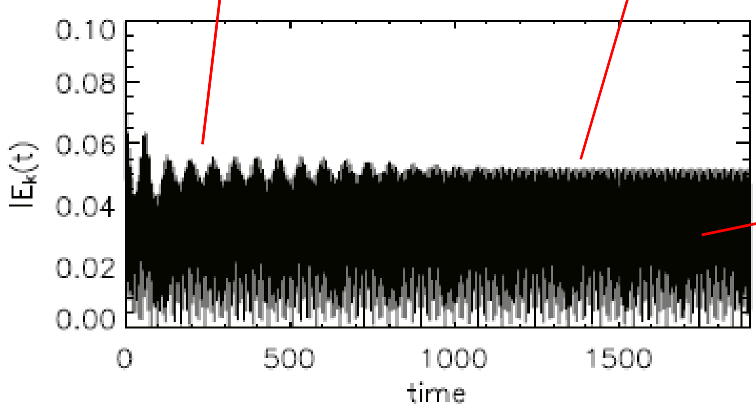
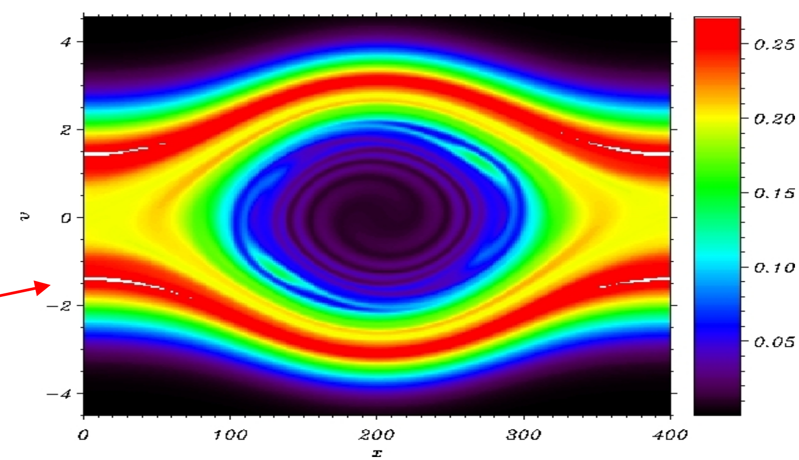
Vlasov simulations



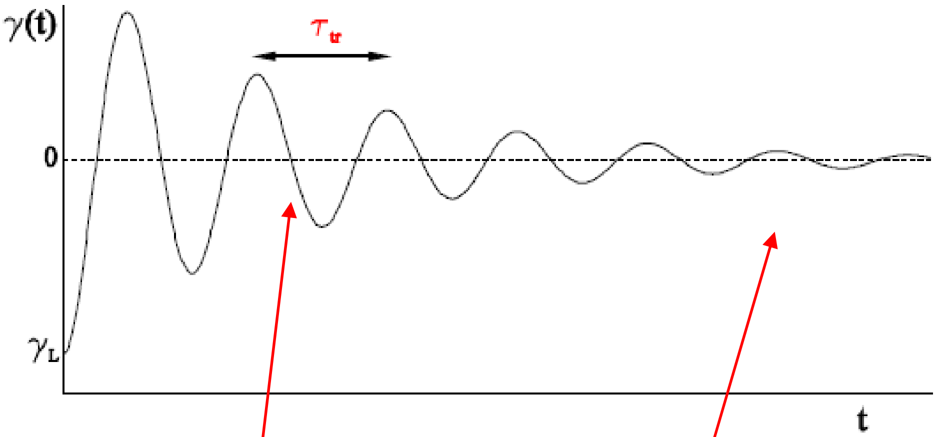
Phase space distribution function

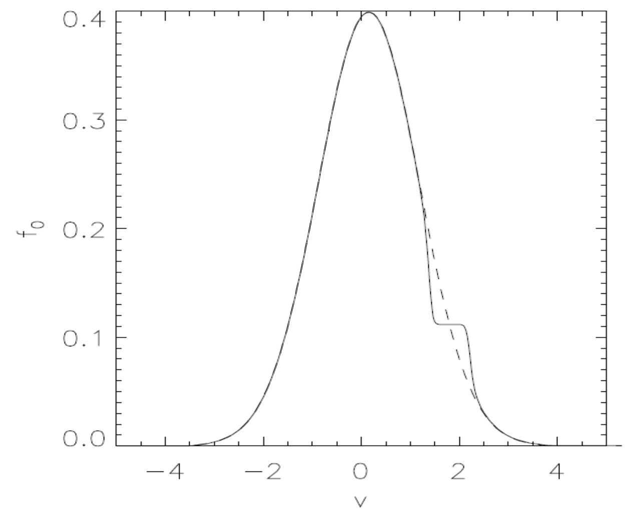


Signature of trapping

Nonlinear saturation of Landau damping

Damping rate time behavior





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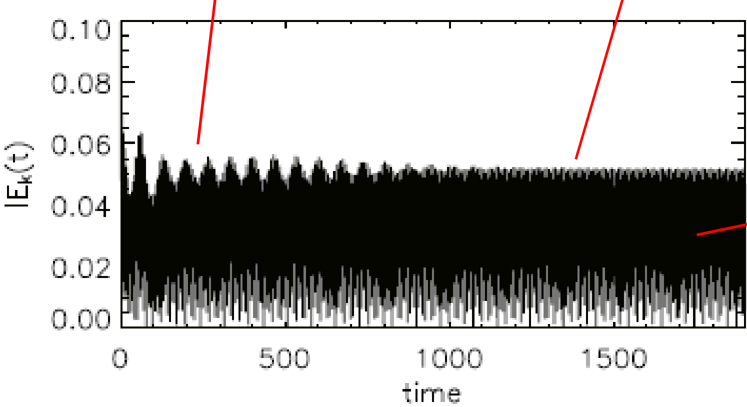
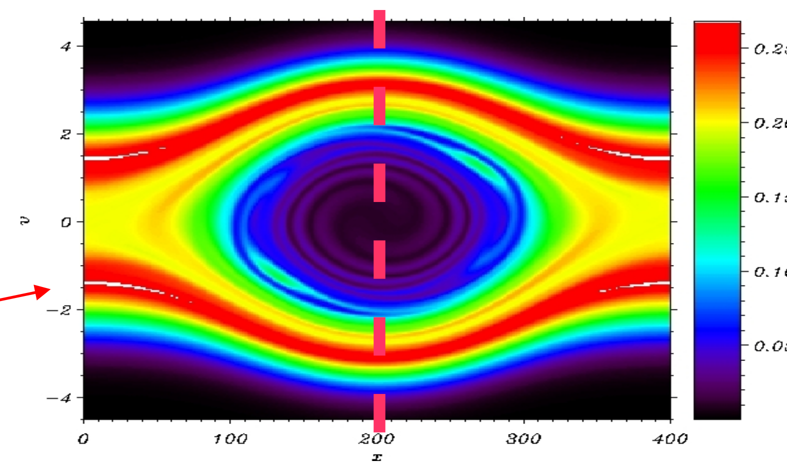
Vlasov simulations

