SOLUZIONI PROVA DI RECUPERO I° COMPITINO 

8/1/2010

1) $\ell = 10.0 \pm 0{,}2$ cm
$L = 500 \pm 2$ cm

n° di mattonelle per lato $= n$

$n = \dfrac{L}{\ell}$ con $n \pm \Delta n$ $\qquad n = \dfrac{500}{10} = 50$

$$\frac{\Delta n}{n} = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta \ell}{\ell}\right)^2}$$

$$\frac{\Delta n}{n} = \sqrt{\left(\frac{2}{500}\right)^2 + \left(\frac{0{,}2}{10}\right)^2} = 0{,}020$$

$$\Delta n = 50 \cdot 0{,}020 = 1{,}0$$

$$n = 50 \pm 1{,}0$$

Il numero di mattonelle necessario è quindi

$N = n^2$ con $N \pm \Delta N$ $\qquad N = 50^2 = 2500$

$$\frac{\Delta N}{N} = 2\,\frac{\Delta n}{n}$$

$$\frac{\Delta N}{N} = 2 \cdot 0{,}020 = 0{,}04$$

$$\Delta N = 2500 \cdot 0{,}04 = 100$$

$$\Rightarrow \boxed{2500 \pm 100}$$

2) Area stanza $= A_s = L^2$

$$\frac{\Delta A_s}{A_s} = 2\,\frac{\Delta L}{L} = 2 \cdot \frac{2}{500} = 0{,}008 \qquad \boxed{0{,}8\%}$$

Area mattonella $= A_m = \ell^2$

$$\frac{\Delta A_m}{A_m} = 2\frac{\Delta \ell}{\ell} = 2 \cdot \frac{0,2}{10} = 0,04 \qquad \boxed{4\%}$$

3) $B = 2xe^x$

$$\Delta B = \left|\frac{\partial B}{\partial x}\right| \cdot \Delta x$$

$$\Delta B = \left|2e^x + 2xe^x\right| \Delta x$$

$$\boxed{\Delta B = 2e^x \left|1+x\right| \Delta x}$$

4) $x = 2.3 \pm 0,1$

$B = 45,8812$

$\Delta B = 6,58 \simeq 7$

$\Rightarrow B \simeq 46 \qquad \boxed{46 \pm 7}$

5) $T_{best} = \frac{\Sigma T_i}{5} = 134,48 \ {}^\circ C \qquad = T_{medio}$

$\Delta T = \frac{\sigma}{\sqrt{N}}$ deviazione standard della media

$$\sigma = \sqrt{\frac{\Sigma (T_i - T_{medio})^2}{N-1}} = 0,2588$$

$$\Delta T = \frac{0,2588}{\sqrt{5}} = 0,1157 \simeq 0,12 \quad {}^\circ C$$

$$\boxed{(134,48 \pm 0,12)\ {}^\circ C}$$

③

6) $\Delta T_{tot} = \sqrt{(\Delta T_{stat})^2 + (\Delta T_{sist})^2}$

con $\Delta T_{stat} = 0,1157$ °C

e con $\frac{\Delta T_{sist}}{T} = 0,001 \Rightarrow \Delta T_{sist} = 0,13448$ °C

$\Delta T_{tot} = 0,1774$ °C $\simeq 0,18$ °C

$\boxed{(134,48 \pm 0,18)\ °C}$

7)

$74,847 \pm 0,004$

$512 \pm 17$

$2,13500 \pm 0,00008$

$55700 \pm 1300$

$61,48 \pm 0,23$

$35,13 \pm 0,05$

$5700 \pm 300$

$815 \pm 2$

8) $p = 0,3$ binomiale $N = 3$
$K = 1, 2, 3$

$1 - P(N=3, k=0) =$

$= 1 - \frac{3!}{0!\,3!}(0,3)^0 \cdot (0,7)^3 = 1 - (0,7)^3 =$

$= \boxed{0,657}$ $\approx 66\%$

9) $p = 0{,}6$

Lanciando due missili $N=2$

$$1 - P(N=2,\ k=0) = 1 - \frac{2!}{0!\,2!}(0{,}6)^0 \cdot (0{,}4)^2 =$$

$$= 1 - (0{,}4)^2 = 0{,}84$$ (è minore di 0,90)

Lanciando 3 missili $N=3$

$$1 - P(N=3,\ k=0) = 1 - \frac{3!}{0!\,3!}(0{,}6)^0 \cdot (0{,}4)^3 =$$

$$= 1 - (0{,}4)^3 = 0{,}94$$ ok.

quindi $\boxed{N=3}$

10) $3R + 4V + 5B = 12$ palline

$P(R) = \frac{3}{12} = \frac{1}{4}$ prob. che estraendo una pallina, questa sia Rossa

