Scuola di Dottorato Leonardo da Vinci – a.a. 2012/13

LASER: CARATTERISTICHE, PRINCIPI FISICI, APPLICAZIONI

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Part 6
From amplifier to oscillator: the laser cavity

OUTLOOK

- We have seen how a quantum system, that is matter with discrete energy levels accessible through interaction with photons, can amplify radiation
- Moreover, such amplification involves stimulated emission, hence is in principle well in agreement with the mechanisms leading to coherence (stimulated emitted photons are ideally indistinguishable each other!)
- However, the laser is not an amplifier, but rather a source, that is an **oscillator**:
  - ✓ Need for **optical cavities**
  - ✓ Need to balance between gain and loss
  - ✓ Need to analyze the optical properties (longitudinal and transverse modes)

Main objective: to describe the optical cavity, a key ingredient for laser operation

Secondary objective: to go deeper into the laser technologies and discuss motivations and consequences of technical choices pertaining to laser operation

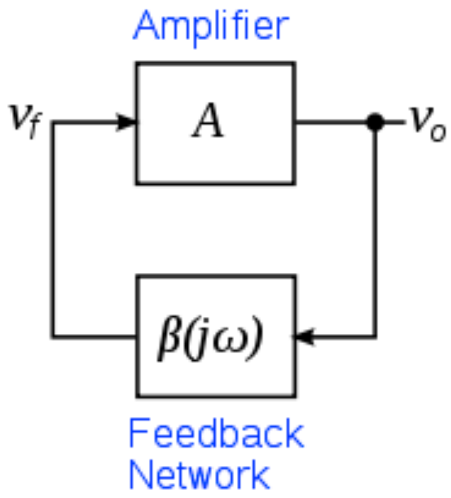
AMPLIFIER→ OSCILLATOR

Vivid example: electronics
amplifier → oscillator through application of a positive feedback

Feedback oscillator [edit]

The most common form of linear oscillator is an electronic amplifier such as a transistor or op amp connected in a feedback loop with its output fed back into its input through a frequency selective electronic filter to provide positive feedback. When the power supply to the amplifier is first switched on, electronic noise in the circuit provides a signal to get oscillations started. The noise travels around the loop and is amplified and filtered until very quickly it becomes a sine wave at a single frequency.

Feedback oscillator circuits can be classified according to the type of frequency selective filter they use in the feedback loop:[2][3]



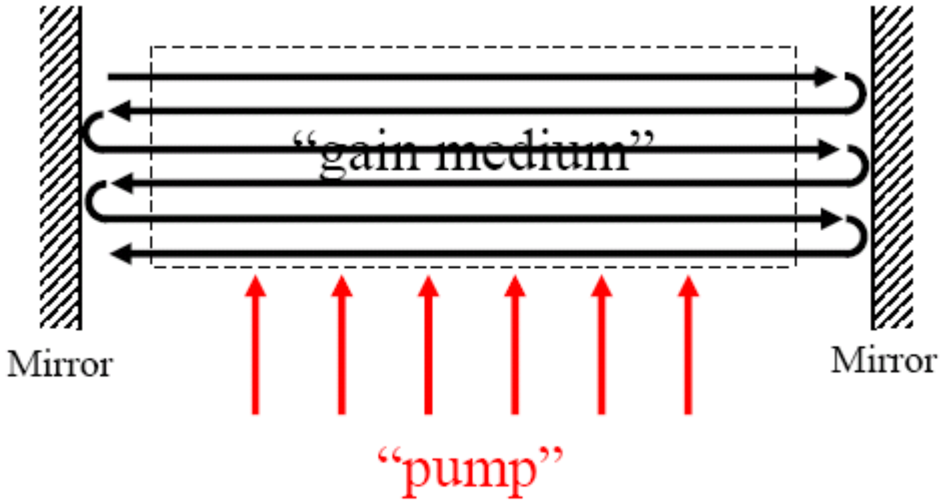
Block diagram of a feedback linear oscillator; an amplifier A with its output v_o fed back into its input v_f through a filter, $\beta(j\omega)$.

(Self) oscillation is obtained by re-sending back to the input part of the (amplified) output

- There is **freqency selection**
- Some kind of "trigger" is needed to start the oscillation

FEEDBACK AND OPTICAL CAVITY

In optics, feedback can be achieved in a very straightforward (apparently…) way:
the active medium is placed within a pair (at least) of mirrors



The set of components providing with optical feedback (intentional!) is called optical resonator or optical cavity

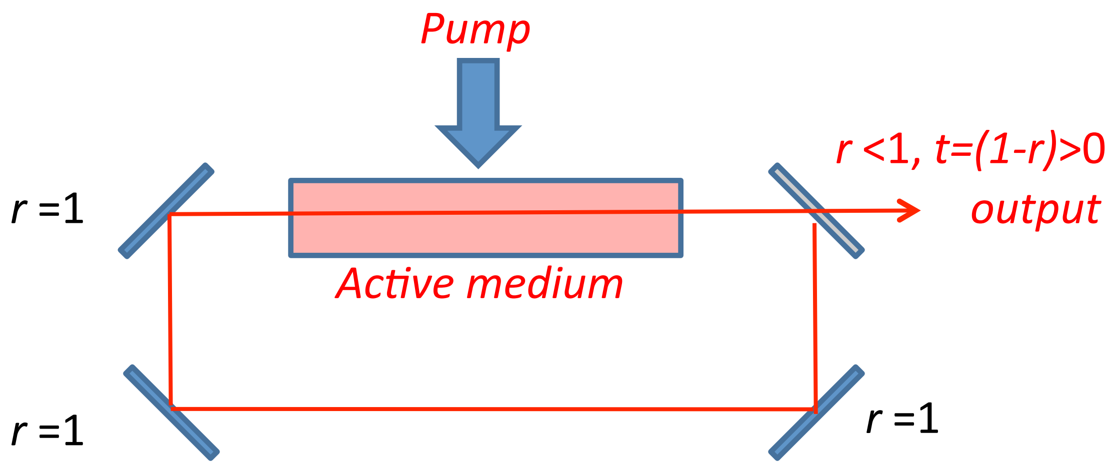
Note: the presence of the cavity will eventually affect the frequency response of the oscillator, that is the wavelength emitted by the laser, as well as its optical properties

