Esame di metodi computazionali SOLUZIONI

18/4/2011

1) $$\rho = \frac{0{,}070}{0{,}003 \cdot (0{,}10)^2} \quad \frac{Kg}{m^3}$$

$$\rho = 2333{,}33 \ \frac{Kg}{m^3}$$

$$\frac{\Delta\rho}{\rho} = \sqrt{\left[\frac{\Delta m}{m}\right]^2 + \left[\frac{\Delta(L^2)}{L^2}\right]^2}$$

con $$\frac{\Delta(L^2)}{L^2} = 2\,\frac{\Delta L}{L}$$

$$\frac{\Delta\rho}{\rho} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + 4\left(\frac{\Delta L}{L}\right)^2}$$

$$\frac{\Delta\rho}{\rho} = \sqrt{\left(\frac{2}{70}\right)^2 + 4\left(\frac{0{,}1}{10}\right)^2} = \sqrt{0{,}001216} = 0{,}0349$$

$$\rightarrow \quad \Delta\rho = 0{,}0349 \cdot \rho$$

$$\Delta\rho = 81 \ \frac{Kg}{m^3}$$

$$\Rightarrow \quad \boxed{\rho = (2330 \pm 80)\ \frac{Kg}{m^3}}$$

2) p = probabilità che un carattere sia sbagliato

$p = \frac{1}{5000}$ $\qquad N = 3500$ caratteri in una pagina

$\mu = Np = \frac{3500}{5000} = 0{,}7$ $\qquad$ n° medio di errori per pagina

$\sigma^2 = \mu$

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a) $P(K) = \frac{\mu^k}{k!} e^{-\mu}$ distrib di Poisson

$K=0$ ovvero che non ci siano errori in una pagina

$$P(0) = \frac{\mu^0}{0!} e^{-\mu} = e^{-\mu} = e^{-0.7} = 0,4966 \simeq \boxed{0,50}$$

c) Se considero 20 pagine, il numero medio di errori è

$$\mu_1 = 20\mu = 20 \cdot 0,7 = 14$$

$$P(k) = \frac{\mu_1^k}{k!} e^{-\mu_1}$$

$K=5$

$$P(5) = \frac{\mu_1^5}{5!} e^{-\mu_1} = \frac{14^5}{5!} e^{-14} = 0,0037 \simeq \boxed{0,004}$$

b) $\boxed{\begin{array}{l} \mu_1 = 14 \\ \sigma^2 = \mu_1 = 14 \end{array}}$

3) 98000 ± 6000
$22,23 \pm 0,08$
249 ± 15
$52,45 \pm 0,22$
3800 ± 500
280 ± 30

4) Distribuzione binomiale

$p = \frac{1}{3}$ prob. di rispondere correttamente ad una domanda

$N=10$ n° di domande

a) $\mu = Np = \frac{10}{3} = 3,33$ $\boxed{\mu = \frac{10}{3}}$

③

$$\sigma^2 = Np(1-p) = 10 \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{20}{9} \qquad \boxed{\sigma^2 = \frac{20}{9}}$$

b) $P(K \geqslant 5) = P(5) + P(6) + P(7) + P(8) + P(9) + P(10) =$

$$= 1 - \left(P(0) + P(1) + P(2) + P(3) + P(4)\right)$$

$$P(K) = \frac{N!}{K!(N-K)!} \; p^K \; (1-p)^{N+K}$$

$$P(0) = \frac{\cancel{10!}}{0!\,\cancel{10!}} \left(\frac{1}{3}\right)^0 \cdot \left(\frac{2}{3}\right)^{10} = \left(\frac{2}{3}\right)^{10}$$

$$P(1) = \frac{10!}{1!\,9!} \; \frac{1}{3} \cdot \left(\frac{2}{3}\right)^9 = 10 \cdot \frac{1}{3} \left(\frac{2}{3}\right)^9 = 5 \cdot \left(\frac{2}{3}\right)^{10}$$

$$P(2) = \frac{10!}{2!\,8!} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^8 = 45 \cdot \frac{2^8}{3^{10}}$$

$$P(3) = \frac{10!}{3!\,7!} \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^7 = 120 \cdot \frac{2^7}{3^{10}}$$

$$P(4) = \frac{10!}{4!\,6!} \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^6 = 210 \cdot \frac{2^6}{3^{10}}$$

