Esame di metodi computazionali 10/2/2012 1

1) $S = ab + \pi r^2$

$S = (18,2 \cdot 123 + \pi\, 25^2)\ m^2 = 4202,1\ m^2$

$\Delta S = \sqrt{[\Delta(ab)]^2 + [\Delta(\pi r^2)]^2}$

$\frac{\Delta(ab)}{ab} = \sqrt{\left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta b}{b}\right)^2} = \sqrt{\left(\frac{0,5}{18,2}\right)^2 + \left(\frac{0,5}{123}\right)^2} = 0,4074$

$\Rightarrow \Delta(ab) = ab \cdot 0,4074 = (18,2 \cdot 123 \cdot 0,4074)\ m^2$

$\Delta(ab) = 912,0\ m^2$

$\frac{\Delta(\pi r^2)}{\pi r^2} = 2\frac{\Delta r}{r} = 2 \cdot \frac{0,5}{25} = 0,04$

$\Rightarrow \Delta(\pi r^2) = \pi r^2 \cdot 0,04 = (\pi \cdot 25^2 \cdot 0,04)\ m^2$

$\Delta(\pi r^2) = 78,5\ m^2$

$\Delta S = \sqrt{(912,0)^2 + (78,5)^2}\ m^2 = 915\ m^2 \simeq 900\ m^2$

$\boxed{S = (4200 \pm 900)\ m^2}$

$S = (4,2 \pm 0,9) \cdot 10^3\ m^2$

2)

18780 ± 190	$(1,878 \pm 0,019) \cdot 10^4$
990 ± 70	$(9,9 \pm 0,7) \cdot 10^2$
$13,23 \pm 0,09$	$(1,329 \pm 0,009) \cdot 10^1$
852 ± 17	$(8,52 \pm 0,17) \cdot 10^2$
$32,347 \pm 0,021$	$(3,2347 \pm 0,0021) \cdot 10^1$
$156,33 \pm 0,07$	$(1,5633 \pm 0,0007) \cdot 10^2$

3) $$\int_{-\infty}^{+\infty} p(x)\,dx = 1$$ assioma della completezza

$$\int_0^{\pi/2} a \sin x\,dx = a\left[-\cos x\right]_0^{\pi/2} = a(0+1) = a$$

$$\boxed{a = 1}$$

4) $$\mu = E[x] = \int_{-\infty}^{+\infty} x\,p(x)\,dx = \int_0^{\pi/2} \underbrace{x}_{g(x)} \cdot \underbrace{\sin x}_{f'(x)}\,dx =$$

$$= \left[x \cdot (-\cos x)\right]_0^{\pi/2} - \int_0^{\pi/2} 1 \cdot (-\cos x)\,dx =$$

avendo integrato per parti

$$= (0-0) + \int_0^{\pi/2} \cos x\,dx = \int_0^{\pi/2} \cos x\,dx = \left[\sin x\right]_0^{\pi/2} =$$

$$= 1 - 0 = 1 \qquad \boxed{\mu = 1}$$

5) $p = 0{,}3$

Binomiale $N = 5$, K = n° di studenti accettati

$$P(K \leq 2) = P(K=0) + P(K=1) + P(K=2)$$

$$P(K) = \frac{N!}{K!(N-K)!} p^K (1-p)^{N-K}$$

$$P(0) = \frac{\cancel{5!}}{0!\,\cancel{5!}} (0{,}3)^0 (1-0{,}3)^5 = (1-0{,}3)^5 = (0{,}7)^5 = 0{,}16807$$

$$P(1) = \frac{5!}{1!\,4!} (0{,}3)^1 (0{,}7)^4 = 5 \cdot 0{,}3 \cdot (0{,}7)^4 = 0{,}36015$$

$$P(2) = \frac{5!}{2!\,3!}(0{,}3)^2 \cdot (0{,}7)^3 = \frac{\cancel{0}}{2}(0{,}3)^2 \cdot (0{,}7)^3 = 0{,}3087$$ ③

$$\Rightarrow P = 0{,}16807 + 0{,}36015 + 0{,}3087 = 0{,}83692$$

$$\boxed{P \simeq 83{,}7\%}$$

6) $P_T = 0{,}5$ $\quad K$ = numero di teste $\quad N = 4$

Binomiale

$K = 3$, $K = 4$

$$P = P(K=3) + P(K=4)$$

$$P(3) = \frac{4!}{3!\,1!}(0{,}5)^3 \cdot (0{,}5) = 4 \cdot (0{,}5)^4$$

$$P(4) = \frac{\cancel{4!}}{\cancel{4!}\,0!}(0{,}5)^4 \cdot (0{,}5)^0 = (0{,}5)^4$$

$$P = 4(0{,}5)^4 + (0{,}5)^4 = 5 \cdot (0{,}5)^4 = 0{,}3125$$

$$\boxed{P \simeq 31{,}2\%}$$

7) $p = 0{,}005$

$N = 500 \qquad \mu = Np = 500 \cdot 0{,}005 = 2{,}5$

Possiamo approssimare la distribuzione binomiale con una poissoniana di media $\mu = 2{,}5$