The optical cavity owns a primarily "longitudinal" character as we have already discussed for the photon in the box and the quantum well systems, both treated as one-dimensional (i.e., longitudinal) problems

However, the transversal behavior is important as well

THE SIMPLEST LASER CAVITY

Remember: the cavity must allow for photon extraction if we want the laser photons to be used!
The simplest way to model the cavity (model, not realization!) is the so-called ring cavity:
4 mirrors forming a ring, one of them partially transmitting (reflectivity < 1)



Roundtrip time $\tau \sim L_{tot}/c$
(L_{tot} total optical length, assuming a refractive index $n \sim 1$)

(*r :* mirror reflectivity, that is the ratio between incident and reflected power; virtually, it can be decided when the mirror is produced)

Trigger of the laser action:
a "fluctuation", e.g., a *spontaneously* emitted photon going *by chance* along the cavity, i.e., along the correct path

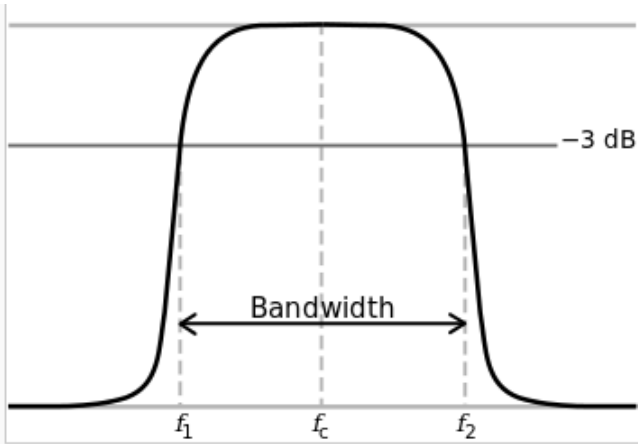
We can assume a constant rate for energy losses from the cavity due to photon leaving the cavity itself to be used as laser photons

$$\frac{dE}{dt} = -E\alpha \rightarrow E(t) = E_0 e^{-\alpha t}$$

E : e.m. energy within the cavity
α : rate of energy loss from the cavity

Q-FACTOR OF THE RESONATOR

In physics and engineering the **quality factor** or **Q factor** is a dimensionless parameter that describes how under-damped an oscillator or resonator is,[1] or equivalently, characterizes a resonator's bandwidth relative to its center frequency.[2] Higher *Q* indicates a lower rate of energy loss relative to the stored energy of the resonator; the oscillations die out more slowly. A pendulum suspended from a high-quality bearing, oscillating in air, has a high *Q*, while a pendulum immersed in oil has a low one. Resonators with high quality factors have low damping so that they ring longer.



The bandwidth, Δf, or f_1 to f_2, of a damped oscillator is shown on a graph of energy versus frequency. The Q factor of the damped oscillator, or filter, is $f_c/\Delta f$. The higher the Q, the narrower and 'sharper' the peak is.

Definition:
Quality-factor: $Q = 2\pi\, E_{stored} / E_{lost\ \ per\ cycle}$

The higher the Q-factor,
the sharper the frequency response

Assuming constant loss rate:
per each roundtrip the energy stored into the cavity decreases by a fraction $\alpha\tau$
per each frequency cycle the decrease will be $\alpha T = \alpha/\nu$

$Q_{roundtrip} = 2\pi c/(\alpha L_{tot})$
$Q_{cycle} = 2\pi\nu/\alpha = \omega/\alpha$

The Q-factor is inversely proportional to the loss rate
Physically, the larger is the dissipation, the lower is the quality and the broader is the bandwidth

IDEAL CAVITY AND LOSSES

e.m. energy within a cavity can be written as: $E_{stored} = SL_{tot}u_\nu$ *(with u_ν e.m. energy density, S cross section of the cavity, i.e., of the beam)*

The intensity (i.e., the module of Poyting vector) onto the output cavity mirror will be:

$I = Scu_\nu$ *(with c speed of light, $n \approx 1$ is assumed)*

Intensity lost at the output:

$I' = Scu_\nu(1-r)$ *(with $r < 1$ mirror reflectivity)*

Energy lost per cycle, i.e., in the period T:

$E_{lost} = I'\,T = I'\,\lambda/c = Scu_\nu(1-r)\lambda/c = S\lambda u_\nu(1-r)$

Examples:
L = 50 cm, r = 0.98, λ = 0.6 μm → Q ~ 2×10^7 (HeNe)
L = 0.5 mm, r = 0.30, λ = 0.8 μm → Q ~ 6×10^3 (diode)

Q-factor of the risonator:
$Q = E_{stored}/E_{lost} = 2\pi SL_{tot}u_\nu/((1-r)S\lambda u_\nu) = 2\pi(L_{tot}/\lambda)/(1-r)$

