A semi-Lagrangian discontinuous Galerkin scheme for Vlasov-Poisson equation

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This work concerns the numerical simulation of the $1D \times 1D$ Vlasov-Poisson equation

$$\begin{aligned} \partial_t f + v\partial_x f + E\partial_v f &= 0 \\ \partial_x E &= \int_{\mathbb{R}} f dv - 1. \end{aligned}$$

using a semi-Lagrangian discontinuous Galerkin scheme. Such a scheme has been developped already in [2] and then more recently in [1, 4, 5] for Vlasov-Maxwell/Poisson applications.

One key point, in such applications, is to use directional splitting which leads to a succession of constant advection problems and the scheme has the advantage of not being restricted to a CFL condition. In the case of constant linear advection equation with periodic boundary conditions and an uniform mesh, this scheme has a superconvergence property as proved in [6] for an arbitrary degree.

An advantage of this method is that it also fits to the case of constant advection on a 1D unstructured mesh. We investigate the case of a non-uniform mesh in velocity. We will compare these results with the simulation of Kinetic Electrostatic Electron Nonlinear (KEEN) Waves for which we already have numerical results on GPU in the case of an uniform mesh in [3].

References

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