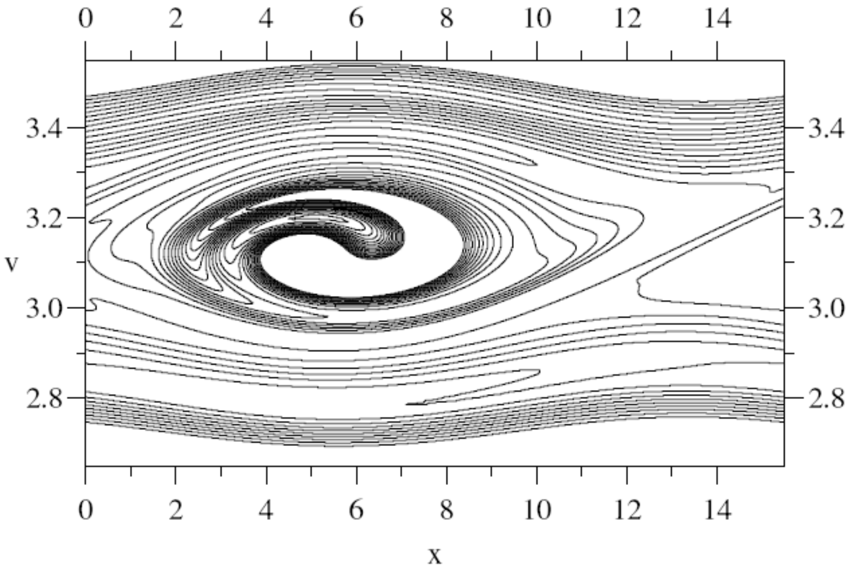


Phase space distribution function



Signature of trapping

Be careful with numerical Vlasov simulations

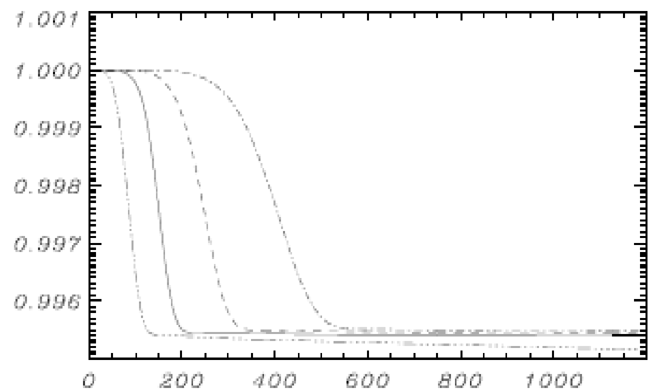


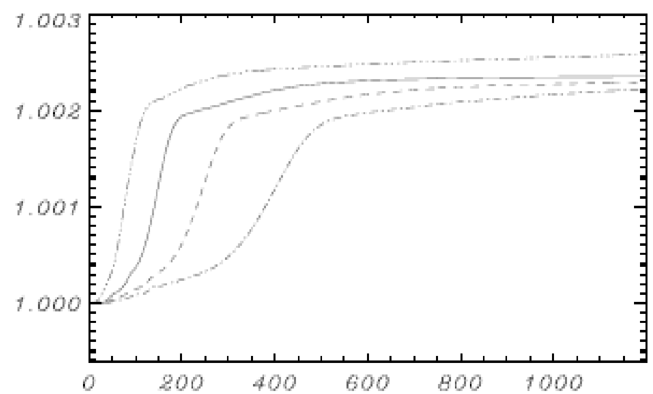
Trapping region

Galeotti and Califano, PRL (2005)

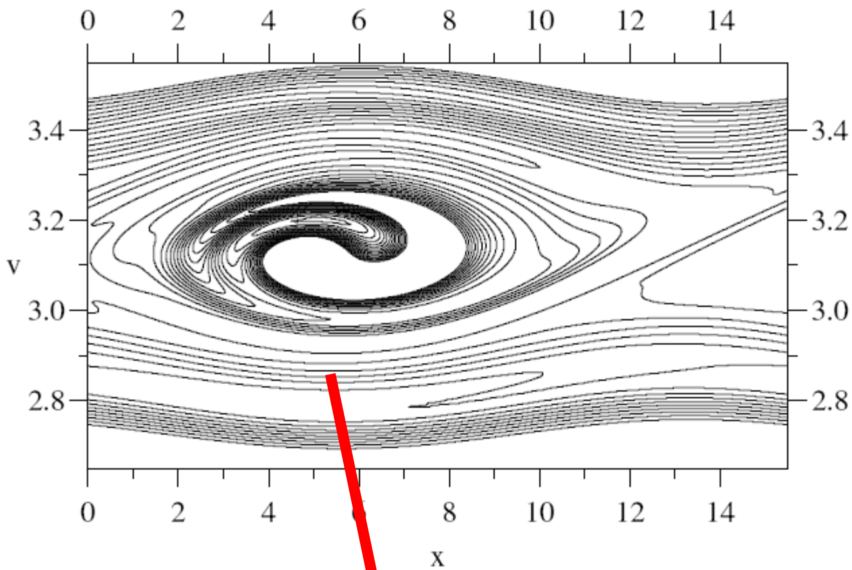
$$N_3(t) = \int f^3(x, v, t)\, dx dv$$

$$S(t) = -\int f \ln(f)\, dx dv$$





Filamentation problem



Small-scale structures are generated in the phase space distribution function due to trapping

