1) $q = 42$

$$\Delta q = \Delta(x^2 y)$$

$$\frac{\Delta(x^2 y)}{(x^2 y)} = \sqrt{\left(2\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

$$\Rightarrow \Delta q = x^2 y \cdot \sqrt{4\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

$$\Delta q = 40 \cdot \sqrt{4 \cdot \left(\frac{0{,}1}{2}\right)^2 + \left(\frac{0{,}5}{10}\right)^2} = 4{,}47$$

$$\Rightarrow \boxed{q = 42 \pm 4}$$

2)

780 ± 50	$(7{,}8 \pm 0{,}5) \cdot 10^2$
$73{,}22 \pm 0{,}04$	$(7{,}322 \pm 0{,}004) \cdot 10^1$
572 ± 21	$(5{,}72 \pm 0{,}21) \cdot 10^2$
$82{,}347 \pm 0{,}017$	$(8{,}2347 \pm 0{,}0017) \cdot 10^1$
28700 ± 300	$(2{,}87 \pm 0{,}03) \cdot 10^4$
$387{,}12 \pm 0{,}08$	$(3{,}8712 \pm 0{,}0008) \cdot 10^2$

3) $P(A) = \frac{42}{100}$ $\quad P(B) = \frac{14}{100}$ $\quad P(C) = \frac{44}{100}$

Prob. che sia difettoso $P(F)$

$P(F|A) = \frac{5}{100}$ $\quad P(F|B) = \frac{4}{100}$ $\quad P(F|C) = \frac{10}{100}$

$$P(F) = P(F|A) \cdot P(A) + P(F|B) \cdot P(B) + P(F|C) \cdot P(C)$$

$$P(F) = \frac{5}{100} \cdot \frac{42}{100} + \frac{4}{100} \cdot \frac{14}{100} + \frac{10}{100} \cdot \frac{44}{100}$$

$$P(F) = \frac{210+56+440}{10000} = \frac{706}{10000} = 0{,}0706$$

$$\boxed{P(F) \cong 0{,}071}$$

4) Processo binomiale $p = \frac{1}{2}$ che un dado abbia uscita pari

$$\mu = Np \qquad \mu = 20 \cdot \frac{1}{2} \qquad \boxed{\mu = 10}$$

$$\sigma^2 = Np(1-p) = 20 \cdot \frac{1}{2} \cdot \frac{1}{2} \qquad \boxed{\sigma^2 = 5}$$

$$P(K=6) = \frac{N!}{K!\,(N-K)!} p^k (1-p)^{N-k} =$$

$$= \frac{20!}{6!\,14!} (0{,}5)^6 \cdot (0{,}5)^{14} = \frac{20!}{6!\,14!} (0{,}5)^{20} =$$

$$= 38760 \cdot (0{,}5)^{20} = 0{,}03696$$

$$\boxed{P \cong 0{,}037}$$

Modo alternativo di calcolare P: usando l'approssimazione gaussiana della binomiale

$$P = \int_{5,5}^{6,5} p(x)\,dx = \int_{z_1}^{z_2} g(z)\,dz \qquad z_1 = \frac{5{,}5-10}{\sqrt{5}} = -2{,}01$$

$$z_2 = \frac{6{,}5-10}{\sqrt{5}} = -1{,}56$$

$$= \int_{-2,01}^{-1,56} g(z)\,dz = \int_{1,56}^{2,01} g(z)\,dz = \int_0^{2,01} g(z)\,dz - \int_0^{1,56} g(z)\,dz =$$

$$= 0{,}4778 - 0{,}4406 = 0{,}0372 \quad \boxed{\cong 0{,}037} \quad \text{stesso risultato}$$

5) $\mu = 34000 \qquad \sigma = 4000$

$$P(x > 40000) = \int_{40000}^{+\infty} p(x)\,dx =$$

$$z_1 = \frac{40000 - 34000}{4000} = 1{,}5$$

$$= \int_{1,5}^{+\infty} g(z)\,dz = 0{,}5 - \int_{0}^{1,5} g(z)\,dz = 0{,}5 - 0{,}4332 = 0{,}0668$$

$$\boxed{P \cong 0{,}067}$$

$$P(30000 < x < 35000) = \int_{30.000}^{35000} p(x)\,dx =$$

$$z_1 = \frac{30.000 - 34000}{4000} \qquad z_2 = \frac{35000 - 34000}{4000}$$

$z_1 = -1$

$z_2 = 0{,}25$

$$= \int_{-1}^{0,25} g(z)\,dz = \int_{-1}^{0} g(z)\,dz + \int_{0}^{0,25} g(z)\,dz =$$

$$= \int_{0}^{1} g(z)\,dz + \int_{0}^{0,25} g(z)\,dz = 0{,}3413 + 0{,}0987 = 0{,}44$$

$$\boxed{P \simeq 0{,}44}$$

6) $$\int_{-\infty}^{+\infty} f(x) = 1$$

$$\Rightarrow \int_0^1 (x^2 + a)\,dx = \left[\frac{x^3}{3}\right]_0^1 + a\left[x\right]_0^1 = \frac{1}{3} + a$$

$$\frac{1}{3} + a = 1$$

$$a = 1 - \frac{1}{3} \qquad \boxed{a = \frac{2}{3}}$$

7) $$\mu = \int_{-\infty}^{+\infty} x\, f(x)\, dx = \int_0^1 x\left(x^2 + \frac{2}{3}\right) dx =$$

$$= \left[\frac{x^4}{4}\right]_0^1 + \frac{2}{3}\left[\frac{x^2}{2}\right]_0^1 = \frac{1}{4} + \frac{\cancel{2}}{3} \cdot \frac{1}{\cancel{2}} = \frac{1}{4} + \frac{1}{3} = \frac{4+3}{12}$$

