Effect of viscosity on propagation of MHD waves in astrophysical plasma

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We determine the general dispersion relation for the propagation of magnetohydrodynamic (MHD) waves in an astrophysical plasma by considering the effect of viscosity with an anisotropic pressure tensor. Basic MHD equations have been derived and linearized by the method of perturbation to develop the general form of the dispersion relation equation. Our result indicates that an astrophysical
plasma with an anisotropic pressure tensor is stable in the presence of viscosity and a strong magnetic field at considerable wavelength.

In the MHD approximation, the behavior of a continuous astrophysical plasma is governed by a simplified form of Maxwell's equations, together with that of hydrodynamics to which one adds the effect of the Lorentz force. Thus, the governing magnetized astrophysical plasma with anisotropic pressure are the continuity equation, the equation of motion, and Faraday's law of induction. We assume the conservation of mass and conservation of momentum as

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho v) = 0,$$

$$\rho \frac{dv}{dt} = \vec{J} X \vec{B} - \nabla.\vec{P},$$

And Faraday's law of induction

$$\frac{\partial \vec{B}}{\partial t} = \nabla X (\vec{V} X \vec{B}) \, \partial \mathbf{B}$$

Introducing the viscosity tensor and neglecting the gravitational acceleration from the equation of motion leadsto (Balbus and Hawley 1991)

$$\rho \frac{dv}{dt} = \vec{J} X \vec{B} - \nabla.\vec{P} + \nabla..\pi$$

where ρ is the mass density, **v** is the fluid velocity, **B** is the magnetic field which is aligned along the z-axis $\vec{B} = B\hat{e}z$, Π is the viscosity tensor, $\vec{P}$ is the anisotropic pressure tensor defined as $\vec{P} = P_{\perp}\hat{I} + (P_{\uparrow\downarrow})\hat{b}\hat{b}$ in which $\hat{I}$ is the unit tensor, $P\perp$ and $P_{\|}$ are the components of pressure perpendicular and parallel to the magnetic field, respectively, and $\hat{b} = \frac{\vec{B}}{B}$ is the direction of the magnetic field, $\vec{J} = \nabla X \frac{\vec{B}}{\mu}$ is the current density.

The stability of the MHD plasma wave has been studied in compressible plasmas in the presence of anisotropic viscosity. The general dispersion equation is derived, and solutions to this dispersion equation and stability criteria are obtained for the limiting cases of incompressible plasmas. We have investigated the sixth-order dispersion relation for the propagation of MHD waves in astrophysical plasma in the presence of viscosity with an anisotropic pressure tensor from the linearized form
of our basic equations. We discussed the solutions of the dispersion relation in various special cases corresponding to the standard wave mode of stability and instability zero β, low β and high β plasmas. Our dispersion relation is a sixth-order ω with different coefficients. Pandey

and Dwivedi (2007) also derived the sixth-order ω dispersion relation but the coefficients are different from our cases since they assumed viscosity with isotropic pressure. However, in our case, we assumed viscosity with an anisotropic pressure tensor with the magnetic field along the z-axis. In the presence

of strong magnetic fields and when the collision rates are sufficiently small, the pressures can be significantly different in parallel and perpendicular to the magnetic field (Axelsson 1998). Keeping this in view, if the field is too strong at the considered wavelength, then magnetic tension will overcome the magnetic torquing and the flow will stabilize.

References

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