ECOLE POLYTECHNIQUE – « INGENIEUR POLYTECHNICIEN » PROGRAM
INTERNATIONAL ADMISSIONS

Recommended knowledge in Physical Sciences

The recommended knowledge in Physical Sciences for the applicants to the "Track 2 - International admissions" is detailed below.

This document is meant to give the applicants directions on the knowledge they are likely to be interviewed upon. It is given for information purposes only and cannot be considered as a basis of the programme for the second track examinations.

Ecole Polytechnique reserves the right to test an applicant's knowledge on other fields of Physical Sciences than those listed in this document.
Besides, Ecole Polytechnique expects the applicants to know the numerical values of the basic constants of physics, as well as the orders of magnitude of the physical phenomena of nature.
The applicants should be able to show excellent standard mathematical skills.

An excellent level in mathematics and in physical sciences is a key to successful studies at Ecole Polytechnique.

I. MECHANICS

- Newtonian mechanics
- Mechanics of solids
- Statics and mechanics of fluids
- Applications of mechanics

II. ELECTRIC CIRCUITS

III. ELECTRICITY AND MAGNETISM

- Electrostatics
- Magnetostatics
- Electromagnetic waves

IV. OPTICS

- Geometrical optics
- Wave optics

V. THERMODYNAMICS

- Perfect gas
- First and second principles of thermodynamics

Physical constants

The values of Planck, Boltzmann and Avogadro constants, the charge and the mass of the electron, the speed of light in vacuum, the electric permittivity and the magnetic permeability of free space, in *SI* system of units (at least two significant digits are required).

Orders of magnitude

The orders of magnitude of quantities such as the magnetic field of the Earth, the radius of the Earth, the acceleration of free fall at the Earth's surface, the concentration of electrons in a typical metal, the wavelengths of the electromagnetic waves of the visible spectrum, the distance between two atoms in a solid or liquid, the Bohr radius of the fundamental state of the hydrogen atom, the size of the nucleus.

Compulsory minimal requirements of calculation skills

Mastering a certain number of calculation skills such as is compulsory

Expansions

Be able to study the behaviour of a physical quantity $A(x)$ in the neighbourhood of a given value of its argument x. The common expansions about $x \approx 0$

$$\sin x \approx x - \frac{x^3}{6}; \quad \cos x \approx 1 - \frac{x^2}{2}; \quad \tan x \approx x + \frac{x^3}{3}; \quad \cot g\, x \approx \frac{1}{x} - \frac{x}{3}$$

$$e^x \approx 1 + x + \frac{x^2}{2}; \quad \ln(1+x) \approx x; \quad (1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha)(\alpha - 1)}{2} x^2$$

Derivatives and primitives of the functions of a single variable

Derivatives of the elementary functions (x^n, $\ln x$, e^x, $\sin x$, $\cos x$, $tg\, x$, $\cot g\, x$) as well as of the composition function $f(g(x))$.
Rules for the derivative of the product and the quotient of two functions of a real variable.
Primitives of the elementary functions above.
Integration by parts.
Conditions for the convergence of an integral in the cases of an infinite integration interval or the presence of points of discontinuity.

Functions of several variables. Common differential operators.

Total differential.
Partial derivatives with respect to an independent variable in the case of a function of several variables.
Nabla operator $\vec{\nabla}$. Gradient $\vec{\nabla} f$ of a function $f(\mathbf{r})$.
Curl $\vec{\nabla} \times \mathbf{A}$ of a vector field $\mathbf{A}(\mathbf{r})$. Divergence $\vec{\nabla}.\mathbf{A}$. Circulation $\oint_{(C)} \mathbf{A}.d\mathbf{l}$.

Laplacian $\nabla^2 f$ and vector Laplacian $\nabla^2 \mathbf{A}$.

Multiple integrals. Stokes, Gauss – Ostrogradski theorems.

Reduction of multiple integrals to simple integrals by using the symmetry properties (cylindrical, spherical) of the integrants and surfaces (volumes) involved
Stokes theorem.
Gauss-Ostrogradski theorem.