$$\boxed{\mu = \frac{7}{12}}$$

$$\sigma^2 = E[x^2] - \mu^2$$

$$E[x^2] = \int_{-\infty}^{+\infty} x^2 f(x)\, dx = \int_0^1 x^2\left(x^2 + \frac{2}{3}\right) dx =$$

$$= \left[\frac{x^5}{5}\right]_0^1 + \frac{2}{3}\left[\frac{x^3}{3}\right]_0^1 = \frac{1}{5} + \frac{2}{3} \cdot \frac{1}{3} = \frac{1}{5} + \frac{2}{9} = \frac{9+10}{45}$$

$$= \frac{19}{45}$$

$$\Rightarrow \sigma^2 = \frac{19}{45} - \left(\frac{7}{12}\right)^2 = \frac{19}{45} - \frac{49}{144} = \frac{304 - 245}{720} =$$

$$\boxed{\sigma^2 = \frac{59}{720}}$$

8) $\sigma^2 = 16 \rightarrow \sigma = 4$ $\qquad N = 10$

di tutta la popolazione

$m = 50$

$$m - A\frac{\sigma}{\sqrt{N}} \leq \mu \leq m + A\frac{\sigma}{\sqrt{N}}$$

con $A = 1{,}96$ (livello di confidenza al 95%)

$$1{,}96 \cdot \frac{\sigma}{\sqrt{N}} = 1{,}96 \cdot \frac{4}{\sqrt{10}} = 2{,}48 \simeq 2{,}5$$

$$50 - 2{,}5 \leq \mu \leq 50 + 2{,}5$$

$$\boxed{47{,}5 \leq \mu \leq 52{,}5}$$

9) Media pesata

$$\bar{x} = \frac{\sum \frac{x_i}{\sigma_i^2}}{\sum \frac{1}{\sigma_i^2}} = \frac{\frac{3{,}3}{(0{,}5)^2} + \frac{3{,}2}{(0{,}2)^2} + \frac{3}{(0{,}25)^2}}{\frac{1}{(0{,}5)^2} + \frac{1}{(0{,}2)^2} + \frac{1}{(0{,}25)^2}} =$$

$$= \frac{13{,}2 + 80 + 48}{4 + 25 + 16} = \frac{141{,}2}{45} = 3{,}1378$$

$$\Delta x = \sqrt{\frac{1}{\sum \frac{1}{\sigma_i^2}}} = \sqrt{\frac{1}{45}} = 0{,}149 \simeq 0{,}15$$

$$\Rightarrow \boxed{3{,}14 \pm 0{,}15}$$

10) Fit dei minimi quadrati $N = 3$

	x_i	y_i	x_i^2	$x_i y_i$
	0	1,3	0	0
	0,5	5,5	0,25	2,75
	1,5	11	2,25	16,5
$\sum_i$	2	17,8	2,5	19,25

$$A = \frac{(\sum y_i)(\sum x_i^2) - (\sum x_i)(\sum x_i y_i)}{\Delta}$$

con $\Delta = N (\sum x_i^2) - (\sum x_i)^2 =$

$$= 3 \cdot 2{,}5 - (2)^4 = 7{,}5 - 4 = 3{,}5$$

$$A = \frac{17,8 \cdot 2,5 - 2 \cdot 19,25}{3,5} = \frac{6}{3,5} = 1,714$$

$$(\Delta A)^2 = (\Delta y)^2 \cdot \frac{\sum x_i^2}{\Delta} = (0,2)^2 \cdot \frac{2,5}{3,5} = 0,0286$$

$$\Delta A = \sqrt{0,0286}$$

$$\boxed{A = 1,71 \quad \pm 0,17}$$

$$B = \frac{N\left(\sum x_i y_i\right) - \left(\sum x_i\right)\left(\sum y_i\right)}{\Delta}$$

$$B = \frac{3 \cdot 19,25 - 2 \cdot 17,8}{3,5} = \frac{22,15}{3,5} = 6,328$$

$$(\Delta B)^2 = (\Delta y)^2 \frac{N}{\Delta} = (0,2)^2 \cdot \frac{3}{3,5} = 0,0343$$

$$\Delta B = 0,185 \simeq 0,19$$

$$\boxed{B = 6,33 \pm 0,19}$$

11) $f(x) = A + Bx \quad = 1,71 + 6,33 \cdot x$

x_i	y_i	$f(x_i)$	$y_i - f(x_i)$
0	1,3	1,71	−0,41
0,5	5,5	4,875	0,625
1,5	11	11,205	−0,205

$$\chi_0^2 = \sum_i \frac{(y_i - f(x_i))^2}{(\Delta y)^2} = \frac{(-0,41)^2 + (0,625)^2 + (-0,205)^2}{(0,2)^2} =$$

$$= \frac{0,60075}{0,04} = 15,02$$

Per calcolare il $\tilde{\chi}_0^2$ devo dividere per il numero di gradi di libertà

$$d = N - 2 = 1$$

$$\tilde{\chi}_0^2 = \frac{\chi_0^2}{d} = 15,02$$

Per giudicare la bontà dell'ipotesi fatta devo calcolare

$$P_1\left(\tilde{\chi}^2 \geq 15,02\right) < P_1\left(\tilde{\chi}^2 \geq 10\right) = 0,2\,\%$$

Avendo posto la soglia del valore di accettazione al 5% si ha che

$$P_1\left(\tilde{\chi}^2 \geq 15,02\right) \leq 0,2\,\% < 5\%$$

l'ipotesi NON È ACCETTABILE