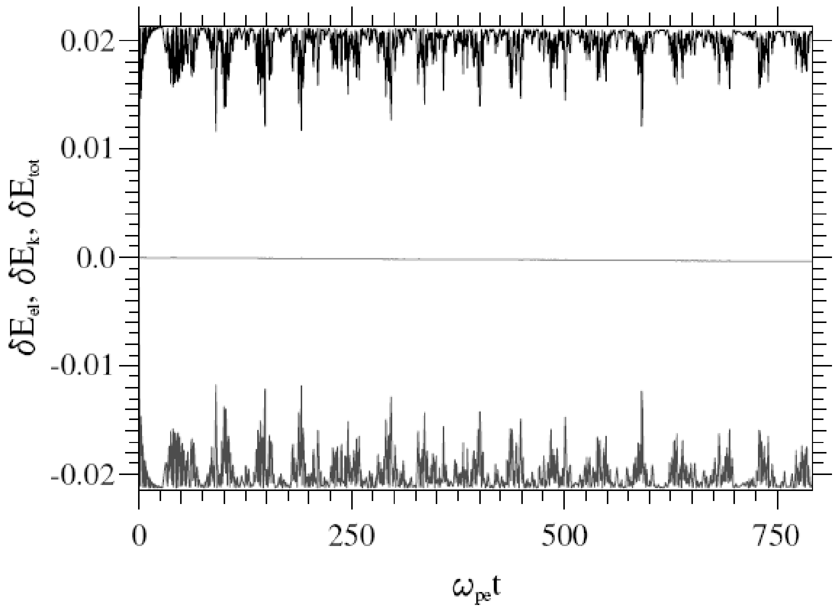
$$L_v \approx \Delta v$$

When the typical size of the velocity structures gets smaller than the numerical resolution…

The numerical algorithm is not able to resolve them anymore…The topology of phase space is changed

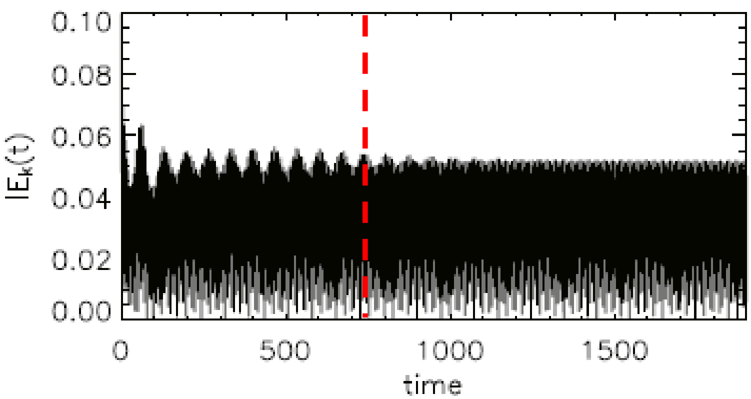
NUMERICAL VISCOSITY

Energy conservation



$$\frac{\partial}{\partial t}\int\left(\frac{1}{2}n\mathbf{V}^2+\frac{E^2}{2}\right)dx=0$$

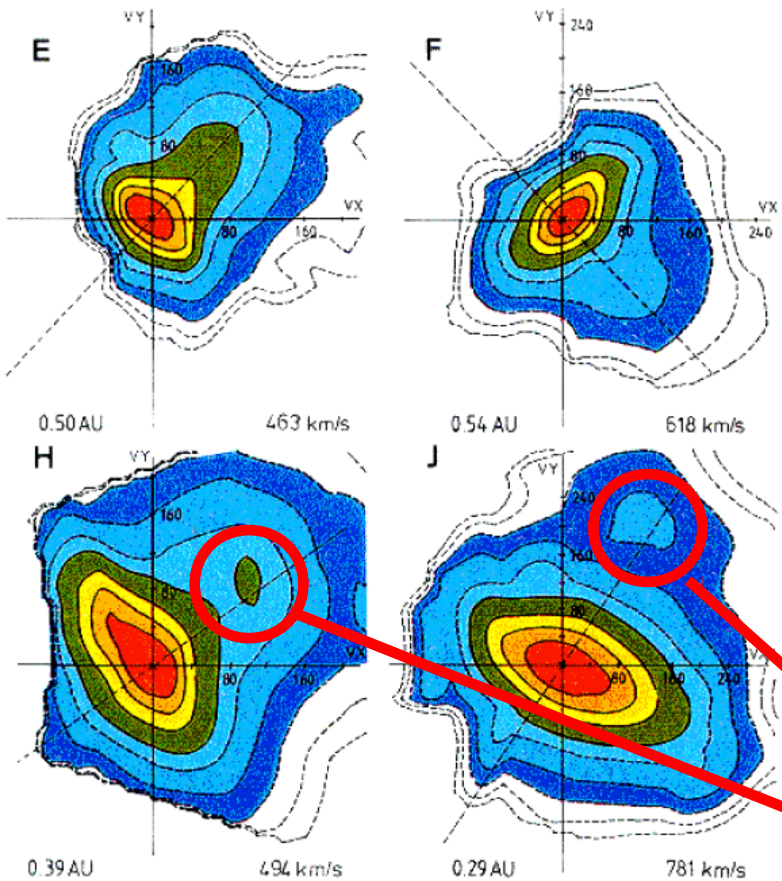
$$\int\left(\frac{1}{2}n\mathbf{V}^2+\frac{E^2}{2}\right)dx=cost$$

