Semi-Lagrangian simulations on polar grids: from diocotron instability to ITG turbulence test cases

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While developing a new semi-Lagrangian solver, the gap between a linear Landau run in $1D \times 1D$ and a $5D$ gyrokinetic simulation in toroidal geometry is quite huge. Intermediate test cases are welcomed for checking the code. We consider here as building block, a $2D$ guiding center type equation on an annulus:

$$\partial_t f - \frac{\partial_\theta \Phi}{r}\partial_r f + \frac{\partial_r \Phi}{r}\partial_\theta f = 0,\ t \in [0,T],\ r \in [r_{\min}, r_{\max}],\ \theta \in [0, 2\pi],$$

where the potential Φ solves a Poisson type equation. Numerical difficulties are added w.r.t. basic Vlasov Poisson simulations, in the sense that the advection is no more constant and we have no more a uniform periodic grid.
We first consider the diocotron instability test case [3]. Conservation of electric energy and mass are obtained for some adapted boundary conditions on the continuous model and checked on the semi-Lagrangian code, which is developed in the environment of the on-going library SELALIB [4]. The linear phase is also validated against analytical growth rates given by the dispersion relation. We then extend the code to the $4D$ slab drift kinetic Vlasov equation [2]. There, analytical growth rates are no more available, but we can check the linear phase with recent results [1].
For the simulations, a classical method is used with cubic splines, as in [2], and extension to a $2D$ conservative method and other interpolation choices, already working for cartesian guiding center test cases are discussed.

References

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