Differential equations

Solution of first order differential equations with separable variables.
Solution of second order linear and homogeneous differential equations with constant coefficients. Characteristic polynomial, number and nature of solutions, critical damping.
Solution of second order linear inhomogeneous equations with constant coefficients. Concepts of forced oscillations and resonance.

Equations with partial derivatives

D'Alembert's solution of the wave equation.
Progressive monochromatic plane waves. Concepts of wave vector, wavelength, frequency and period.
Principal phenomenological laws (Fick, Fourier and diffusion equations). Energy, mass, etc. balance within an elementary volume.

Linear algebra

Calculation of a determinant, diagonalization of a matrix, concepts of eigenvalues and eigenvectors of a linear operator.

Trigonometry

Definitions and properties of the basic trigonometric functions (sine, cosine, tangent, cotangent).
Common trigonometric formulas ($\cos 2x = \cos^2 x - \sin^2 x$; $\sin 2x = 2 \sin x \cos x$; $\sin \alpha + \sin \beta = 2 \sin [(\alpha + \beta) / 2] \cos [(\alpha - \beta) / 2]$; $\cos \alpha + \cos \beta = 2 \cos [(\alpha + \beta) / 2] \cos [(\alpha - \beta) / 2]$, etc.).
Fourier series of a regular enough periodic function.

I. MECHANICS

Newtonian mechanics

Newton's laws: the principle of inertia, the principle of action and reaction, the fundamental equation of dynamics.
Galilean relativity. Concept of non-inertial reference frames and forces referred to as « inertia » forces (in particular, in the case of linear acceleration and uniform rotation frames)
Angular momentum theorem. Kinetic energy theorem. Momentum theorem.

A two particle system. Central force motion, bound states, scattering states.
Expressions for the velocity and the acceleration of a material point in cylindrical and spherical co-ordinates.
Concept of potential energy. Independence on the path of the work done by a potential-derived force.
Conservation of mechanical energy of an isolated material system in the case of conservative forces.
Conservation of angular momentum in the case of central forces. First and second Kepler’s laws (the law of conical sections and the law of areas).
Conservation of momentum in the case of an isolated system. Elastic and inelastic collision problems. Concept of a centre of mass of a system.
Expressions for the potential, kinetic and total energy of a particle in the case of a circular trajectory.

Mechanics of solids

Rigid bodies (non-deformable solids). Solids rotating about a fixed axis. Moment of inertia of a rigid body. Expression for the kinetic energy of a rigid body as a sum of a translational term of its centre of mass and of a rotational term referred to the centre-of-mass reference frame (Koenig’s theorem). The problem of the compound pendulum.

Statics and mechanics of fluids

Euler’s description (the concept of a velocity field) of a fluid. Concepts of flow density, mass flow rate and volume flow rate. Mass balance. Equation of the conservation of mass in its local form.
Definitions of a stationary flow, of an incompressible flow, of a non-rotational flow.
Perfect flows: Euler’s equation, Bernoulli’s relationship on incompressible and homogeneous flows.
Calculation of the resulting force of the pressure forces exerted upon an object, in fluid statics.
The Archimedes’ principle (the buoyancy force applied to an object immersed in a fluid).

Applications of mechanics

Lorentz force (force exerted on a charged particle in constant electric and magnetic fields).
Trajectory of a charged particle in a static and uniform magnetic field.
Linear oscillations; damped harmonic oscillations. Forced oscillations, resonance.

II. ELECTRIC CIRCUITS

Electric voltage. Kirchoff’s laws of knots and meshes. Electric current. Ohm’s law. Superposition theorem.

Basic circuit components: resistor, capacitor, coil. Their impedances in sinusoidal regime.

Transient regime of charging and discharging a capacitor.
Sinusoidal currents and voltages. Maximum value, rms (root mean square) value. Impedances in series and in parallel.
Study of resonances in circuits in sinusoidal regime. *RLC* circuit. Relation to resonance in mechanics.

III. ELECTRICITY AND MAGNETISM

Electrostatics

Coulomb's law. The concept of electric field. Electrostatic field **E**. Circulation and flow of **E**. Gauss' theorem. Symmetry properties of **E**.