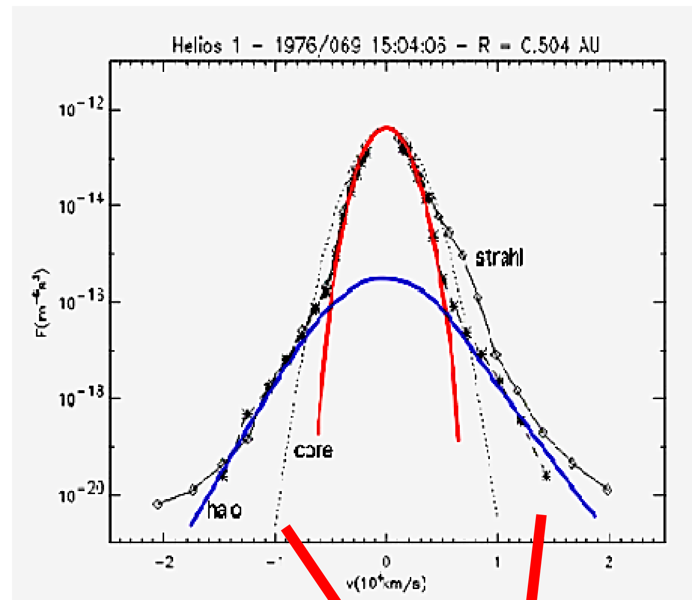


Valentini et al. PRE (2005)
Valentini et al. JCP (2005)

Velocity distributions in space plasmas

The velocity distributions in space plasmas are source of free energy for many instabilities and kinetic processes

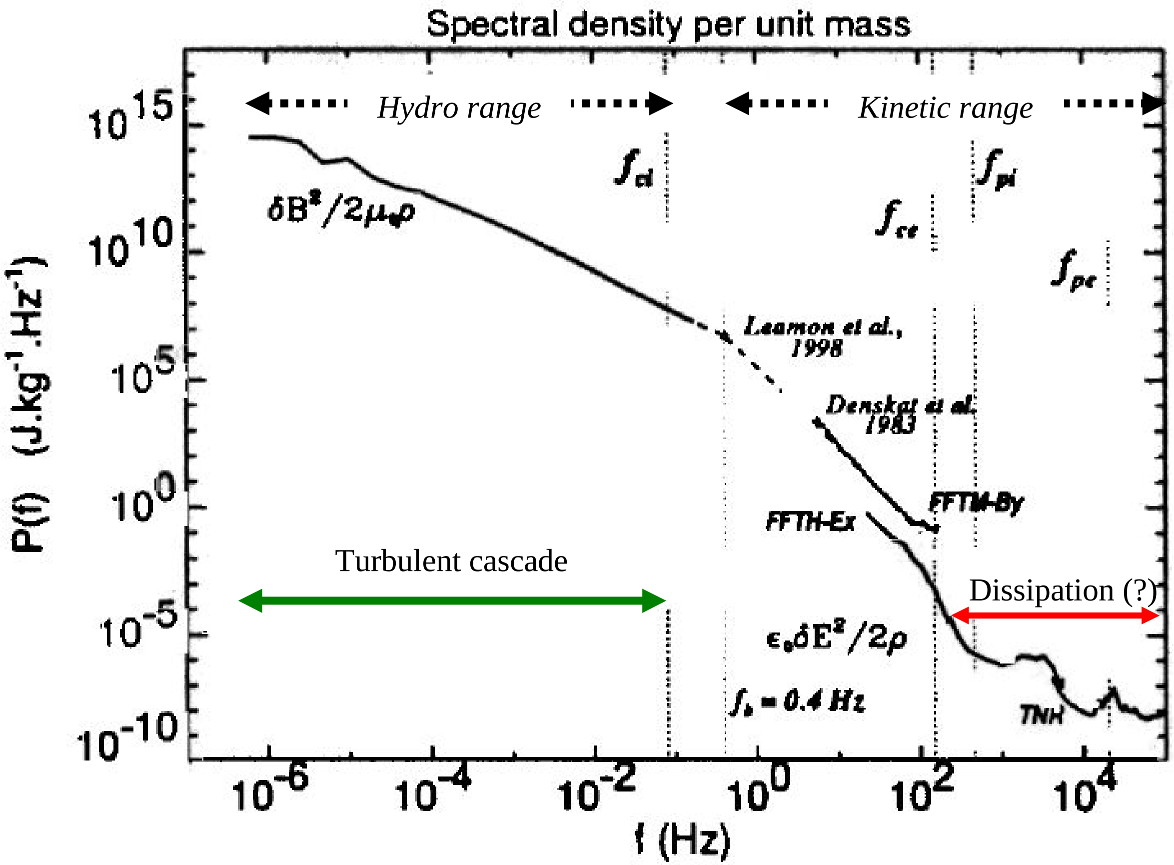




Generation of accelerated beams

Generation of suprathermal tails

Turbulence and kinetic effects in space plasmas



Energy cascades towards small scales

the understanding of the physical processes that govern the system dynamics at frequency of the order and beyond the ion-cyclotron frequency is of key relevance in space physics

