Soluzioni esame scritto di METODI COMPUTAZIONALI 11/7/2011

1) $x = 6{,}0 \pm 0{,}1 \qquad y = 3{,}0 \pm 0{,}1$

$$q = xy + x^2/y$$

$$q = 18 + 36/3 = 18 + 12 = 30$$

$$\Delta q = \Delta(xy) + \Delta\left(\frac{x^2}{y}\right)$$

$$\frac{\Delta(xy)}{xy} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

$$\Rightarrow \left[\Delta(xy)\right]^2 = (xy)^2 \cdot \left[\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2\right] =$$

$$= (18)^2 \left[\left(\frac{0{,}1}{6}\right)^2 + \left(\frac{0{,}1}{3}\right)^2\right] = 0{,}45$$

$$\frac{\Delta\left(\frac{x^2}{y}\right)}{\left(\frac{x^2}{y}\right)} = \sqrt{\left(\frac{\Delta x^2}{x^2}\right)^2 + \left(\frac{\Delta y}{y}\right)^2} = \sqrt{\left(2\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

$$\Rightarrow \left[\Delta\left(\frac{x^2}{y}\right)\right]^2 = \left(\frac{x^2}{y}\right)^2 \cdot \left[4\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2\right] =$$

$$= \left(\frac{36}{3}\right)^2 \cdot \left[4\left(\frac{0{,}1}{6}\right)^2 + \left(\frac{0{,}1}{3}\right)^2\right] = 0{,}32$$

$$\Rightarrow \Delta q = \sqrt{0{,}45} + \sqrt{0{,}32} = 0{,}671 + 0{,}566 = 1{,}237 \simeq 1{,}2$$

$$\boxed{q = 30{,}0 \pm 1{,}2}$$

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2) Probab. di estrarre una pallina rossa $p = \frac{10}{100} = 0{,}1$

$N = 4$ tentativi

$K = 1$ n° di successi

$$P(N,k) = \frac{N!}{k!\,(N-k)!}\, p^k\, (1-p)^{N-k}$$

$$P(4,1) = \frac{4!}{1!\,3!}\,(0{,}1)^1 \cdot (0{,}9)^3 = 4 \cdot 0{,}1 \cdot (0{,}9)^3 = 0{,}2916$$

$$\boxed{P \simeq 0{,}29}$$

3) Distrib. binomiale, di media $\mu = Np$

$N = 11$

$p = 1/2$

$$\mu = 11 \cdot \frac{1}{2} = 5{,}5$$

$$\sigma = \sqrt{Np(1-p)} = \sqrt{11 \cdot 0{,}5 \cdot 0{,}5} = 1{,}658$$

$$\sigma^2 = 2{,}75$$

a) $\boxed{\mu = 5{,}5 \qquad \sigma^2 = 2{,}75}$

b) $$P(11,5) = \frac{11!}{5!\,6!}\,(0{,}5)^5 \cdot (0{,}5)^6 =$$

$$= \frac{11 \cdot \cancel{10}^{2} \cdot \cancel{9}^{3} \cdot \cancel{8}^{2} \cdot 7}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2}} \cdot (0{,}5)^{11} = 462 \cdot (0{,}5)^{11} = 0{,}22558$$

$$\boxed{P \simeq 0{,}23}$$

4) $990 \pm 60 \Rightarrow (9{,}9 \pm 0{,}6) \cdot 10^2$

$43{,}23 \pm 0{,}09$

253 ± 22

$52{,}448 \pm 0{,}022$

$78800 \pm 400 \Rightarrow (7{,}88 \pm 0{,}04) \cdot 10^4$

$276{,}32 \pm 0{,}08$

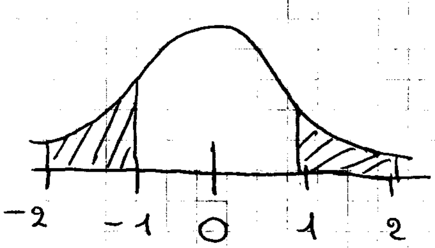
5) $\begin{cases} \mu = 172{,}5 \\ \sigma = 5 \end{cases}$ $\qquad p(x) = \dfrac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

a) $P(162{,}5 < x < 167{,}5) = \displaystyle\int_{162{,}5}^{167{,}5} p(x)\,dx = \int_{z_1}^{z_2} g(z)\,dz$