$$1 - \frac{1}{3^{10}} \left[2^{10} + 5 \cdot 2^{10} + 45 \cdot 2^8 + 120 \cdot 2^7 + 210 \cdot 2^6 \right] =$$

$$= 1 - \frac{2^6}{3^{10}} \left[2^4 + 5 \cdot 2^4 + 45 \cdot 2^2 + 120 \cdot 2 + 210 \right] =$$

$$= 1 - \frac{2^6}{3^{10}} \left[16 + 80 + 180 + 240 + 210 \right] =$$

$$= 1 - \frac{2^6}{3^{10}} \cdot 726 = 1 - 0{,}7869 \simeq 0{,}2131 \simeq \boxed{0{,}21} = P$$

$P \simeq 21\,\%$

5) $$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

$$P(-10 \leq x \leq 10) = \int_{-10}^{10} p(x)\,dx = \int_{z_1}^{z_2} g(z)\,dz = 0{,}70$$

con $$g(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{-10}{\sigma}$$

$$z_2 = \frac{10}{\sigma}$$

$$\int_{-\frac{10}{\sigma}}^{\frac{10}{\sigma}} g(z)\,dz) = 2\int_{0}^{\frac{10}{\sigma}} g(z)\,dz = 0{,}7$$

$$\int_{0}^{10/\sigma} g(z)\,dz = \frac{0{,}7}{2} = 0{,}35$$

Guardando le tavole, si ha che $\frac{10}{\sigma} = 1{,}04$

$$\Rightarrow \sigma = \frac{10}{1{,}04} = 9{,}615 \qquad \boxed{\sigma^2 = 92}$$

6) $$\int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$\int_{0}^{2} \left(ax + \frac{1}{3}\right) dx = a\left[\frac{x^2}{2}\right]_0^2 + \frac{1}{3}\left[x\right]_0^2 =$$

$$= a \cdot 2 + \frac{2}{3}$$

$$2a + \frac{2}{3} = 1 \rightarrow 2a = \frac{1}{3} \qquad \boxed{a = \frac{1}{6}}$$

7) $$\mu = \int_{-\infty}^{+\infty} x f(x)\, dx = \int_0^2 x\left(\frac{x}{6}+\frac{1}{3}\right)dx =$$

$$= \int_0^2 \left(\frac{x^2}{6}+\frac{x}{3}\right)dx = \frac{1}{6}\left[\frac{x^3}{3}\right]_0^2 + \frac{1}{3}\left[\frac{x^2}{2}\right]_0^2 =$$

$$= \frac{1}{\cancel{6}_{3}}\cdot\frac{\cancel{8}^{4}}{3} + \frac{1}{3}\cdot 2 = \frac{4}{9}+\frac{2}{3} = \frac{4+6}{9} = \frac{10}{9} \qquad \boxed{\mu = \frac{10}{9}}$$

$$\sigma^2 = E[x^2] - \mu^2$$

$$E[x^2] = \int_0^2 x^2\left(\frac{x}{6}+\frac{1}{3}\right)dx = \int_0^2\left(\frac{x^3}{6}+\frac{x^2}{3}\right)dx =$$

$$= \frac{1}{6}\left[\frac{x^4}{4}\right]_0^2 + \frac{1}{3}\left[\frac{x^3}{3}\right]_0^2 = \frac{1}{6}\cdot\frac{\cancel{16}^{4}}{\cancel{4}_{1}} + \frac{1}{3}\cdot\frac{8}{3} = \frac{\cancel{4}^{2}}{\cancel{6}_{3}} + \frac{8}{9} =$$

$$= \frac{6+8}{9} = \frac{14}{9}$$

$$\Rightarrow \sigma^2 = \frac{14}{9} - \left(\frac{10}{9}\right)^2 = \frac{126-100}{81} = \frac{26}{81} \qquad \boxed{\sigma^2 = \frac{26}{81}}$$