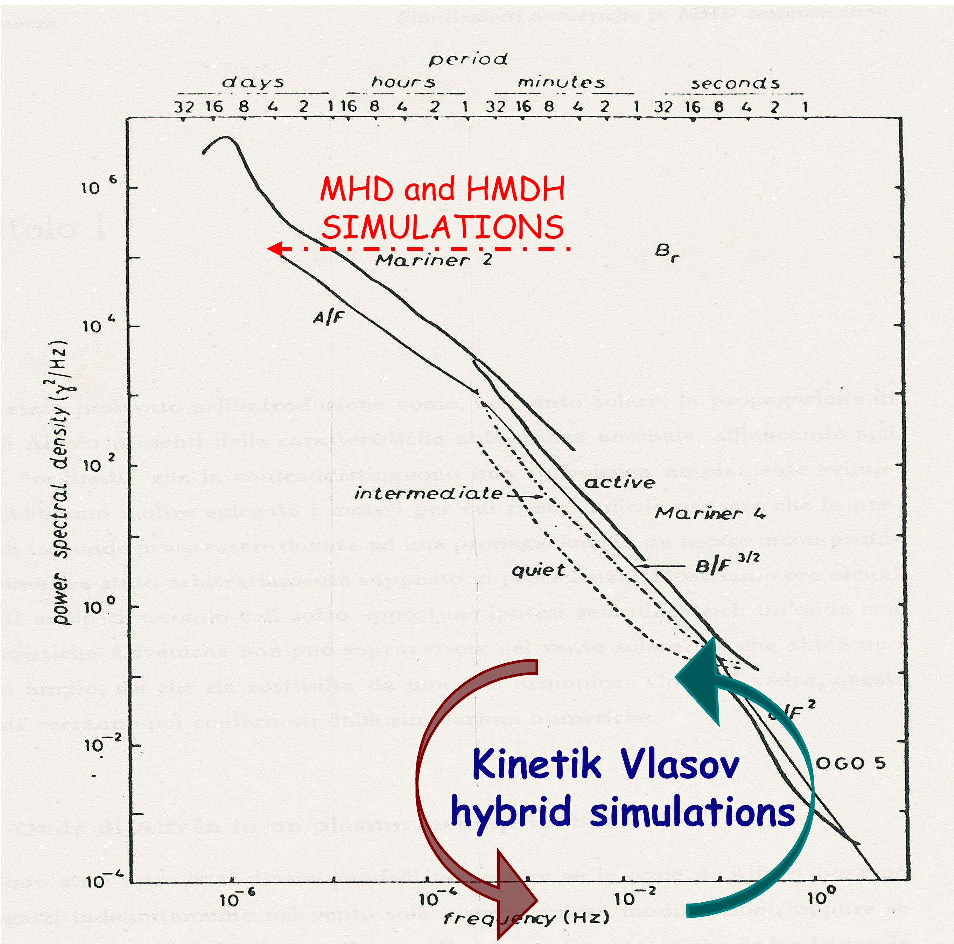
$\omega << \Omega_i$ FLUID REGIME

$\omega \geq \Omega_i$ KINETIC REGIME

Ω_i Link beween low frequency (fluid) dynamics and high frequency (kinetic) dynamics

Frontier problem in plasma physics

Investigation of cross-scale interactions: from fluid to kinetic scales
"A challenge in computational plasma physics"



OBSERVATION DATA: Magnetic & electric field, density, velocity, PDF, …

Kinetic effects in the energy transfer across ion cyclotron frequency

Today (Giga-teraflop): Vlasov-hybrid

Future (tera-petaflop): full Vlasov!

Hybrid Vlasov-Maxwell equations (dimensionless units)

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \left[\mathbf{E} + \mathbf{v} \times \mathbf{B}\right] \cdot \nabla_v f = 0$$

Vlasov equation for ions

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \frac{1}{n}\mathbf{j} \times \mathbf{B} - \frac{1}{n}\nabla P_e + d_e^2[\nabla^2 \mathbf{E} + \cdots]$$

Ohm's law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Faraday equation

$$\nabla \times \mathbf{B} = \mathbf{j}$$

Current Density (low frequency approximation)

$$n_e \simeq n_i \simeq n$$

Quasi-neutrality

$$P_e = n^\gamma$$

Equation of state

$$V_A = \frac{B}{(4\pi\rho)^{1/2}}$$

$$\Omega_i = \frac{eB}{m_i c}$$

$$d_i = \frac{V_A}{\Omega_i}; \quad d_e = \frac{m_e}{m_i}$$

Valentini et al, JCP 2007

Numerical algorithm

Splitting scheme (Chen and Knorr, JCP, 1977), for the time advance of the Vlasov equation; third order **Van Leer scheme** for spatial and velocity derivatives

For details, see Valentini et al. JPC, 2007; and Mangeney et al. JCP, 2002

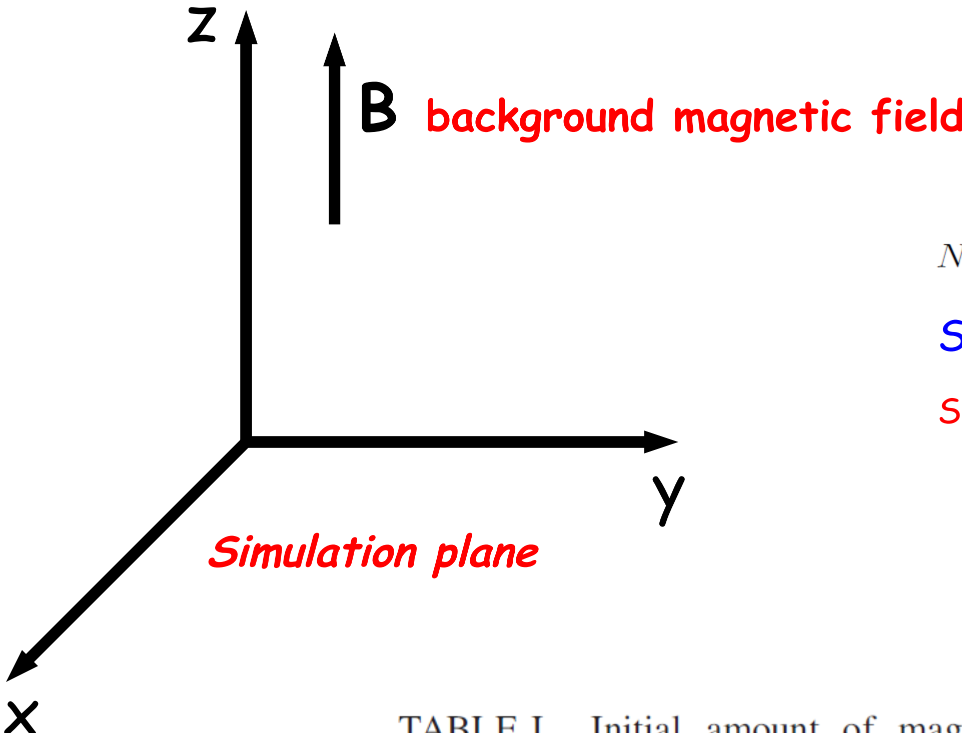
Hybrid Current Advance Method for the evolution of the electromagnetic fields

Parallelization strategy

The code is parallelized in physical space using the MPI directives.

