A 4D semi-Lagrangian Vlasov solver based on an arbitrary curvilinear grid in physical space

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The large magnetic field in a Tokamak generates a huge anisotropy in the physics along and across magnetic field lines. For this reason aligning the grid on the magnetic surfaces and possibly on the magnetic field lines can considerably increase the accuracy for a given resolution. For Tokamaks with a circular poloidal cross section, magnetic surfaces are circular and thus standard toroidal coordinates are naturally aligned on magnetic surfaces. This is not the case for more general equilibria with an X-point, where a numerical definition of the mesh is required. For this reason, we need a Vlasov solver that can handle such a mesh.

The method of CAO-DAO can help us describe the geometry of magnetic surfaces including the X-point. It will give us a mesh of the poloidal plane as a spline surface. This is defined as a small collection of patches where a logical grid is defined and mapped to the actual computational domain. Each patch defines a curvilinear grid. We solve all equations in the patch using the coordinate transformation defined by the mapping. So we must develop a numerical method for the Vlasov equation in four dimensional phase-space that can handle this change of coordinates. Note that the velocity grid remains cartesian.

The semi-Lagrangian method consists in two steps: 1) Following the characteristics, which are particle trajectories, originating from grid points, 2) Interpolating back on the grid. The second step needs to be performed on the patch. For the first part, we have three options. The first is to move the particles in the physical domain. Then we need to know the inverse of the change of coordinates in order to transform back on the patch for interpolation. That could become complicated and costly because we don't know in general an analytical form for the inverse of the change of coordinates. The second is to move directly all the particles in the patch. In this case, we need to solve the equations of motion in the patch coordinates, where they become very involved and more costly to solve. The third option proposed in [1], and that we have chosen, is to move the positions of the particles in the patch and the velocities of the particles in the physical domain. With this strategy, we do not need to know the inverse of the change of coordinates and the equations of motion are simpler.

The Vlasov equation (or gyrokinetic equation in the Tokamak) needs to be coupled with a field solver, in our case the quasi-neutral equation, that will also be defined on our patch. For this we use a Finite Element method using B-spline basis functions. The particularity of our method is its flexibility with respect to the mesh. It can handle as well analytical curvilinear coordinates (as polar coordinates) or numerical defined curvilinear coordinate systems. We will present first results obtained with this new code and compare them with our classical semi-Lagrangian code.

References

[1] T. Bagdonat and U. Motschmann, 3D Hybrid Simulation Code Using Curvilinear Coordinates, *Journal of Computational Physics* **183**, 470485 (2002)