Effects of local field fluctuations on Vlasov dynamics

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We intend to merge two different theories concerning N-body systems interacting via Coulomb electrostatic or Newton gravitational forces. The first theory is the kinetic description of such systems based on the Vlasov equation. The second one is the Chandrasekhar-Holtsmark static description of total field fluctuations.
The latter is a static approach of the statistical fluctuations of the total field acting on a given particle interacting with N-1 other identical particles. The interaction corresponds either to the Coulomb or the gravitational potential. Assuming that the position variables of the particles are uniformly distributed and uncorrelated, Chandrasekhar and Holtsmark showed [1, 2] that these fluctuations follow a Levy distribution of index $3/2$, a particular distribution in which the nearest neighbor plays a predominant role. One of its particular features is the divergence of its second and higher order moments due to its algebraic long tail. This divergence is essentially caused by the divergence at short distances of the force in $1/r^2$.
It is well-known that the action of the mean-field on the time evolution of the one-particle phase-space distribution results in the Vlasov kinetic equation. However, the fluctuations of the total field around the mean-field should also have consequences on that evolution. This is the main object of our work. It has been established, moreover, that Levy processes are connected to anomalous diffusion and fractional kinetics [3]. Thus, we develop a new mathematical framework, inspired by the approach developed by Chandrasekhar and Holtsmark, to include the effect on the dynamics of these fluctuations around mean-field. In this framework, by introducing a particular coarse graining of the one-particle phase-space distribution function, the interaction felt by any given particle of the system due to the others can be decomposed into two parts that can be treated separately. One part, within a given set of assumptions, can be treated perturbatively and leads back to the Vlasov term. The other, which corresponds to local field fluctuations, leads to a fractional Laplacian, whose order, is a direct consequence of the $1/r^2$ behavior of the force at short distances.

References

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