Kinematic and fluid reductions for fusion plasmas: a Hamiltonian approach

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Due to the extreme temperature reached inside fusion devices, core plasma is mainly collisionless. As a consequence, its dynamics can be described by the set of VLASOV-MAXWELL equations:

$$\begin{cases} \partial_t f_\alpha = -\mathbf{v}\cdot\nabla f_\alpha - \dfrac{q_\alpha}{m_\alpha}\left(\mathbf{E}+\dfrac{\mathbf{v}}{c}\times\mathbf{B}\right)\cdot\partial_{\mathbf{v}} f_\alpha \\ \partial_t \mathbf{E} = c\nabla\times\mathbf{B} - 4\pi\displaystyle\sum_\alpha q_\alpha \int_{\mathbb{R}^3} f_\alpha \mathbf{v}\,\mathrm{d}v \\ \partial_t \mathbf{B} = -c\nabla\times\mathbf{E}. \end{cases} \tag{1}$$

These equations can be expressed using Hamiltonian formalism such that $\dot{F} = \{F, H\}$, where the dot refers to the time derivative and $F = (f_\alpha, \mathbf{E}, \mathbf{B})$. Here, H is the Hamiltonian of the system and $\{\cdot,\cdot\}$ the associated POISSON bracket [1][2]. However, one can not in general solve this system, either analytically or numerically. Indeed, it would require presently inaccessible computing power and data storage capacity to perform numerical simulations of equations (1) with realistic values of the physical parameters. Therefore, one has to use reduced models to be able to compute the evolution of such complicated systems.

Dynamical reduction of the VLASOV-MAXWELL system is a well known procedure in plasma physics that can lead to various descriptions (e.g. gyrokinetics, multi-fluid model, magnetohydrodynamics, etc). However, one has to ensure, during this reduction, not to lose the initial Hamiltonian structure of the equations. This is required to preserve intrinsic physical properties such as conserved quantities.

The aim of this work is to develop theoretical tools to connect all these descriptions (either kinetic or fluid) in the framework of Hamiltonian dynamics. We will in particular base our study on DIRAC's theory of constraints [3][4], which provides a strong formalism adapted to plasma physics equations. The emphasis will be put on the role of long range interactions during the dynamical reduction. Numerical simulations will be performed to compare the reduced model with the parent one. Eventually, these tools will be applied to standard models in fusion physics such as MHD or gyrokinetic equations.

References

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