For details, see Mangeney et al. JPC, 2002

Hybrid Vlasov-Maxwell simulations of 2D plasma turbulence



$$\beta = 2v_{ti}^2/V_A^2 = 2$$
$$T_e/T_i = 1$$

$$N_x = 512; N_y = 512; N_{V_x} = 51; N_{V_y} = 51; N_{V_z} = 51$$

Simulation of decay turbulence

Servidio, Valentini, Califano and Veltri, PRL (2012)

TABLE I. Initial amount of magnetic fluctuations (second column), system size (third column), and time of the peak of the turbulent activity (last column).

	$\delta b/B_0$	L_0/d_i	$\tau^\star\ (\Omega_{ci}^{-1})$
Run I	$1/7$	$2\pi \times 30$	150
Run II	$1/3$	$2\pi \times 20$	50
Run III	$1/3$	$2\pi \times 10$	20

Hybrid Vlasov-Maxwell simulations of 2D plasma turbulence

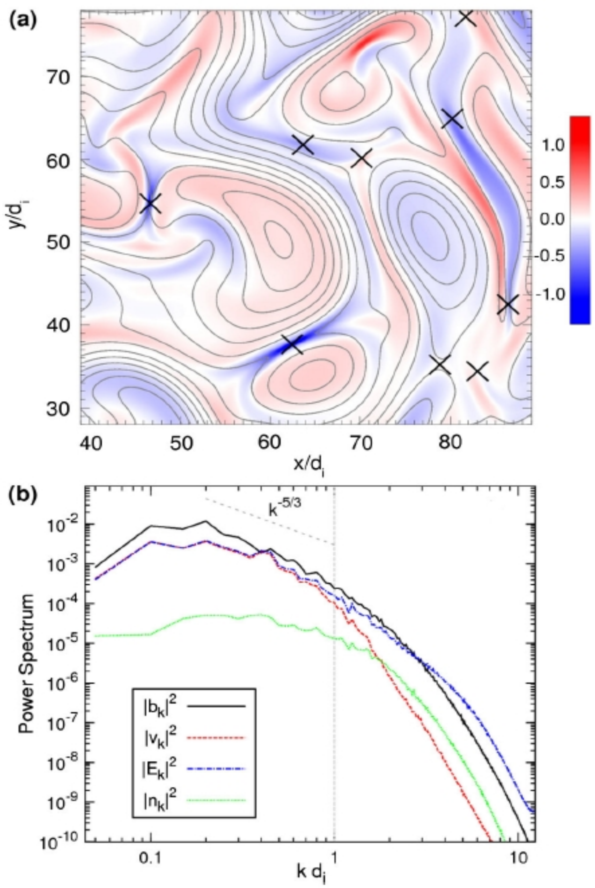


FIG. 1 (color online). (a) Shaded contours (zoom) of j_z together with a_z (isolines) and its X points (black crosses). (b) Power spectra of ion density (green dotted), ion bulk velocity (red dashed), magnetic field (black solid), and electric field (dot-dashed blue). The Kolmogorov expectation $k^{-5/3}$ (gray dashed) is reported as a reference, while the vertical dashed line represents the ion skin depth wave number.

The contour plot of the out of plane current density shows the presence of magnetic reconnection events (black crosses)

Local temperature anisotropy $A = \frac{T_\perp}{T_\parallel}$

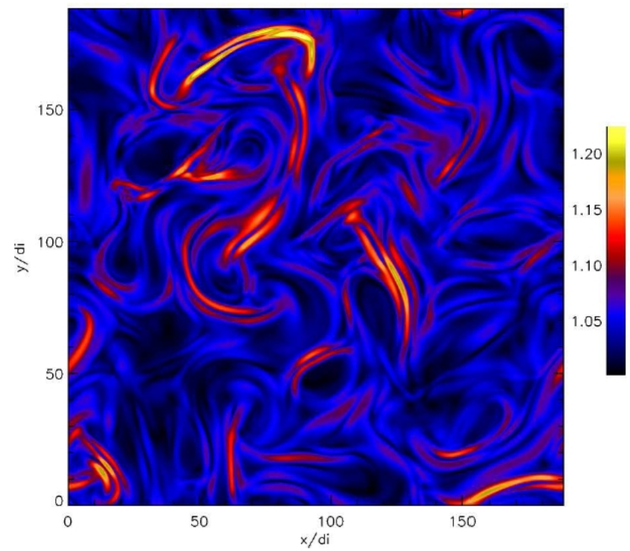


FIG. 8: Temperature anisotropy, defined as λ_1/λ_3 [see Eq. (7) and text below], in turbulence.

