# Brownian corrections to particle motion in the XY Hamiltonian Mean Field Model

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We study the dynamics of the $N$-particle system evolving according to the Hamiltonian

$$H = \sum_{i}^{N} \frac{p_i^2}{2} + \frac{K}{2N} \sum_{i,j}^{N} [1 - \cos(\theta_i - \theta_j)], \tag{1}$$

commonly known as the XY Hamiltonian Mean Field (XY-HMF) model. $\theta_i$ is the position of the $i$-th particle on the circle $S_{2\pi} = \mathbb{R}/2\pi$ and $p_i$ its conjugate momentum. This model has two behaviours, viz. *(i)* for an attractive potential ($K = 1$), a phase transition occurs from an inhomogeneous, clustered phase to a homogeneous phase for a given value of temperature ; *(ii)* for a repulsive potential ($K = -1$), no phase transition occurs.

In the second case, particle motion may be approximated by the ballistic motion with small corrections. For particles with initial positions uniformly distributed in $S_{2\pi}$, while initial velocities are distributed in equally spaced (by $\Delta v_0$) beams containing one particle each, i.e., $v_{j0} \sim (j/N - 1/2)\Delta v_0$ for the initial velocity of the $j$-th particle, it is shown that corrections to the ballistic velocities are in the form of independent Brownian noises. Moreover, we also estimate a time validity for this approximation. Molecular dynamics simulations of the XY-HMF model with the proposed "particles in monokinetic beams" initial conditions are presented to confirm our preliminary theoretical results.

For the attractive case, we model the system, in presence of the ordered phase, as composed of two sets of particles: $N_{\mathrm{p}}$ *passing particles* move according to a ballistic motion corrected by the presence of $N_{\mathrm{c}}$ *cluster particles*, that lay inside the cat's eye and are assumed to have fixed positions. Thus, we focus on the dynamics of passing particles with the same strategy as above. The presence of cluster particles, however, no longer allows us to admit small corrections to a ballistic motion. Preliminary numerical simulations for this case show that these corrections diverge from a Brownian noise in very short times.

We are still carrying out the calculations for the attractive case. As a first order correction to the velocities, we expect to get Brownian noise and non-negligible corrections for higher orders.

## References

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