"Soluzioni I^a prova in itinere (A) 16/12/2009 (1)
Metodi computazionali
applicati alla geologia"

1) $\frac{\Delta t}{t} = \frac{0,1}{1,6} = 0,0625 \simeq \boxed{0,06}$ 6%

$\frac{\Delta h}{h} = \frac{0,1}{14,1} = 0,00709 \simeq \boxed{0,007}$ 0,7%

$g = \frac{2h}{t^2}$ $g = 11,0156 \ m/s^2$

$$\frac{\Delta g}{g} = \sqrt{\left(\frac{\Delta h}{h}\right)^2 + \left(\frac{2\Delta t}{t}\right)^2} = \sqrt{(0,00709)^2 + 4\cdot(0,0625)^2} =$$

$= 0,1252$

$\Rightarrow \Delta g = 1,379 \ m/s^2 \simeq 1.4 \ m/s^2$

$$\boxed{g = (11 \pm 1,4)\ m/s^2}$$

2) $v = \sqrt{2gh} = f(g,h)$

$$\Delta v = \left|\frac{\partial f}{\partial g}\right| \Delta g + \left|\frac{\partial f}{\partial h}\right| \Delta h$$

oppure $v = 2^{1/2} \cdot g^{1/2} \cdot h^{1/2}$

g e h non hanno errori indipendenti

$$\frac{\Delta v}{v} = \frac{\Delta(g^{1/2})}{g^{1/2}} + \frac{\Delta(h^{1/2})}{h^{1/2}}$$

$$\Delta(\sqrt{g}) = \frac{1}{2\sqrt{g}} \cdot \Delta g$$

$$\Rightarrow \frac{\Delta v}{v} = \frac{1}{2\sqrt{g}} \frac{\Delta g}{\sqrt{g}} + \frac{1}{2\sqrt{h}} \frac{\Delta h}{\sqrt{h}}$$

$$\frac{\Delta v}{v} = \frac{1}{2} \frac{\Delta g}{g} + \frac{1}{2} \frac{\Delta h}{h}$$

$$\frac{\Delta v}{v} = \frac{1}{2} \cdot 0,1252 + \frac{1}{2} \cdot 0,00709 =$$

$$= 0,0661$$

$$v = \sqrt{2 \cdot 11,0156 \cdot 14,1} \ m/s = 17,625 \ m/s$$

$$\Delta v = 1,166 \ m/s \simeq 1,2 \ m/s$$

$$\boxed{v = (17,6 \pm 1,2) \ m/s}$$

3) $n_2 = n_1 \cdot \frac{\sin\theta_1}{\sin\theta_2}$ $\sin\theta_1 = f(\theta_1)$

$$\Delta \sin\theta_1 = \left|\frac{\partial f}{\partial \theta_1}\right| \Delta\theta_1 = \left|\frac{\partial \sin\theta_1}{\partial \theta_1}\right| \Delta\theta_1 =$$

$$= |\cos\theta_1| \cdot \Delta\theta_1$$

con $\Delta\theta_1 = 0,5 \cdot \frac{\pi}{180}$ rad $= 0,0087$ rad

$$\Delta\theta_1 = \Delta\theta_2 = 0,009 \text{ rad}$$

$$\Delta \sin\theta_2 = |\cos 30°| \cdot 0{,}009 = 0{,}00779$$

$$\boxed{\Delta(\sin\theta_2) = 0{,}008}$$

$$\Delta \sin\theta_1 = |\cos 70°| \cdot 0{,}009 = 0{,}00309$$

$$\boxed{\Delta(\sin\theta_1) = 0{,}003}$$

4) $$n_2 = 1 \cdot \frac{\sin(70°)}{\sin(30°)} = 1{,}879385$$

$$n_2 = f(\theta_1, \theta_2)$$

$$\Delta n_2 = \left|\frac{\partial f}{\partial \theta_1}\right| \Delta\theta_1 + \left|\frac{\Delta f}{\partial \theta_2}\right| \Delta\theta_2$$

oppure

$$\frac{\Delta n_2}{n_2} = \sqrt{\left(\frac{\Delta(\sin\theta_1)}{\sin\theta_1}\right)^2 + \left(\frac{\Delta(\sin\theta_2)}{\sin\theta_2}\right)^2}$$

$$\frac{\Delta n_2}{n_2} = \sqrt{\left(\frac{0.008}{0{,}93969}\right)^2 + \left(\frac{0{,}003}{0{,}5}\right)^2} = 0{,}0104$$

$$\Delta n_2 = 0{,}01957 \simeq 0{,}02$$

$$\boxed{n_2 = 1{,}88 \pm 0{,}02}$$

5) $$m_{best} = \frac{\sum_{i=1}^{6} x_i}{6} = 24{,}5 = \bar{x}$$

$\Delta m = \sigma_{\bar{x}}$ deviazione standard della media

$$\sigma_x = \sqrt{\frac{\sum (x-\bar{x})^2}{6-1}} = \sqrt{\frac{0{,}28}{5}} = 0{,}23664$$

