Non-mean-field critical exponent in a mean-field model: comparison between dynamics and statistical mechanics

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Many interesting physical systems can be described by the Vlasov equation [1], including plasmas and self-gravitating systems. In particular, for N-particle systems with long-range interactions the rigorous mean-field limit [2] leads to a description of the system in the continuum, $N \to \infty$, limit in which the Vlasov equation rules the dynamics. This equation shows an infinity of stationary solutions, for example, all distribution functions that depend on phase-space variables only through the single-particle energy do not evolve in time, as proven by Jeans [3]. Moreover, for all finite values of N, such systems evolve towards Boltzmann-Gibbs (BG) equilibrium on a time scale which diverges with N and the system is trapped in long-living quasistationary states (QSS) with out-of-equilibrium properties [5].

Close to critical stability thresholds of these QSSs the system shows a scaling behavior that mimics the one observed near second order phase transitions [4]. To tackle this problem, we consider a magnetic mean-field model, called Hamiltonian Mean-Field (HMF), whose critical behavior is described by the Curie-Weiss law. The equilibrium susceptibility, χ_T, of this model can be obtained using standard methods of statistical mechanics. It diverges at the critical point with the classical exponent $\gamma_T = -1$. We compare this divergence with that obtained from the Vlasov equation. In this context, Vlasov susceptibility, here χ_V, can be derived from the linear response theory derived in Ref. [6]. While in the paramagnetic phase the critical exponent is the same in both contexts, in the ferromagnetic phase the exponents do not coincide. Both analytically and numerically we find that the Vlasov critical exponent is $\gamma_V^{ferr} = -1/4$, to compared with the equilibrium value $\gamma_T^{ferr} = -1$.

References

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