The Q-factor is limited by the need of taking out photons from the cavity
In *ideal cavities*, the main dependence is on the mirror reflectivity (and on cavity length)
⇓
Q-factor depends on the optical and geometrical features of the cavity

Qualitatively: to get lasing action ***gain > losses***
(the Q-factor is not limiting the lasing action in high gain active media)

GAIN AND LOSSES IN THE THREE LEVEL SYSTEM

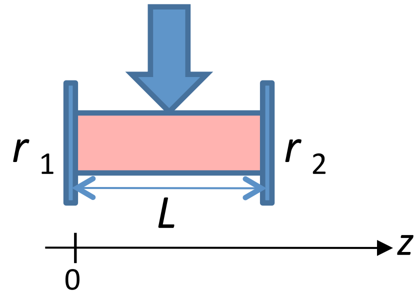
Let's now look for a quantitative expression of the gain vs losses concept
We will use the three level system whose master equations we have already mentioned

Gain of the three level medium

$$G = \frac{R\left(2A_{32}+A_{21}\right)-A_{21}\left(A_{31}+A_{32}\right)}{A_{21}\left[A_{31}+A_{32}\right]}\left(\frac{c}{n}\right)\frac{N\,\hbar\omega\,B_{21}\,F(\omega)}{V}$$

A_{32}>>A_{21}
R : pumping rate
F(ω): lineshape (we will see!)

For $I_o << I_{crit}$	$I(z,\omega)=I_o\exp(G\,z)$	"Inverse Beer's Law"
For $I_o >> I_{crit}$	$I(z,\omega)=I_o+I_{crit}\,G\,z$	Linear regime



Consistency condition: at the end of the cavity *(z=L)* I want to retrieve *at least* the intensity I_0 present at the beginning of the cavity --> **threshold in the gain**

$$I_o\exp(-\alpha\,2L)\exp(G_{th}\,2L)\,r_1\,r_2=I_o \qquad \text{[V-9a]}$$

or

$$\boxed{G_{th}=\alpha+\frac{1}{2L}\ln\left(\frac{1}{r_1\,r_2}\right)} \qquad \text{[V-9b]}$$

where α is included to account for residual cavity losses and the r_i 's are the respective reflectivity's of the two mirrors.

Here considered:
two mirror cavity with reflect. r_1,r_2 and length *L*
Further losses assumed with rate α

A **threshold** value is found in the gain in order to allow for consistency

GAIN AND LOSSES II

$$I_o \approx r_1\, r_2\, [I_o + I_{crit}\, \mathcal{G}\, 2L]$$

$$\Downarrow$$

$$I_o \approx 2 I_{crit}\, \mathcal{G}\, L \left[\frac{1}{r_1\, r_2} - 1\right]^{-1}$$

***At large intensity ($I_0 >> I_{crit}$), that is well above threshold:* the "linear regime" applies**

Let's now transfer the threshold from the gain *G* on the pumping rate *R*

We can write:

$$\frac{\mathcal{G}}{\mathcal{G}_{th}} = \frac{R - R_o}{R_{th} - R_o}$$

With:

$$R_o = \frac{A_{21}\,(A_{31} + A_{32})}{(2A_{32} + A_{21})}$$

Losses due to the spontaneous emission!

$$R_{th} - R_o = \mathcal{G}_{th}\, \frac{A_{21}\,[A_{31} + A_{32}]}{(2A_{32} + A_{21})} \left(\frac{n}{c}\right) \frac{V}{N\, \hbar\omega\, B_{21}\, F(\omega)}$$

Therefore Equation [V-10b] becomes

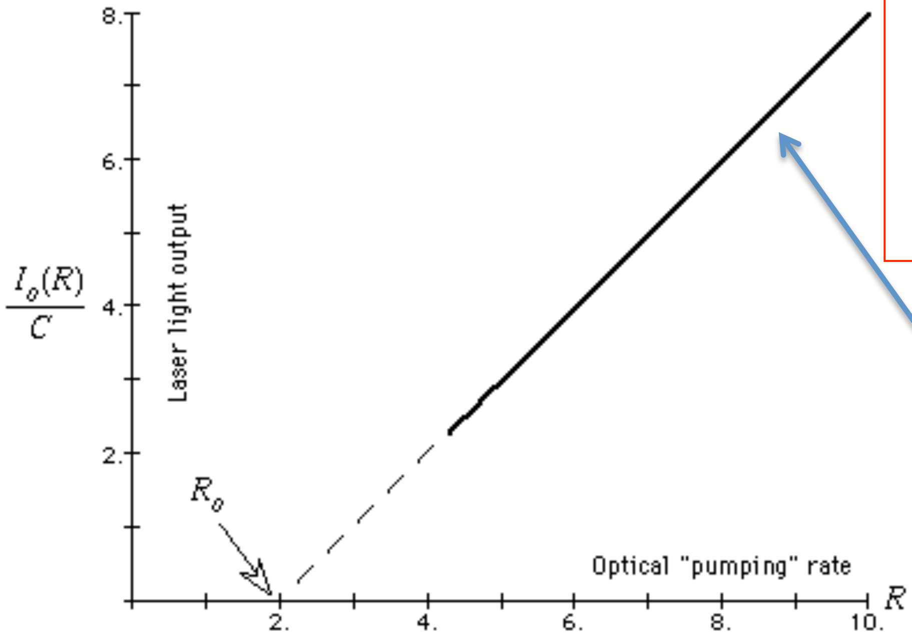
$$I_o(R) \approx \frac{2 I_{crit}\, \mathcal{G}_{th}\, L}{\left[\frac{1}{r_1\, r_2} - 1\right]} \left\{\frac{R - R_o}{R_{th} - R_o}\right\} = \left\{\frac{2 I_{crit}\, \mathcal{G}_{th}\, L}{\left[\frac{1}{r_1\, r_2} - 1\right][R_{th} - R_o]}\right\}(R - R_o) \equiv C\,(R - R_o).$$

e.m. intensity (at the beginning or the end of the cavity, thanks to consistency)