$$P(K) = \frac{\mu^K}{K!} e^{-\mu}$$

$$P(A) = P(K=2) = \frac{\mu^2}{2!} e^{-\mu} = \frac{(2{,}5)^2}{2} \cdot e^{-2{,}5} = 3{,}125\, e^{-2{,}5} =$$

$$= 0{,}2565$$

$$\boxed{P(A) \cong 25{,}7\%}$$

$$P(B) = P(K>2) = 1 - P(0) - P(1) - P(2)$$

$$P(0) = \frac{2,5^0}{0!} e^{-2,5} = e^{-2,5}$$

$$P(1) = \frac{2,5^1}{1!} e^{-2,5} = 2,5\, e^{-2,5}$$

$$P(B) = 1 - e^{-2,5} - 2,5 e^{-2,5} - 3,125\, e^{-2,5} = 1 - 6,625\, e^{-2,5} =$$

$$= 0,4562$$

$$\boxed{P(B) \simeq 45,6\%}$$

8) $m = 1$ $\qquad N = 10$

$\sigma = 0,05$

di tutta la popolazione

$$m - A \frac{\sigma}{\sqrt{N}} \leq \mu \leq m + A \frac{\sigma}{\sqrt{N}}$$

con $A = 2,576$

$$A \frac{\sigma}{\sqrt{N}} = 2,576 \cdot \frac{0,05}{\sqrt{10}} = 0,0407 \simeq 0,04$$

$$1 - 0,04 \leq \mu \leq 1 + 0,04$$

$$\boxed{0,96 \leq \mu \leq 1,04}$$

9) In questo caso $S = 0,05$

$$m - t_{crit} \frac{S}{\sqrt{N}} \leq \mu \leq m + t_{crit} \frac{S}{\sqrt{N}}$$

con $t_{crit} = 3,250$

9 gradi di libertà

$$t_{crit} \cdot \frac{s}{\sqrt{N}} = 3{,}25 \cdot \frac{0{,}05}{\sqrt{10}} = 0{,}0514 \simeq 0{,}05$$

$$1 - 0{,}05 \leq \mu \leq 1 + 0{,}05$$

$$\boxed{0{,}95 \leq \mu \leq 1{,}05}$$

10) Metodo di fit dei minimi quadrati:

	x	y	x^2	xy
	0	2	0	0
	1	1,5	1	1,5
	5	-5	25	-25
	10	-10	100	-100
Σ	16	-11,5	126	-123,5

$N = 4$

$$A = \frac{\sum y \sum x^2 - \sum x \sum xy}{\Delta}$$

con $\Delta = N \sum x^2 - \left(\sum x\right)^2$

$$\Delta = 4 \cdot 126 - (16)^2 = 248$$

$$A = \frac{-11{,}5 \cdot 126 - 16 \cdot (-123{,}5)}{248} = \frac{-1461{,}6 + 1976}{248} = 2{,}074$$

$$(\Delta A)^2 = (\Delta y)^2 \cdot \frac{\sum x^2}{\Delta} = (0{,}5)^2 \cdot \frac{126}{248} = 0{,}1270$$

$$\Delta A = \sqrt{0{,}1270} = 0{,}356 \simeq 0{,}4$$

$$\boxed{A = 2{,}1 \pm 0{,}4}$$

$$B = \frac{N \sum xy - \sum x \sum y}{\Delta}$$

$$B = \frac{4 \cdot (-123{,}5) - 16 \cdot (-11{,}5)}{248} = \frac{-494 + 184}{248} = -1{,}25$$

$$(\Delta B)^2 = (\Delta y)^2 \cdot \frac{N}{\Delta} = (0{,}5)^2 \cdot \frac{4}{248} = 0{,}00403$$

$$\Delta B = 0{,}0635 \simeq 0{,}06$$

$$\boxed{B = -1,25 \pm 0,06}$$

11)

x	y	f(x)	y - f(x)
0	2	2,1	-0,1
1	1,5	0,85	0,65
5	-5	-4,15	-0,85
10	-10	-10,4	0,4

$f(x) = A + Bx$

$f(x) = 2,1 - 1,25\,x$

$$\chi_0^2 = \frac{\sum (y_i - f(x_i))^2}{(\Delta y)^2} = \frac{(-0,1)^2 + (0,65)^2 + (-0,85)^2 + (0,4)^2}{(0,5)^2} =$$

$$= \frac{1,315}{0,25} = 5,26$$

$d = 4 - 2$ gradi di libertà

$$\tilde{\chi}_0^2 = \frac{\chi_0^2}{d} = \frac{5,26}{2} = 2,63$$

$$P_2\left(\tilde{\chi}^2 \geq 2,63\right) \cong 7,4\%$$

L'ipotesi è accettabile