$g(z) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$ $\qquad z_1 = \dfrac{162{,}5 - 172{,}5}{5} = -2$

$z_2 = \dfrac{167{,}5 - 172{,}5}{5} = -1$

$P = \displaystyle\int_{-2}^{-1} g(z)\,dz = \int_{1}^{2} g(z)\,dz = \int_{0}^{2} g(z)\,dz - \int_{0}^{1} g(z)\,dz =$



$= 0{,}4772 - 0{,}3413 = 0{,}1359$

$\boxed{P \simeq 0{,}136}$

b) $P(x > 187{,}4) = \displaystyle\int_{187{,}4}^{+\infty} p(x)\,dx = \int_{z_1}^{+\infty} g(z)\,dz$

$z_1 = \dfrac{187{,}4 - 172{,}5}{5} = 2{,}98$

$= \displaystyle\int_{2{,}98}^{+\infty} g(z)\,dz = 0{,}5 - \int_{0}^{2{,}98} g(z)\,dz = 0{,}5 - 0{,}4986 = 0{,}0014$

$\boxed{P = 0{,}0014}$

6) $$f(x) = \begin{cases} \frac{x}{3} + a & 0 \leq x \leq 2 \\ 0 & x < 0,\ x > 2 \end{cases}$$

$$\int_{-\infty}^{+\infty} f(x)dx = 1$$ assioma della completezza

$$\Rightarrow \int_0^2 \left(\frac{x}{3} + a\right)dx = 1$$

$$\frac{1}{3}\left[\frac{x^2}{2}\right]_0^2 + a\left[x\right]_0^2 = \frac{1}{3}\cdot 2 + 2a = \frac{2}{3} + 2a$$

$$\frac{2}{3} + 2a = 1$$

$$2a = \frac{1}{3}$$

$$\boxed{a = 1/6}$$

7) $$\mu = \int_{-\infty}^{+\infty} x \cdot p(x)\,dx$$ con $p(x) = \frac{x}{3} + \frac{1}{6}$ per $0 \leq x \leq 2$

$$\mu = \int_0^2 x\left(\frac{x}{3} + \frac{1}{6}\right)dx = \frac{1}{3}\left[\frac{x^3}{3}\right]_0^2 + \frac{1}{6}\left[\frac{x^2}{2}\right]_0^2 =$$

$$= \frac{1}{3}\cdot\frac{8}{3} + \frac{1}{\cancel{6}_3}\cancel{2} = \frac{8}{9} + \frac{1}{3} = \frac{8+3}{9}$$

$$\boxed{\mu = \frac{11}{9}}$$

$$\sigma^2 = E[x^2] - \mu^2$$

$$E[x^2] = \int_0^2 x^2\left(\frac{x}{3} + \frac{1}{6}\right)dx = \frac{1}{3}\left[\frac{x^4}{4}\right]_0^2 + \frac{1}{6}\left[\frac{x^3}{3}\right]_0^2 =$$

$$= \frac{1}{3}\cdot\frac{\cancel{16}^4}{\cancel{4}_1} + \frac{1}{\cancel{6}_3}\cdot\frac{\cancel{8}^4}{3} = \frac{4}{3} + \frac{4}{9} = \frac{12+4}{9} = \frac{16}{9}$$

$$\sigma^2 = \frac{16}{9} - \left(\frac{11}{9}\right)^2 = \frac{16}{9} - \frac{121}{81} = \frac{144-121}{81} = \frac{23}{81}$$

$$\boxed{\sigma^2 = \frac{23}{81}}$$

8) $m = \frac{\sum x_i}{N} = 1{,}21429 \simeq 1{,}214$ $N=7$

$S = \sqrt{\frac{\sum_i (x_i - m)^2}{6}} \cong 0{,}195$

$N-1 = 6$ gradi di libertà

$t_{crit} = 2{,}447$

$$m - t_{crit} \frac{S}{\sqrt{N}} \leq \mu \leq m + t_{crit} \frac{S}{\sqrt{N}}$$

$$t_{crit} \cdot \frac{S}{\sqrt{N}} = 2{,}447 \cdot \frac{0{,}195}{\sqrt{7}} = 0{,}180$$