8) Calcolo media

$$m = \frac{\sum x_i}{N} \qquad \text{con } N = 8$$

$$m = 2{,}5$$

$$s = \sqrt{\frac{\sum (x_i - m)^2}{N-1}} = \sqrt{\frac{0{,}36}{7}} = 0{,}227$$

$$\frac{s}{\sqrt{N}} = \frac{0{,}227}{\sqrt{8}} = 0{,}080$$

t di Student con $N-1 = 7$ gradi di libertà

$t_{critico} = 2,365$ dalla tabella

$$t_{critico} \cdot \frac{s}{\sqrt{N}} = 0,189 \simeq 0,19$$

$$m - 0,19 \leq \mu \leq m + 0,19$$

$$\boxed{2,31 \leq \mu \leq 2,69}$$

9) $\bar{x} = \frac{\sum x_i}{N}$ con $N = 36$

$\mu_x = 3,5$

$\sigma_x^2 = 0,1$

$$\mu_{\bar{x}} = \mu_x = 3,5$$

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{N} = \frac{0,1}{36} = 0,00278 \longrightarrow \sigma_{\bar{x}} = 0,053$$

$$P(3,55 \leq \bar{x} \leq 3,60) = \int_{3,55}^{3,60} \frac{1}{\sqrt{2\pi}\,\sigma_{\bar{x}}} e^{-\frac{(\bar{x}-\mu_{\bar{x}})^2}{2\sigma_{\bar{x}}^2}} \, d\bar{x} =$$

$$= \int_{z_1}^{z_2} g(z)\,dz = \quad \text{con} \quad g(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

$$z_1 = \frac{3,55 - 3,5}{0,053} = 0,94$$

$$z_2 = \frac{3,60 - 3,5}{0,053} = 1,89$$

$$= \int_{0,94}^{1,89} g(z)\,dz = \int_0^{1,89} g(z)\,dz - \int_0^{0,94} g(z)\,dz =$$

$= 0{,}4706 - 0{,}3264 = \boxed{0{,}1442}$ $P \simeq 14\%$

10) Fit dei minimi quadrati

$N = 3$

x_i	y_i	Δy_i	x_i^2	$x_i y_i$
1	3,4	0,5	1	3,4
1,5	4,1	0,5	2,25	6,15
2	5	0,5	4	10
$\sum_i$ 4,5	12,5		7,25	19,55

$$A = \frac{(\sum y_i)(\sum x_i^2) - (\sum x_i)(\sum x_i y_i)}{\Delta}$$

con $\Delta = N(\sum x_i^2) - (\sum x_i)^2$

$$\Delta = 3 \cdot 7{,}25 - (4{,}5)^2 = 1{,}5$$

$$\Rightarrow A = \frac{12{,}5 \cdot 7{,}25 - 4{,}5 \cdot 19{,}55}{1{,}5} = \frac{2{,}65}{1{,}5} = 1{,}7667$$

$$(\Delta A)^2 = (\Delta y)^2 \cdot \frac{\sum x_i^2}{\Delta} = (0{,}5)^2 \cdot \frac{7{,}25}{1{,}5} = 1{,}208$$

$$\Delta A = 1{,}099 \simeq 1{,}1$$

$$\Rightarrow \boxed{A = (1{,}8 \pm 1{,}1)}$$

$$B = \frac{N(\sum x_i y_i) - (\sum x_i)(\sum y_i)}{\Delta}$$

$$B = \frac{3 \cdot 19{,}55 - 4{,}5 \cdot 12{,}5}{1{,}5} = 1{,}6$$

$$(\Delta B)^2 = (\Delta y)^2 \frac{N}{\Delta} = (0{,}5)^2 \cdot \frac{3}{1{,}5} = 0{,}5 \quad \rightarrow \Delta B = 0{,}71$$

$$\rightarrow \quad \boxed{B = 1,6 \pm 0,7}$$

11)

x_i	y_i	$f(x_i)$
1	3,4	3,4
1,5	4,1	4,2
2	5	5

con $f(x_i) = A + B x_i$

$$f(x_i) = 1,8 + 1,6 \cdot x_i$$

$$\chi_0^2 = \frac{\sum (y_i - f(x_i))^2}{(\Delta y)^2} = \frac{0,01}{0,25} = 0,04$$

$d = 3 - 2 = 1$ numero di gradi di libertà

$$\tilde{\chi}_0^2 = \frac{\chi_0^2}{d} = 0,04$$

$$P_1(\tilde{\chi}^2 \geq 0,04) \simeq 90\%$$ guardando le tavole

Poiché $90\% > 5\%$ l'ipotesi può essere accettata.

Si tratta di una probabilità alta in modo sospetto...