Depends on the cavity design and on the active medium

The intensity depends on pumping, but a *threshold* exists

THRESHOLD FOR LASER EMISSION

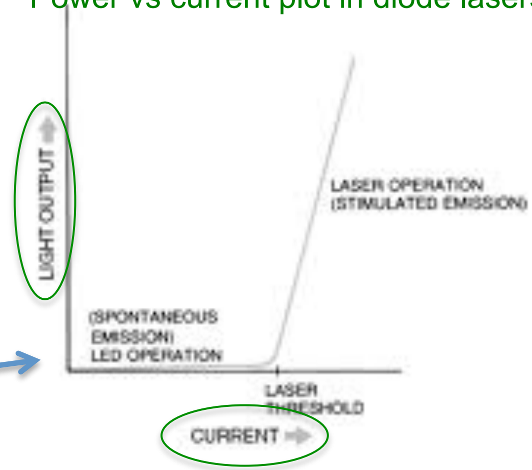


Gain must surpass losses
The output intensity depends on pumping rate *R*

Lasing action possible only *above a threshold*

Note: the linear behavior tends to saturate, at some point, in real laser systems

Power vs current plot in diode lasers



Note:
the shown behavior assumes the ability to externally control the pumping rate *R* . This is not always possible, but in the cases where it is possible, the appearence of the threshold is clearly seen (e.g., in diode lasers by increasing the injected current)

GAIN AND FREQUENCY

Once more we recall the gain of the active medium (in the three level system):

$$\mathcal{G} = \frac{R(2A_{32} + A_{21}) - A_{21}(A_{31} + A_{32})}{A_{21}[A_{31} + A_{32}]} \left(\frac{c}{n}\right) \frac{N \hbar \omega B_{21} F(\omega)}{V}$$

Lineshape of the active medium

The gain of the active medium is a **function of the frequency**

<u>Assuming</u> an ideal quantum system, the energy conservation involved in the stimulated emission would lead to a lineshape strongly peaked around the transition frequency ("delta"-like behavior)

In real systems the energy conservation must be "relaxed"

The transition "lines" get broader!!

(By the way, we saw this in classical terms, remember that the Lorentzian-like lineshape in absorption was broad, with a width proportional to the friction (viscous) coefficient)

Main mechanisms for "line broadening":
- Homogeneous broadening (all elements of the system behave statistically in the same way)
- Inhomogeneous broadening (every element behaves in its own way and the final result implies average)

HOMOGENEOUS BROADENING

Let's consider the simplest system, i.e., an atom in a gas (most results will hold true in other systems, as well)

We mentioned that all excited states have a natural lifetime τ_{sp} ultimately ruling the spontaneous emission ($\tau_{sp} = 1/A_{21}$ for a two level system)

According to Heisenberg: $\Delta\omega\Delta\tau_{sp} \geq 2\pi$ $\rightarrow$ $\Delta\omega \geq 2\pi/\tau_{sp}$ (that is $\Delta\nu \geq 1/\tau_{sp} \neq 0$)

(typical values for an atomic gas and electric dipole transitions: $\Delta\nu \approx 0.1$-1 GHz)

More realistic picture, accounting for **relaxation** processes (typically, **non radiative**):

$1/\tau_{eff} = 1/\tau_{sp} + 1/\tau_{nonrad}$ $\rightarrow$ $\Delta\omega \sim 1$-5 GHz, and even more

Note: non radiative de-excitation processes are frequently associated with collisions
Collisional rate depends also on the density (and temperature)

$\Downarrow$

In active media at the solid or liquid state very short non radiative lifetimes can be esaily found leading to remarkable broadening (up to tens of GHz, at room temperature!)

Other homogeneous broadening effects may occur, related for instance to the saturation effects ($I >> I_{sat}$) or to finite interaction times, and so on

3.2 Doppler Width

Generally the Lorentzian-line profile with the natural linewidth $\delta\nu_n$, as discussed in the previous section, cannot be observed without special techniques, because it is completely concealed by other broadening effects. One of the major contributions to the spectral linewidth in gases at low pressures is the Doppler width, which is due to the thermal motion of the absorbing or emitting molecules.

Consider an excited molecule with a velocity $\mathbf{v} = \{v_x, v_y, v_z\}$ relative to the rest frame of the observer. The central frequency of a molecular emission line that is ω_0 in the coordinate system of the molecule, is Doppler shifted to

$$\omega_e = \omega_0 + \mathbf{k}\cdot\mathbf{v} \tag{3.38}$$

for an observer looking towards the emitting molecule (that is, against the direction of the wave vector $\mathbf{k}$ of the emitted radiation; Fig.3.6a). The apparent emission frequency ω_e is increased if the molecule moves towards the observer ($\mathbf{k}\cdot\mathbf{v} > 0$), and decreased if the molecule moves away ($\mathbf{k}\cdot\mathbf{v} < 0$).