$$\boxed{1{,}034 \leq \mu \leq 1{,}394}$$

9) Media pesata:

$$\bar{x} = \frac{\sum \frac{x_i}{(\Delta x_i)^2}}{\sum \frac{1}{(\Delta x_i)^2}} \qquad \Delta x = \frac{1}{\sqrt{\sum \frac{1}{(\Delta x_i)^2}}}$$

$$\bar{x} = \frac{\frac{1{,}4}{(0{,}5)^2} + \frac{1{,}2}{(0{,}2)^2} + \frac{1}{(0{,}25)^2} + \frac{1{,}3}{(0{,}2)^2}}{\frac{1}{(0{,}5)^2} + \frac{1}{(0{,}2)^2} + \frac{1}{(0{,}25)^2} + \frac{1}{(0{,}2)^2}} = \frac{84{,}1}{70} = 1{,}2014$$

$$\Delta x = \frac{1}{\sqrt{70}} = 0{,}119 \simeq 0{,}12$$

$$\Rightarrow \quad \boxed{1{,}20 \pm 0{,}12}$$

10) Fit dei minimi quadrati

	x_i	y_i	Δy_i	x_i^2	$x_i y_i$
	0	-2,8	0,3	0	0
	1	-5,7	0,3	1	-5,7
	3	-9	0,3	9	-27
$\sum_i$	4	-17,5		10	-32,7

$N = 3$

$$A = \frac{(\sum y_i)(\sum x_i^2) - (\sum x_i)(\sum x_i y_i)}{\underline{\Delta}}$$

con $\underline{\Delta} = N(\sum x_i^2) - (\sum x_i)^2$

$$\underline{\Delta} = 3 \cdot 10 - (4)^2 = 30 - 16 = 14$$

$$A = \frac{-17,5 \cdot 10 - 4 \cdot (-32,7)}{14} = \frac{-175 + 130,8}{14} = -3,1571$$

$$(\Delta A)^2 = (\Delta y)^2 \frac{(\sum x_i^2)}{\underline{\Delta}} = (0,3)^2 \cdot \frac{10}{14} = 0,06428$$

$\Rightarrow \Delta A = 0,25$

$$\boxed{A = -3,16 \pm 0,25}$$

$$B = \frac{N(\sum x_i y_i) - (\sum x_i)(\sum y_i)}{\underline{\Delta}}$$

$$B = \frac{3 \cdot (-32,7) - 4 \cdot (-17,5)}{14} = \frac{-98,1 + 70}{14} = -2,007$$

$$(\Delta B)^2 = (\Delta y)^2 \frac{N}{\underline{\Delta}} = (0,3)^2 \cdot \frac{3}{14} = 0,01928$$

$\Rightarrow \Delta B = 0,139 \simeq 0,14$

$$\boxed{B = -2{,}01 \pm 0{,}14}$$

11)

x_i	y_i	$f(x_i)$	$y_i - f(x_i)$
0	$-2{,}8$	$-3{,}16$	$0{,}36$
1	$-5{,}7$	$-5{,}17$	$-0{,}53$
3	-9	$-9{,}19$	$0{,}19$

$f(x) = A + Bx$

$f(x) = -3{,}16 - 2{,}01\,x$

$$\chi_0^2 = \sum_i \frac{(y_i - f(x_i))^2}{(\Delta y)^2} = \frac{(0{,}36)^2 + (-0{,}53)^2 + (0{,}19)^2}{(0{,}3)^2}$$

$$\Rightarrow \quad \chi_0^2 = 4{,}962$$

Per calcolare il chi quadro ridotto, devo dividere per il n° di gradi di libertà

$$d = \underset{\text{n° di punti}}{3} - \underset{\text{ho calcolato A e B dai dati}}{2} = 1$$

$$\tilde{\chi}_0^2 = \frac{1}{d}\chi_0^2 = 4{,}962$$

Per giudicare la bontà dell'ipotesi fatta devo calcolare

$$P_1\left(\tilde{\chi}^2 \geq 4{,}962\right) \underset{\text{guardo sulle tavole}}{\cong} 2{,}5\,\%$$

Avendo posto la soglia del valore di accettazione al 5% si ha quindi che:

l'ipotesi <u>non è accettabile</u>.