Dissipation induced by spectral-like filters in the numerical integration of Hamiltonian fluids

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The nonlinear development of high wave numbers induced by the nonlinear wave-wave coupling is a universal phenomenon relevant to the numerical modeling of both thermonuclear and astrophysical plasmas, of hydrodynamics and atmospheric fluids, and of the integration of transport equations in general. It becomes crucial when dealing with non dissipative systems which are described by Hamiltonian equations, like e.g. Vlasov plasmas. Indeed, the discretization of the numerical grid poses a limit to the highest wavenumber which is allowed during the numerical integration, since aliasing effects due to the nonlinear wave-wave coupling lead to the accumulation of energy at the grid scale and eventually to a blow-up of the solution (nonlinear computational instability [1, 2]).

A common way to avoid this problem is the introduction in the model of a small viscous dissipation, which however represents an artifact in intrinsically Hamiltonian systems. Alternative ways may rely on dissipative integration schemes (e.g. Lax-Friedrich scheme [3]) or in the implementation of some kind of selective cut-off of high wavenumbers only. Such cut-off is realized by introducing some numerical filters in the integration algorithm. Despite these cut-offs are generally devised to affect the least possible the ideally non-dissipative character of the analytical Hamiltonian model, some effects are bound to occur. A natural question arises about which of these methods of smoothing may be more physical with respect to the phenomenon in consideration.

We here consider a class of spectral-like filters ([4]) quite used in the numerical simulation of Vlasov systems and collisionless magnetized plasmas. In particular we characterize their dissipative behaviour by studying their effect on the onset and linear and non linear growth rate modification of some fluid and MHD instabilities (e.g. Kelvin-Helmholtz and magnetic reconnection). To provide a quantitative estimate we compare this effect to the one of the classical viscous or resistive dissipation.

References

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