$P(V) = \frac{4}{12} = \frac{1}{3}$ " " Verde

$P(B) = \frac{5}{12}$ " " Blu

$$P(E_1) = P(R) \cdot P(R) \cdot P(R) = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$$

$$\boxed{P(E_1) = \frac{1}{64}}$$

$$P(E_2) = P(R)P(V)P(B) = \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{5}{12} = \frac{5}{144}$$

$$\boxed{P(E_2) = \frac{5}{144}}$$

11) $\mu = 1,8$

$\sigma^2 = 0,16 \quad \rightarrow \quad \sigma = 0,4$

$$1 - \int_{1,74}^{1,86} p(x)\,dx = 1 - \int_{z_1}^{z_2} g(z)\,dz$$

con $z_1 = \dfrac{1,74 - 1,8}{0,4} = -0,15$

$z_2 = \dfrac{1,86 - 1,8}{0,4} = 0,15$

$$= 1 - 2\int_0^{0,15} g(z)\,dz = 1 - 2 \cdot 0,0596 =$$

$= \boxed{0,881} \qquad \approx 88\%$

12) Deve essere

$$\int_0^{\pi} f(x)\,dx = 1$$

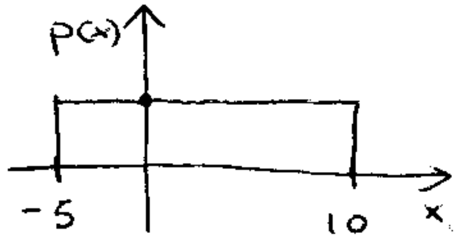
$$\int_0^{\pi} a\,(2x - \cos x)\,dx = 2a\left[\frac{x^2}{2}\right]_0^{\pi} - a\left[\sin x\right]_0^{\pi} =$$

$$= a\pi^2 - a \cdot \cancel{0} = a\pi^2$$

$a\pi^2 = 1$

$\boxed{a = \dfrac{1}{\pi^2}}$

13)




$p(x) = K$

con $K = \frac{1}{15}$

$$\mu = \frac{-5+10}{2} = \frac{5}{2} = \boxed{2,5}$$

$$\sigma^2 = E[(x-\mu)^2] = \int_{-5}^{10} (x-2,5)^2 \cdot p(x)\,dx =$$

$$= \int_{-5}^{10} (x-2,5)^2 \cdot \frac{1}{15}\,dx = \frac{1}{15}\int_{-5}^{10} (x^2 - 5x + 6,25)\,dx =$$

$$= \frac{1}{15}\left[\frac{x^3}{3}\right]_{-5}^{10} - \frac{1}{3}\left[\frac{x^2}{2}\right]_{-5}^{10} + \frac{6,25}{15}\cdot[x]_{-5}^{10} =$$

$$= \frac{1}{15}\left(\frac{1000}{3} + \frac{125}{3}\right) - \frac{1}{6}(100-25) + \frac{6,25}{\cancel{15}}\cdot\cancel{15} =$$

$$= \frac{\cancel{1125}^{25}}{\cancel{45}_1} - \frac{25}{2} + 6,25 = 25 - \frac{25}{2} + 6,25 =$$

$$= \boxed{18.75}$$

$$\boxed{\text{nota: } \sigma^2 = \frac{(10-(-5))^2}{12} = \frac{15^2}{12} = 18.75}$$

14) Approssimazione Gaussiana della binomiale di media

$\mu = Np$ $\qquad\qquad N = 200$, $p = 0,5$

$\rightarrow \mu = 100$

e varianza $\sigma^2 = Np(1-p) = 200(0,5)^2 = 50$

$\rightarrow \sigma = \sqrt{50} = 7,07$

$$P = \int_{99,5}^{100,5} p(x)\,dx = \int_{z_1}^{z_2} g(z)\,dz$$

con $z_1 = \dfrac{99,5-100}{7,07} = -0,07$

$z_2 = \dfrac{100,5-100}{7,07} = 0,07$

$$P = 2\int_0^{0,07} g(z)\,dz = 2 \cdot 0,0279 = \boxed{0,0558}$$

$\approx 5,6\%$

15) 4 persone/minuto

$\mu = 2$ in mezzo minuto

$$1 - P(\phi \text{ clienti}) = 1 - e^{-2} \cdot \frac{2^0}{0!} = 1 - e^{-2} =$$

$$= \boxed{0,864}$$

16) $\mu = 20$ in due settimane

$$e^{-20} \cdot \frac{20^{12}}{12!} = \frac{8,44249 \cdot 10^6}{4,79002 \cdot 10^8} = 1,762 \cdot 10^{-2} =$$

$$= \boxed{0,0176}$$

Si può anche risolvere utilizzando l'approssimazione gaussiana con $\sigma = \sqrt{20}$, $\mu = 20$

$$\int_{11,5}^{12,5} p(x)\,dx = \int_{z_1}^{z_2} g(z)\,dz$$

con $z_1 = -1,90$
$z_2 = -1,68$