24.5	$\rightarrow (x-\bar{x})^2 = 0$
24.8	0,09
24,1	0,16
24,6	0,01
24,4	0,01
24,6	0,01
	tot 0,28

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$$

$$\sigma_{\bar{x}} = \frac{0{,}23664}{\sqrt{6}} = 0{,}0966 \simeq 0{,}1$$

$$\boxed{(24{,}5 \pm 0{,}1)\ g}$$

6) $\frac{\Delta m_{sist}}{m} = 0{,}01 \Rightarrow \Delta m_{sist} = 0{,}24\ g$

$$\Delta m_{tot} = \sqrt{(\Delta m)^2 + (\Delta m_{sist})^2} =$$

$$= \sqrt{(0{,}1)^2 + (0{,}24)^2} = 0{,}26\ g$$

$$\boxed{(24{,}5 \pm 0{,}3)\ g}$$

7) $34{,}847 \pm 0{,}005$

$1{,}37000 \pm 0{,}00003$

$81{,}48 \pm 0{,}24$ oppure $81{,}5 \pm 0{,}2$

$25{,}13 \pm 0{,}04$

4800 ± 500

49000 ± 4000
366 ± 7
370 ± 40

8) $P = 1/2$ = prob di avere Testa

poiché $P(A) = P(3\text{Teste}) + P(3\text{ Croci})$

$$P(A) = 2 \cdot P(K=3,\ N=3) =$$

$$= 2 \cdot \frac{3!}{3!\,0!}\left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^0 = 2 \cdot \left(\frac{1}{2}\right)^3 = 2 \cdot \frac{1}{8}$$

$$\boxed{P(A) = \frac{1}{4}}$$

$$P(B) = P(K \geq 2,\ N=3) = P(2, N=3) + P(3, N=3) =$$

$$= \frac{3!}{2!\,1!}\left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^1 + \frac{3!}{3!\,0!}\left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^0 =$$

$$= 3 \cdot \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 = 4 \cdot \frac{1}{8} = \frac{1}{2}$$

$$\boxed{P(B) = \frac{1}{2}}$$

9) $P(B/A) = P(B)$ se A, B sono indipendenti

$P(B/A)$ = prob. di avere Testa in almeno due lanci sapendo che si presenta la stessa faccia in tutti e 3 i lanci

$$P(B/A) = \frac{1}{2}$$

1 → casi favorevoli (TTT)

2 → casi possibili TTT CCC

quindi $P(B/A) = P(B)$

10) $P(A) = P(MMMMMFFFFF) = \left(\frac{1}{2}\right)^{10}$

$P(A) = 0,00097656$

$$\boxed{P(A) = 0,00098}$$

$P(B) = P(K=5, N=10) =$ con $p = 1/2$

$$= \frac{10!}{5!\,5!} \left(\frac{1}{2}\right)^5 \cdot \left(\frac{1}{2}\right)^5 =$$

$$= \frac{\cancel{10}^{2} \cdot \cancel{9}^{3} \cdot \cancel{8}^{2} \cdot 7 \cdot 6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2}} \cdot \left(\frac{1}{2}\right)^{10} =$$

$$= 252 \cdot \left(\frac{1}{2}\right)^{10} = 0,246 \qquad \boxed{P(B) = 0,246}$$

$$P(C) = P(MFMFMFMFMF) = \left(\frac{1}{2}\right)^{10}$$

$$\boxed{P(C) = 0,00098}$$

11) $\mu = 23$

$$P(x \leq 28) = \int_{-\infty}^{28} p(x)\,dx = \int_{-\infty}^{z_1} g(z)\,dz =$$

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$z_1 = \frac{28-23}{\sigma} = \frac{5}{\sigma}$$

$$= 0.5 + \int_0^{z_1} g(z)\,dz$$

$$0.5 + \int_0^{z_1} g(z)\,dz = 0,94$$

$$\rightarrow \int_0^{z_1} g(z)\,dz = 0,44$$

Dalle tavole $z_1 = 1,56$

quindi $\frac{5}{\sigma} = 1,56$ $\sigma = \frac{5}{1,56}$

$\boxed{\sigma = 3,2}$ $\sigma = 3,2$ Kg

12) $$\int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$\int_0^3 a(3x-x^2)\,dx = 3a\left[\frac{1}{2}x^2\right]_0^3 - a\left[\frac{x^3}{3}\right]_0^3 =$$