Hybrid Vlasov-Maxwell simulations of 2D plasma turbulence

Statistical analysis of the local temperature anisotropy

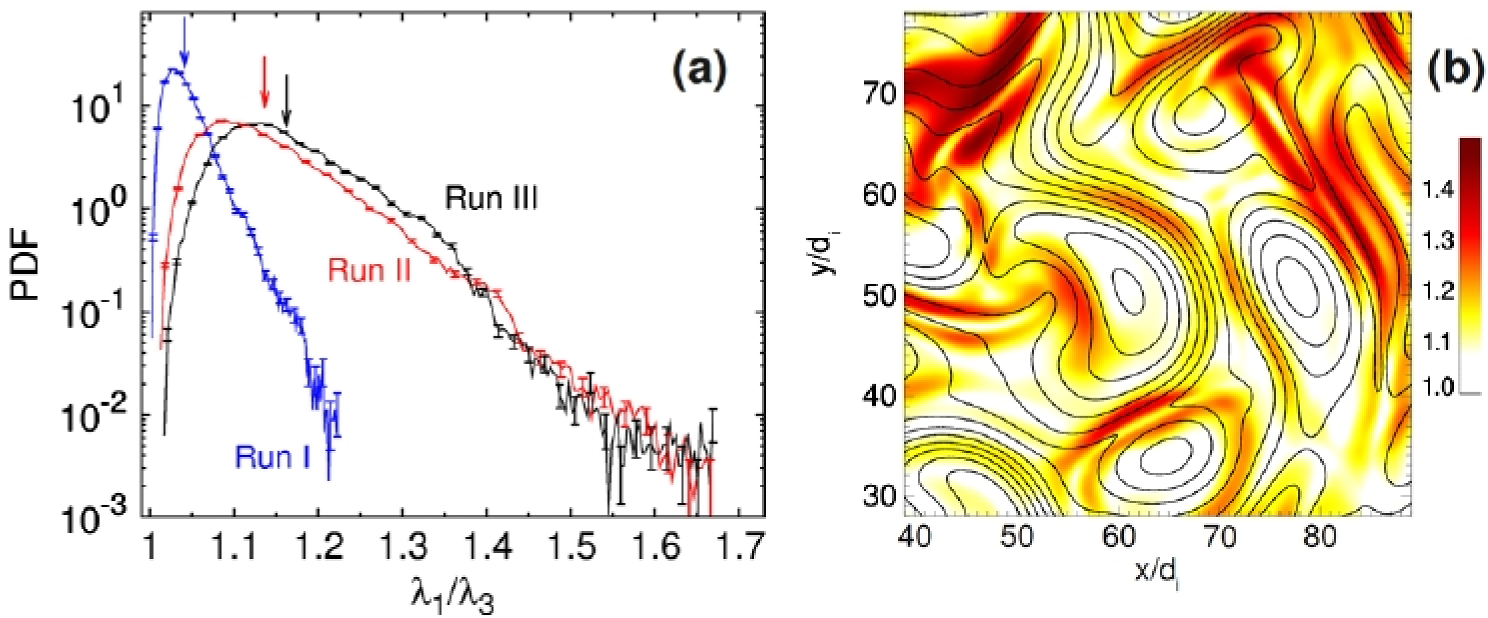


FIG. 2 (color online). (a) PDF of the temperature anisotropy λ_1/λ_3 for all runs (arrows represent averages); (b) shaded-contour (zoom) of the anisotropy together with the inplane magnetic field lines (black):

Hybrid Vlasov-Maxwell simulations of 2D plasma turbulence

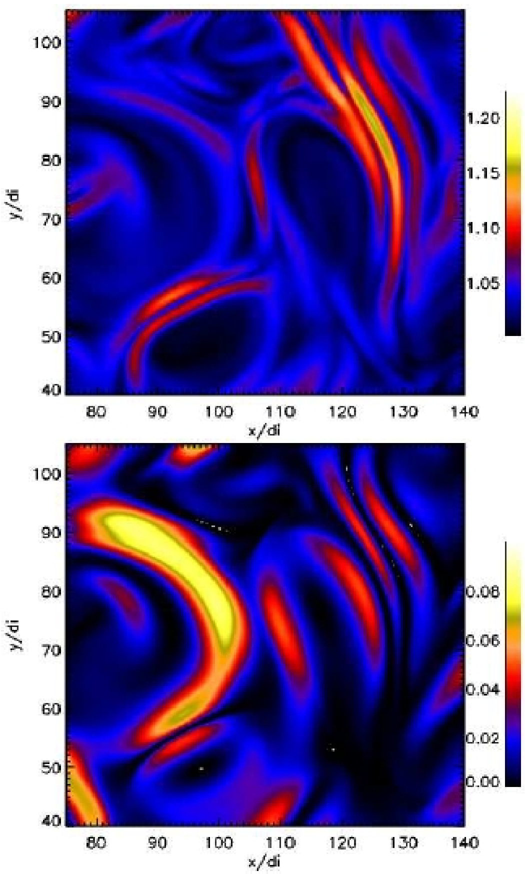


FIG. 10: Zoom of Fig. 8 (top) together with $|\delta \boldsymbol{b}|^2$ (bottom). A correlation between the anisotropy and the magnetic field strength is evident.

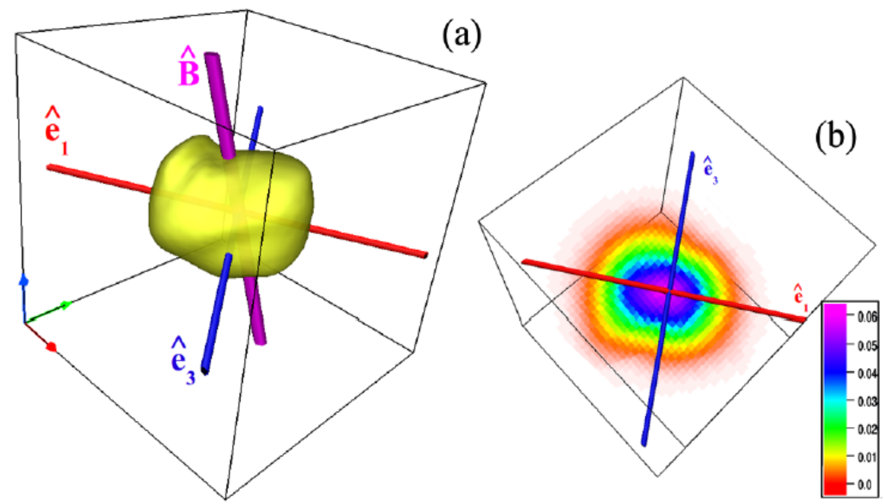


FIG. 3 (color online). (a) Isosurfaces of the velocity distribution function $f(\boldsymbol{x}^\star, \boldsymbol{v})$, at a given spatial position $\boldsymbol{x}^* \simeq (60, 119)d_i$. (b) Two-dimensional cut of f in the minimum variance frame. Thin (red) and thicker (blue) axis indicate $\hat{e}_1$ and $\hat{e}_3$, respectively. The magnetic field direction $\hat{\boldsymbol{B}}$ is represented with a thick (magenta) tube.