Similarly, one can see that the absorption frequency ω_0 of a molecule moving with the velocity $\mathbf{v}$ across a plane EM wave $\mathbf{E} = \mathbf{E}_0\exp(i\omega t - \mathbf{k}\cdot\mathbf{r})$ is shifted. The wave frequency ω in the rest frame appears in the frame of the moving molecule as

$$\omega' = \omega - \mathbf{k}\cdot\mathbf{v}\ .$$

The molecule can only absorb if ω' coincides with its eigenfrequency ω_0. The absorption frequency $\omega = \omega_a$ is then

$$\omega_a = \omega_0 + \mathbf{k}\cdot\mathbf{v}\ . \tag{3.39a}$$

As in the emission case the absorption frequency ω_a is increased for $\mathbf{k}\cdot\mathbf{v} > 0$ (Fig.3.6b). This happens, for example, if the molecule moves parallel to the

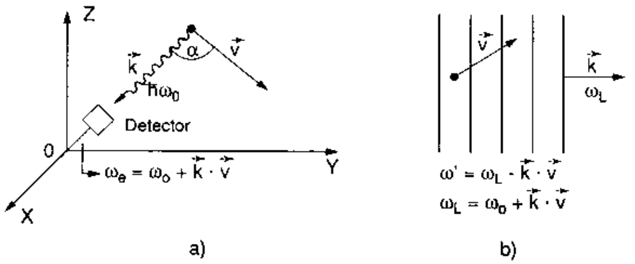


Fig.3.6. (a) Doppler shift of a monochromatic emission line and (b) absorption line

wave propagation. It is decreased if $\mathbf{k}\cdot\mathbf{v} < 0$; e.g., when the molecule moves against the light propagation. If we choose the +z direction to coincide with the light propagation, (3.39a) becomes with $\mathbf{k} = \{0,0,k_z\}$ and $|k| = 2\pi/\lambda$,

$$\omega_a = \omega_0(1 + v_z/c)\ . \tag{3.39b}$$

Note: Eqs.(3.38 and 39) describe the *linear* Doppler shift. For higher accuracies the quadratic Doppler effect has to be considered in addition (Sect.14.1).

At thermal equilibrium, the molecules of a gas follow a Maxwellian velocity distribution. At the temperature T, the number of molecules $n_i(v_z)dv_z$ in the level E_i per unit volume with a velocity component between v_z and v_z+dv_z is

$$n_i(v_z)dv_z = \frac{N_i}{v_p\sqrt{\pi}} e^{-(v_z/v_p)^2}\, dv_z\ , \tag{3.40}$$

where $N_i = \int n_i(v_z)dv_z$ is the density of all molecules in level E_i, $v_p = (2kT/m)^{1/2}$ is the most probable velocity, m is the mass of a molecule, and k is Boltzmann's constant. Inserting the relation (3.39b) between velocity component and frequency shift with $dv_z = (c/\omega_0)d\omega$ into (3.40) gives the number of molecules with absorption frequencies shifted from ω_0 into the interval from ω to $\omega+d\omega$

$$n_i(\omega)d\omega = N_i\,\frac{c}{\omega_0 v_p\sqrt{\pi}}\exp\left[-\left(\frac{c(\omega-\omega_0)}{\omega_0 v_p}\right)^2\right] d\omega. \tag{3.41}$$

Since the emitted or absorbed radiant power $P(\omega)d\omega$ is proportional to the density $n_i(\omega)d\omega$ of molecules emitting or absorbing in the intervall $d\omega$, the intensity profile of a Doppler-broadened spectral line becomes

$$I(\omega) = I_0\exp\left[-\left(\frac{c(\omega-\omega_0)}{\omega_0 v_p}\right)^2\right]. \tag{3.42}$$

This is a Gaussian profile with a full halfwidth

$$\delta\omega_D = 2\sqrt{\ln 2}\,\omega_0 v_p/c = \left(\frac{\omega_0}{c}\right)\sqrt{8kT\ln 2/m}\ , \tag{3.43a}$$

which is called the *Doppler width*. Inserting (3.43) into (3.42) yields with $1/(4\ln 2) = 0.36$

$$\boxed{I(\omega) = I_0\exp\left(-\frac{(\omega-\omega_0)^2}{0.36\,\delta\omega_D{}^2}\right).} \tag{3.44}$$

INHOMOGENEOUS BROADENING II

Note that $\delta\omega_D$ increases linearly with the frequency ω_0 and is proportional to $(T/m)^{1/2}$. The largest Doppler width is thus expected for hydrogen (M = 1) at high temperatures and a large frequency ω for the Lyman α line.

Equation (3.43) can be written more conveniently in terms of the Avogadro number N_A (the number of molecules per mole), the mass of a mole, $M = N_A m$, and the gas constant $R = N_A k$. Inserting these relations into (3.43) gives for the Doppler width

$$\delta\omega_D = (2\omega_0/c)\sqrt{2RT\ln 2/M}\ . \qquad (3.43b)$$

or in frequency units, using the values for c and R,

$$\delta\nu_D = 7.16\cdot 10^{-7}\,\nu_0\sqrt{T/M}\quad [\mathrm{Hz}]\ . \qquad (3.43c)$$

$\Delta\nu$ [Hz] ≅ $7.16\mathrm{x}10^{-7}\ c/\lambda\ \sqrt{T\,[\mathrm{K}]/M\,[\mathrm{a.m.u.}]}$

Example 3.2

a) Vacuum ultraviolet: For the Lyman α line (2p → 1s transition in the H atom) in a discharge with temperature T = 1000 K, M = 1, λ = 1216 Å, $\nu_0 = 2.47\cdot 10^{15}\ \mathrm{s}^{-1} \rightarrow \delta\nu_D = 5.6\cdot 10^{10}$ Hz, $\delta\lambda_D = 2.8\cdot 10^{-2}$ Å.

