Fully-implicit Fourier-Hermite spectral method for the Vlasov equation

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We discuss a spectral method to solve the Vlasov equation for collisionless plasmas, by means of an expansion of the distribution function into a Fourier-Hermite basis [1,2] which reduces the Vlasov equation to an infinite system for the moments of the expansion. The proof-of-principle results are obtained for the 1D-1V Vlasov-Poisson equation, and the focus of this work is to compare the performance of this approach with a standard Particle-in-Cell (PIC) method.

Previous Hermite-based approaches [2] used an explicit time integrator scheme which did not conserve the total energy in the system (for periodic boundary conditions). With a fully-implicit time integrator, we show that the Fourier-Hermite method can instead conserve charge, momentum, and energy exactly. On the other hand, currently no PIC code is able to conserve these three quantities simultaneously. The nonlinear system for the moments of the expansion is then solved numerically with a Jacobian Free Newton-Krylov solver.

We show results for several cases routinely used as benchmarks in computational plasma physics: Langmuir wave, Landau damping, two-stream instability, ion-acoustic wave and plasma echoes. It is shown that the Fourier-Hermite method can achieve a much more accurate solution in a tiny fraction of the computational time relative to PIC, with orders of magnitude higher efficacy (a measure of the cost-effectiveness of the algorithm). However, the Fourier-Hermite method tends to be numerically unstable for strongly non-linear problems, and the effectiveness of various strategies (such as introducing a collisional operator to handle filamentation, changing the Hermite basis or using an adaptive time-stepping) to mitigate this instability is discussed.

References

[1] T. P. Armstrong, R. C. Harding, G. Knorr, D. Montgomery, 'Solution of Vlasov's equation by transform methods'. In: Alder, B., Fernbach, S., Rotenberg, M., Eds. Methods in Computational Physics. Vol. 9, pp. 29–86. New York: Academic Press (1970).

[2] J. W. Schumer and J. P. Holloway, 'Vlasov simulation using velocity-scaled Hermite representations', J. Comput. Phys. 144(2), 626 (1998).