Electrostatic potential ϕ and Poisson's equation.
Calculation of **E** and ϕ for a simple charge distribution ρ. Electrostatic potential between the plates of a planar capacitor.
Concept of electric dipole, field created by a dipole at large distances, interaction energy of a permanent dipole with the electric field. Definition of the electric polarization vector.
Electric field in a conductor at equilibrium. Equipotential surfaces.
Electric field in the vicinity of a metal surface.
Coulomb's law between two charges immersed in a homogenous linear and isotropic dielectric medium.

Magnetostatics

Magnetic field **B**. Symmetry properties of **B**.
Magnetic field created by a thin wire carrying a current (Biot-Savart law), the two Maxwell equations (the divergence of **B** and Ampère's law), vector potential **A**.
Non-unicity of the electrostatic potential ϕ and the vector potential **A**, unicity of the electric field **E** and the magnetic field **B**.
Circulation of **B**. Relationship between the circulation of **B** and the encircled currents (theorem of the total current).
Calculation of **B** created by straight wires and circular loops. Field along the axis of a circular loop and of a coil (solenoid) having a circular cross-section.
Magnetic dipole and magnetic moment **M**. Expression for the interaction energy between a magnetic moment and a magnetic field **B**.
Flux of **B**. Electromagnetic induction phenomenon, Faraday's law, Lenz' rule.

Electromagnetic waves

Electromagnetic waves in vacuum.
Maxwell's equations in vacuum. Progressive harmonic plane waves as solutions of the Maxwell's equations in vacuum. Frequency, wavelength, wave vector. The concept of phase velocity.
Transversality of the electric and magnetic fields.
The state of polarization state an electromagnetic wave. Linear and circular polarizations.
Volume density of the electromagnetic energy, Poynting vector.
Concept of wave packet. Group velocity.

Electromagnetic waves in matter (linear and isotropic medium).
Macroscopic **E** and **B** fields. Constitutive relationships complementing Maxwell's equations.
Frequency-dependent complex dielectric constant $\varepsilon(\omega)$.
Concepts of complex refraction index, dispersion and absorption.
Microscopic models describing the material polarization of the medium: Drude model, model of the elastically bound electron (Lorentz model).

IV. OPTICS

Geometric optics

Concept of light ray. Reflection and refraction by a plane mirror. Snell-Descartes' laws. Limit angle. The total reflection phenomenon.
Spherical mirrors, lenses, conjugation and magnification relations.

Wave optics

Reflection and refraction of a harmonic progressive polarized plane wave at the interface between two dielectric media. Proof of Snell-Descartes laws.
Concept of optical path**.** Interference between two totally coherent waves. Michelson's interferometer. Thin slabs. Fabry-Pérot cavity.
Diffraction at infinity. Huyghens-Fresnel principle. Diffraction by a rectangular slit. Diffraction at infinity by two slits (Young's slits), by a row of slits.

V. THERMODYNAMICS

Thermodynamic state functions: internal energy, entropy, enthalpy, free energy, free enthalpy, as well as their differentials.
Extensive and intensive variables, thermodynamic equilibrium.
Heat capacities at a constant volume and at a constant pressure.

Perfect gas

Perfect monoatomic gas model. Maxwell-Boltzmann distribution of velocities for a monoatomic perfect gas. The equipartition theorem.
Collisions against a wall. Relationship between pressure and mean square velocity.
Perfect gas in a field of forces having a potential energy $V(\mathbf{r})$. The barometric formula.
Limitations of the perfect gas model. Real gases. The van der Waals gas.

First and second principles of thermodynamics

First principle. Internal energy U. Heat transfer. Work exchanged by a system. The work of pressure forces. Enthalpy and Joule-Thomson expansion. The enthalpy of a perfect gas.
Second principle. The entropy S. Entropy balance. Reversible and irreversible processes. Thermodynamic definition of temperature.
The entropy of a perfect gas (for a condensed and idalatable phase).
Heat machines. Dithermal cycle. Efficiency. Carnot's theorem.
Equilibrium between the phases of a pure substance. Triple point, critical point, enthalpy and entropy of phase changes. Clapeyron's formula.
Free energy and free enthalpy: definitions and differentials. Chemical potential. The perfect gas case. Equilibrium between two phases. Generalization, Gibbs' phase rules.