Vlasovian features of N body models : a new look at Landau damping *via* Debye shielding

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We revisit the connection between N-body dynamics and the Vlasov equation for the Coulomb interaction, where the standard mean-field derivation [2] does not apply. Specifically, we consider [1] the one component plasma model, made up of N electrons in a cube with spatial periodicity L in three orthogonal directions. Ions are treated as a uniform neutralizing background, enabling periodic boundary conditions. The electric potential φ solves the Poisson equation, and electrons move according to the Newton equations of motion.

Using the Fourier and Laplace transforms in a way similar to that of the Vlasov-based derivation of Landau damping, a rigorous equation is derived for a linearized version of φ along with the deviation $\delta\mathbf{r}_j(t) = \mathbf{r}_j(t) - \mathbf{r}_j(0) - \mathbf{v}_j(0)t$ of the electrons from ballistic motion. This equation is of the type $\mathcal{E}\hat{\varphi} = S$, where $\mathcal{E}$ is a linear operator, acting on the infinite dimensional array $\hat{\varphi}$ whose components are all the Doppler shifted Fourier-Laplace components of the potential. Both $\mathcal{E}$ and the source term S are sums in which the N particles define a singular measure $\mu_t^{(N)}(\mathrm{d}\mathbf{r}\,\mathrm{d}\mathbf{v})$.

Following the strategy leading to the Vlasov equation for smooth interactions [2], in the large N limit, the sums in $\mathcal{E}$ are substituted with integrals over a smooth distribution function $f_t(\mathbf{r}, \mathbf{v})$. Then $\mathcal{E}$ becomes diagonal, and the new approximate potential turns out to be the sum of the shielded Coulomb potentials of the individual particles. This derivation of Debye shielding does not emphasize pair correlation function, and holds for an individual realisation of the plasma.

Substituting discrete sums in the source S also with integrals for large N, we recover the standard expression for the Landau growth or damping of Langmuir waves, usually obtained from the Vlasov-Poisson system. However, here the electrostatic potential turns out to be the smoothed version of the actual shielded potential in the plasma, unveiling an unexpected connection between Debye shielding and Landau damping.

Shielding so appears as a fast cooperative dynamical process: it results from the accumulation of almost independent repulsive deflections with the same qualitative impact on the effective electric field of particle j (if point-like ions were present, the attractive deflection of charges with opposite signs would have the same effect). It is a cooperative effect, but not a collective one. So, shielding and collisional transport are two aspects of the same two-body repulsive process.

References

[1] D.F. Escande, F. Doveil, Y. Elskens, http://hal.archives-ouvertes.fr/hal-00827759 (2013).

[2] H. Spohn, *Large scale dynamics of interacting particles* (Springer, Berlin, 1991).