b) Visible spectral region: For the sodium D line (3p → 3s transition of the Na atom) in a sodium-vapor cell at T = 500 K, λ = 5891 Å, $\nu_0 = 5.1\cdot 10^{14}\ \mathrm{s}^{-1} \rightarrow \delta\nu_D = 1.7\cdot 10^{9}$ Hz, $\delta\lambda_D = 1\cdot 10^{-2}$ Å.

c) Infrared region: For a vibrational transition $(J_i, v_i) \leftrightarrow (J_k, v_k)$ between two rovibronic levels with the quantum numbers J, v of the CO_2 molecule in a CO_2 cell at room temperature (T = 300K), λ = 10 μm, $\nu = 3\cdot 10^{13}\ \mathrm{s}^{-1}$, M = 44 → $\delta\nu_D = 5.6\cdot 10^{7}$ Hz, $\delta\lambda_D$ = 0.19 Å.

Inhomogeneous broadening dominates when the absorbing species are in (almost free) motion, such as in gas

Typically $\Delta\nu$ ~ several GHz (at room temperature)

Doppler effect produces a shift in the resonant frequency of each single species of the sample depending on the velocity

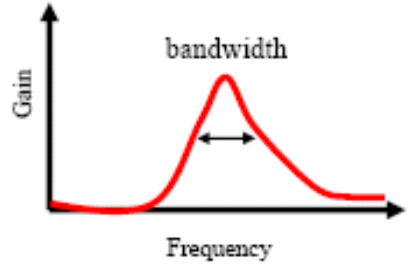
- At the thermal equilibrium the velocity is distributed according to Maxwell-Boltzmann
- A Gaussian lineshape is typical in inhomogeneous broadening

EXAMPLE OF WIDTHS FOR COMMON ACTIVE MEDIA

Typical gain bandwidths:

Full-width at half-maximum (FWHM) in typical operating conditions

Gain medium	Wavelength	Bandwidth (Hz)
HeNe	6328 Å	$1.5\ 10^9$
Nd:glass	1.06 μm	$3\ 10^{12}$
Nd:YAG	1.06 μm	$120\ 10^9$
Ruby	6943 Å	$6\ 10^{10}$
Argon ions	350 – 520 Å	$3.5\ 10^9$
Ti:Saphire	0.7-1.1 μm	$100\ 10^{12}$
Rh.-6G dye	0.56-0.64 μm	$5\ 10^{12}$
CO_2 gas	10 μm	$60\ 10^6$
Er doped fiber	1.55 μm	$4\ 10^{12}$
Diode lasers	0.7-1.6 μm	10^{13}



Typically, gas media have narrower bandwidth (GHz) than liquid or solid state (up to THz)

In solid/liquids the broadening may be due to intense non radiative de-excitation or to the appearence of "bands" rather than discrete levels (we'll see more)

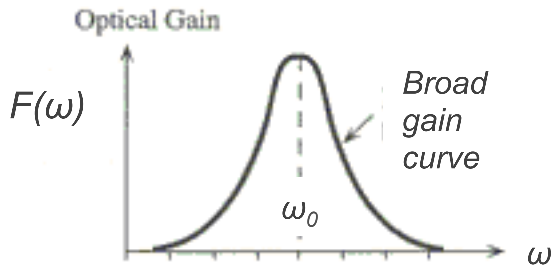
The large bandwidth of some active media (e.g., Ti:Sa, dye solution, diode lasers) open the possibility to *tune* the emission on a large range

Remember! Since $\nu = c/\lambda$ (in the vacuum),
$\Delta\nu = \Delta\lambda\ (c/\lambda^2)$ --> $\Delta\lambda = \lambda\ \Delta\nu/\nu$

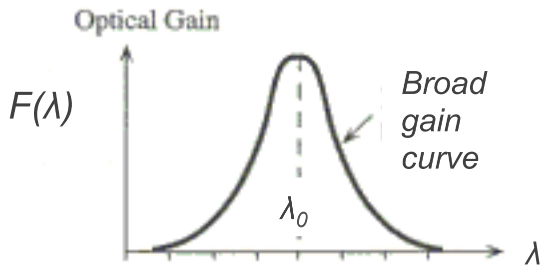
INTERPLAY BETWEEN ACTIVE MEDIUM AND CAVITY

Because of unavoidable broadening effects (e.g., in gases) or thanks to the use of specific active media (e.g., liquid or solid-state), *the active medium can amplify in a rather broad frequency range*

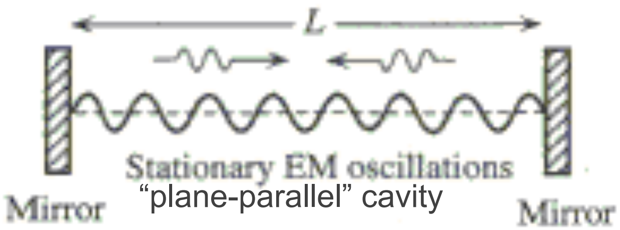
A characteristic optical ***gain curve (or linewidth, or lineshape)*** *$F(\omega)$, or $F(\lambda)$, appears*



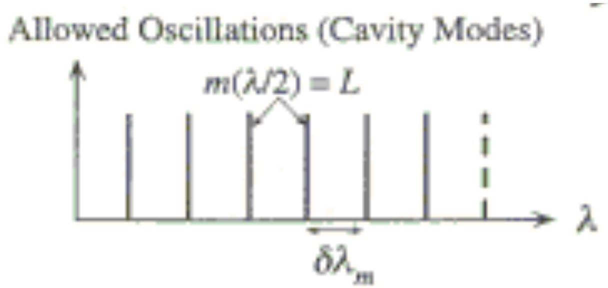
Or (the shape will not always be the same, since $\lambda = c/(2\pi\omega)$)



This will "compete" with the allowed modes supported by the cavity!!



