Stability of Inhomogeneous Equilibria of Hamiltonian Continuous Media Field Theories

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There are a wide variety of $1 + 1$ Hamiltonian continuous media field theories that exhibit phase space pattern formation. In plasma physics, the most famous of these is the Vlasov-Poisson equation, but other examples include the incompressible Euler equation in two-dimensions and the Hamiltonian Mean Field (or XY) model. One of the characteristic phenomenon that occurs in systems described by these equations is the formation of cat's eye patterns in phase space as a result of the nonlinear saturation of instabilities. Corresponding to each of these cat's eyes is a spatially inhomogeneous equilibrium solution of the underlying model, in plasma physics these are called BGK modes, but analogous solutions exist in all of the above systems. We analyze the stability of inhomogeneous equilibria in the Hamiltonian Mean Field model and in the Single Wave model, which is an equation that was derived to provide a model of the formation of electron holes in plasmas. We use action angle variables and the properties of elliptic functions to analyze the resulting dispersion relation construct linearly stable inhomogeneous equilibria for in the limit of small numbers of particles and study the behavior of solutions near these equilibria.