First attempt of 3D-3V Vlasov turbulence

The first example of Valov turbulence has been realized with relatively low resolution both in physical and velocity space

$$N_x = 32; N_y = 32; N_z = 32; N_{V_y} = 31; N_{V_y} = 31; N_{V_z} = 31$$

This was a preparatory run performed on SP6 Machine at CINECA

On the new FERMI machine at CINECA, within a PRACE research project, we plan to run Hybrid Vlasov-Maxwell simulations wih a phase space resolution:

$$N_x = 128; N_y = 128; N_z = 128; N_{V_y} = 51; N_{V_y} = 51; N_{V_z} = 51$$

OR LARGER?!? ASK CAVAZZONI!!!

$$f \simeq 2500Gb = 2.5Tb \text{ !!!}$$

Fig. 8

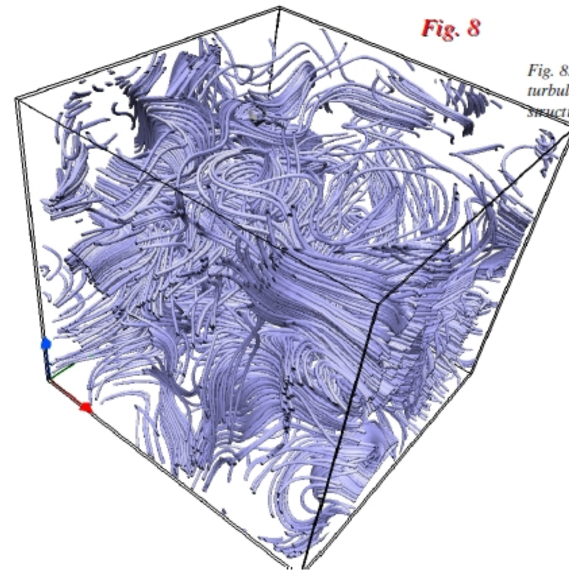


Fig. 8. Velocity field in Vlasov turbulence. Many vortical structures can be recognized...

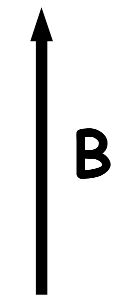


Fig. 9

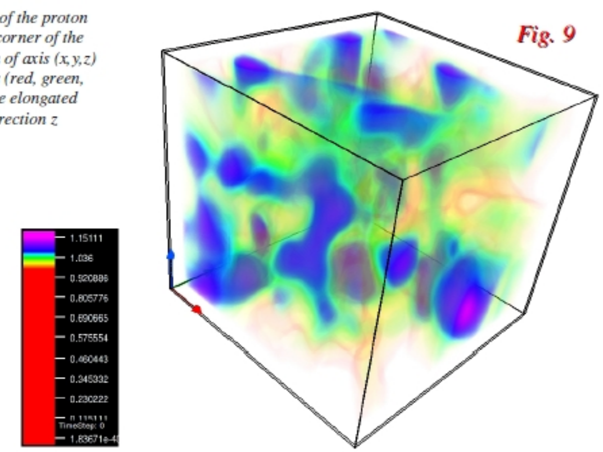


Fig. 9. 3D shaded contour of the proton density. In the bottom-left corner of the cube is reported the system of axis (x,y,z) with their respective colors (red, green, blue). The plasma blobs are elongated along the magnetic field direction z (blue).

First attempt of 3D-3V Vlasov turbulence

*Fig. 10. Total magnetic field **B** (red spaghetti), from different perspectives, together with the high-intensity isosurfaces of j^2 (blue bubbles).*

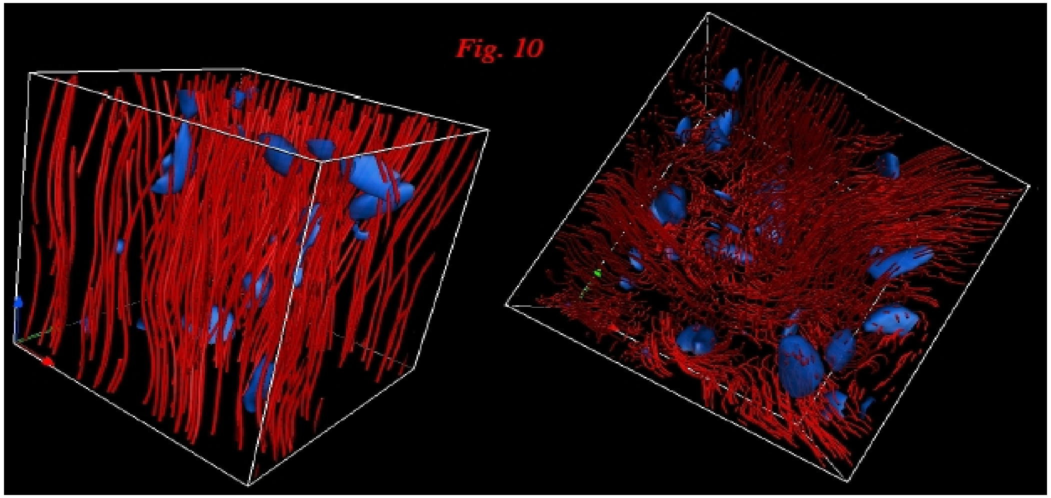


Fig. 11

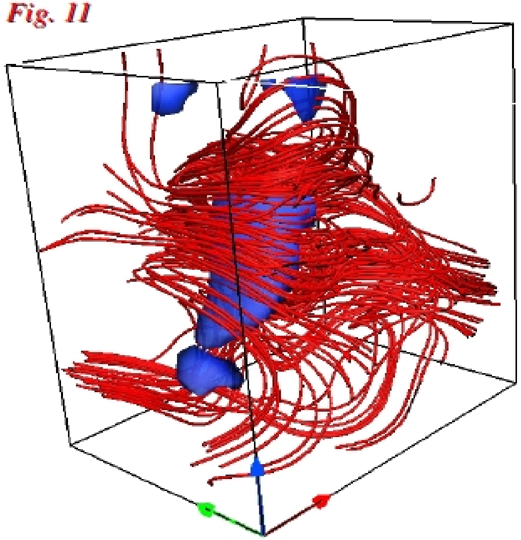


*Fig. 11. Zoom of the magnetic field fluctuations **b** (without mean field), together with j^2 (blue isosurfaces). A magnetic structure (3D vortex, or flux-tube) is identified.*

That's all !!!

Francesco Valentini
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Università della Calabria