Remember:
$k_m = m\pi/L$
$m\lambda/2 = L$



LONGITUDINAL MODES OF THE CAVITY

Modi lungitudinali

Longitudinal modes involve light waves which travel parallel to the laser cavity axis. When such a wave is reflected from a cavity mirror, the reflected wave combines with the incident wave to give rise to a standing or stationary wave as indicated in Figure 5.1. To simplify the diagram, the amplifying medium has been omitted and, provided that it can sustain several modes of oscillation at the same time, it is not necessary to consider it to understand the origin of multiple longitudinal modes. Amplifying media that can support the simultaneous oscillation of a number of longitudinal modes are referred to as being inhomogeneously broadened.

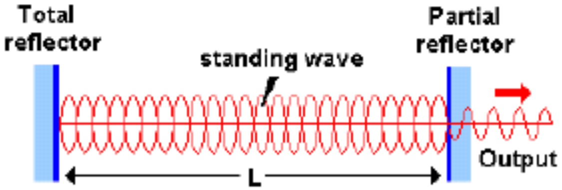


Figure 5.1.

Standing waves can be sustained within the laser cavity only if the length of the cavity is equal to an integral number of half wavelengths. This condition can be written as:

$$nL = s\,\lambda/2, \text{ con } s \text{ intero} \qquad (5.1)$$

Quantizzazione modi

Nella 5.1 n indica l'indice di rifrazione del mezzo che riempie la cavità. For most lasers, values of s larger than a million are typical. Because of this, different longitudinal modes corresponding to successive values of s have wavelengths that are different, but only slightly different. It is a simple matter to write down the above condition in terms of the frequency of a mode rather than its wavelength and, from the resulting expression, calculate the difference in frequency between two successive longitudinal modes. La separazione in frequenza tra due modi longitinali adiacenti si ottiene dalla (5.1) e vale

$$\Delta\nu = c/2nL \qquad (5.2)$$

Free spectral range
Plane parallel cavity

In a plane parallel cavity:
$\Delta\nu_{fsr}$ [GHz] ~ 1.5/(nL [m]) , with n refractive index

Examples: $n = 1$, L = 50 cm -->$\Delta\nu_{fsr} \approx$ 3 GHz (HeNe laser)
$n = 3.3$, L = 0.5 mm -->$\Delta\nu_{fsr} \approx 10^3$ GHz (diode laser)

Definition of ***free spectral range***: distance in frequency (or in wavelength) between two subsequent modes supported by the cavity

For a **plane parallel cavity** (plane parallel mirrors, spaced by L), assuming an active medium with refractive index n (hence, $v_{fase} = c/n$):

Allowed modes:
$k_{m+1} = (m+1)\pi/L = \omega_{m+1}/v_{fase}$
$k_m = m\pi/L = \omega_m/v_{fase}$
$\Delta\omega_{fsr} = \omega_{m+1} - \omega_m = v_{fase}\pi/L$

$\Delta\nu_{fsr} = \Delta\omega_{fsr}/(2\pi) = c/(2nL)$

The cavity behaves like a frequency filter

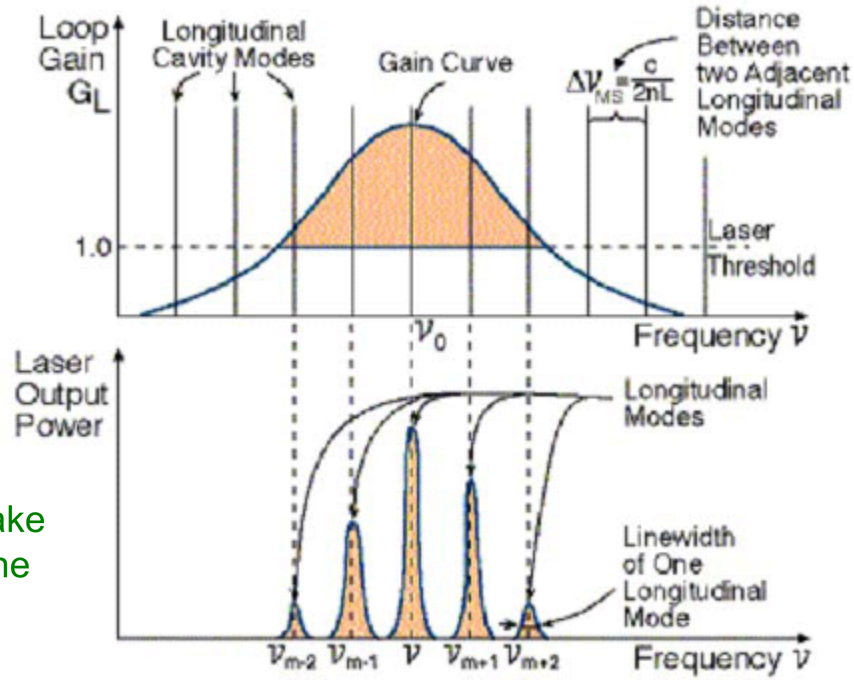
Typically, $L \gg \lambda$ --> m huge
--> the separation is rather small compared to the emitted frequency

