Invariant solutions of the Vlasov equation for one-dimensional models with long-range interactions

Zolacir Trindadre[1,2], Tarcísio M. Rocha Filho[1]

[1] Instituto de Física and International Center for Condensed Matter Physics, Universidade de Brasília, CP: 04455, 70919-970 - Brasília, Brazil
[2] Depto. de Ciências Exatas e Tecnológicas, Universidade Estadual de Santa Cruz, 45662-900, Ilhéus, Brazil.

An interaction potential is considered long-range if it scales at greater distances as $r^{-\alpha}$ with $\alpha < d$, with r the inter-particle distance and d the spatial dimension, and their phenomenology includes unusual features such as negative heat capacity, ensemble inequivalence and long lasting non-Gaussian quasi-stationary states [1]. and stay in such regime for times that diverge with number of particles.

The Vlasov Equation (VE) describes the statistical state of a Hamiltonian system of N classical particles in the $N \to \infty$ limit. Numerical solutions of the VE are extensively used for plasmas and gravitational systems, among others. Analytical solutions are difficult to obtain since it is a non-linear integro-differential equation, but on can look for particular solutions using generalizations of the Lie symmetries methods to such equations [2]. In this work we present a classification of group-invariant solutions of the VE for some one-dimensional representative models of interest in the field of statistical mechanics of long-range interacting systems. The models considered are: The Hamiltonian Mean Field, the Ring and the self-gravitating sheets models [4, 3, 5]. A great deal of the computations required are performed using the package SADE in computer algebra for the determination of infinitesimal Lie symmetries and its generalizations [6]. A discussion and physical interpretation of the obtained solutions are provided.

References

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