Space-only hyperbolic approximation of the Vlasov equation

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We condider the Vlasov equation coupled with the Maxwell equations for the electromagnetic field or with the Poisson equation for electric field only.

In the general three-dimensional (3D) case, the system is very complicated with 7 variables (three velocities, three positions and the time) so leads to very heavy numerical simulations.

The Particle-In-Cell (PIC) method (see for instance [1, 2]) is a popular method for computing collisionless plasma, because it allows performing simulations in complex configurations with a relatively low amount of memory and CPU ressource. However the PIC method is based on an initial random choice of the particles and thus presents numerical noise. Also, it is difficult to ensure the energy conservation. Therefore, Eulerian methods for solving kinetic equations are becoming more and more popular. They allow a better control of the conservation and numerical errors (for a review of eulerian methods, see [4]).

We construct an hyperbolic approximation of the Vlasov equation in which the dependency on the velocity variable is removed. The resulting model enjoys interesting conservation and entropy properties. We propose a finite volume scheme in order to solve this hyperbolic system.

We apply our approach to the simple case of the one-dimensional (1D) Vlasov-Poisson system. We present numerical results for one-dimensional classical test cases in plasma physics: Landau damping, two-stream instability.

The same approach can be applied to the model with a Fourier transformation with respect to the velocity variable [3]. We also obtain a reduced hyperbolic system with a proper discretization by the finite element method. The choice of the boundary condition is essential to ensure the hyperbolic and the stability of the scheme.

References

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