$$= 3\frac{a}{2}(9-0) - \frac{a}{3}(27-0) = a\frac{27}{2} - a\cdot 9 =$$

$$= a\,\frac{27-18}{2} = \frac{9}{2}a$$

$$\Rightarrow \quad \frac{9}{2}a = 1$$

$$\boxed{a = \frac{2}{9}}$$

13) $$\mu = E[x] = \int_{-\infty}^{+\infty} x f(x)\,dx =$$

$$= \int_0^3 a x (3x-x^2)\,dx =$$

$$= a\int_0^3 (3x^2 - x^3)\,dx = \cancel{3}a\left[\frac{x^3}{\cancel{3}}\right]_0^3 - a\left[\frac{x^4}{4}\right]_0^3 =$$

$$= a(27-0) - \frac{a}{4}(81-0) = 27a - \frac{81}{4}a$$

$$= \frac{108-81}{4}a = \frac{27}{4}a = \frac{\cancel{27}^{3}}{\cancel{4}_{2}}\cdot\left(\frac{\cancel{2}^{1}}{\cancel{9}_{1}}\right) = \frac{3}{2}$$

$$\boxed{\mu = \frac{3}{2}}$$

$$\sigma^2 = E\left[(x-\mu)^2\right] = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\, dx =$$

$$= E[x^2] - \mu^2 = E[x^2] - \frac{9}{4}$$

$$E[x^2] = \int_0^3 x^2 \cdot a(3x - x^2)\, dx =$$

$$= \frac{2}{9}\int_0^3 \left(3x^3 - x^4\right) dx = \frac{2}{3}\left[\frac{x^4}{4}\right]_0^3 - \frac{2}{9}\left[\frac{x^5}{5}\right]_0^3 =$$

$$= \frac{\cancel{2}^{1}}{\cancel{3}_{1}} \cdot \frac{\cancel{81}^{27}}{\cancel{4}_{2}} - \frac{2}{\cancel{9}_{1}} \cdot \frac{\cancel{243}^{27}}{5} = \frac{27}{2} - \frac{54}{5} = \frac{135 - 108}{10} =$$

$$= \frac{27}{10}$$

$$\sigma^2 = \frac{27}{10} - \frac{9}{4} = \frac{54 - 45}{20} = \frac{9}{20}$$

$$\boxed{\sigma^2 = \frac{9}{20}}$$

14) Approssimazione gaussiana (della binomiale)

$p = \frac{1}{6}$ $\qquad \mu = Np = 180 \cdot \frac{1}{6} = 30$

$N = 180$ $\qquad \sigma = \sqrt{Np(1-p)}$

$$\sigma = \sqrt{\cancel{180}_{\cancel{30}\,5} \cdot \frac{1}{\cancel{6}_1} \cdot \frac{5}{\cancel{6}_1}} = \sqrt{25} = 5$$

$$\begin{cases} \mu = 30 \\ \sigma = 5 \end{cases} \qquad p(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$P(29{,}5 \leq x \leq 30{,}5) = \int_{29{,}5}^{30{,}5} p(x)\,dx = \int_{-0{,}1}^{0{,}1} g(z)\,dz =$$

$$= 2\int_{0}^{0{,}1} g(z)\,dz = 2 \cdot 0{,}0398 = \boxed{0{,}0796} \simeq 8\%$$

essendo $z_1 = \dfrac{29{,}5-30}{5} = \dfrac{-0{,}5}{5} = -0{,}1$

$$z_2 = \frac{30{,}5-30}{5} = \frac{0{,}5}{5} = 0{,}1$$

15) $\mu = 3$

$$P(k) = \frac{\mu^k}{k!} e^{-\mu}$$

$k = 0$

$$P(0) = \frac{\mu^0}{0!} e^{-\mu} = e^{-\mu} = e^{-3} = 0{,}0497$$

$$\boxed{P(0) = 0{,}050} \qquad 5\%$$

16) $P(k > 4) = 1 - P(0) - P(1) - P(2) - P(3) - P(4)$

$$P(0) = e^{-3}$$

$$P(1) = \mu e^{-\mu} = 3e^{-3}$$

$$P(2) = \frac{\mu^2}{2} e^{-\mu} = \frac{9}{2} e^{-3}$$

$$P(3) = \frac{\mu^3}{6} e^{-\mu} = \frac{27}{6} e^{-3} = \frac{9}{2} e^{-3}$$

$$P(4) = \frac{\mu^4}{24} e^{-\mu} = \frac{81}{24} e^{-3} = \frac{27}{8} e^{-3}$$

$$P(k>4) = 1 - e^{-3}\left(1 + 3 + 2 \cdot \frac{9}{2} + \frac{27}{8}\right) =$$

$$= 1 - e^{-3}\left(1 + 3 + 9 + \frac{27}{8}\right) = 1 - 0{,}8153$$

$$\boxed{P(k>4) = 0{,}185} \qquad 18{,}5\%$$