<u>**Note**</u>: the frequency filter operation can be exploited also to build frequency-selective mirrors

LASER EMISSION SPECTRUM I

Let's assume a gain curve $F(\omega)$, or $G_L(\nu)$, as mentioned here, broader that the $\Delta\nu_{fsr}$ free spectral range of the cavity, as customarily occurring

Lasing will occur at those frequency which are **allowed** by the cavity **and** correspond to a gain **above threshold**



There might be a *competition* between different longitudinal modes above threshold: either lasing will take place at a selected frequency (usually, the mode with the highest gain) or there will be **multimode** operation

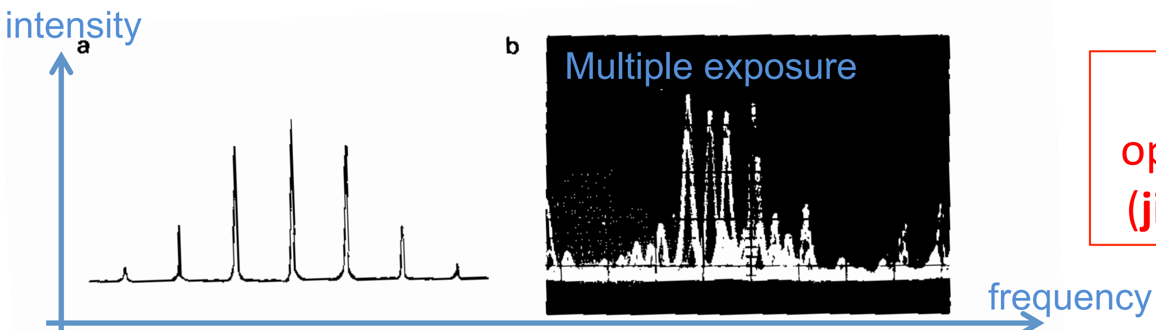


Fig.5.26. (a) Stable multimode operation of a HeNe laser. (Exposure time 1s): (b) Tv short-time exposures of the multimode spectrum of an argon laser superimposed the same film to demonstrate the randomly fluctuating mode distribution

Multimode (longitudinal) operation leads to random jumps (**jitter**) in the emission frequency

LASER EMISSION SPECTRUM II

ser emission:

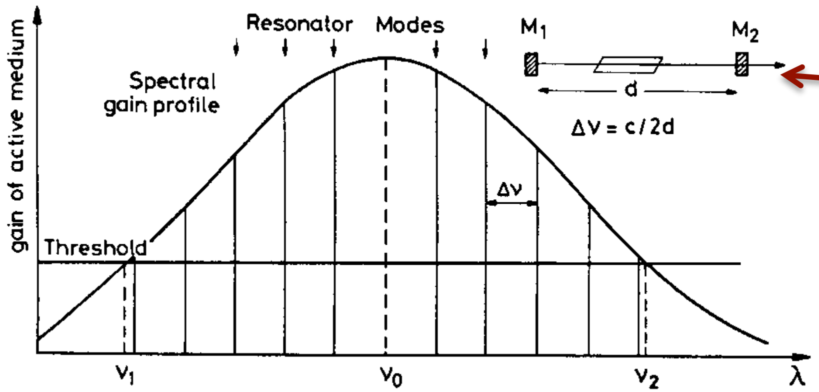


Fig.5.21. Gain profile of a laser transition with resonator eigenfrequencies c modes

According to (5.8) the gain profile $G_0(\nu) = \exp[-2\alpha(\nu)L]$ depends o the line profile $g(\nu-\nu_0)$ of the molecular transition $E_i \to E_k$. The threshol condition can be illustrated graphically by subtracting the frequency-de pendent losses from the gain profile. Laser oscillation is possible at all fre quencies ν_L where this subtraction gives a positive net gain (Fig.5.23).

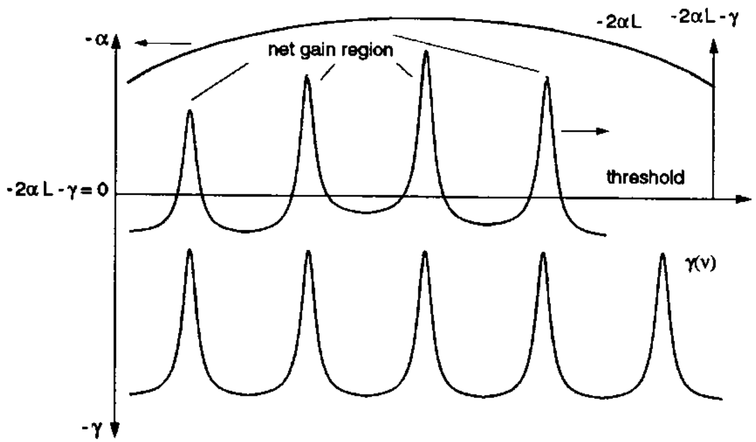


Fig.5.23. Reflection losses of a resonator (*lower curve*), gain curve $\alpha(\nu)$ (*upper curve* and net gain $\Delta\alpha(\nu) = -2L\alpha(\nu)-\gamma(\nu)$ as difference between gain ($\alpha < 0$) and losse (*middle curve*). Only frequencies with $\Delta\alpha(\nu) > 0$ reach oscillation threshold

5.3.1 Active Resonators and Laser Modes

Introducing the amplifying medium into the resonator changes the refractive index between the mirrors and with it the eigenfrequencies of the resonator. We obtain the frequencies of the *active resonator* by replacing the mirror separation d in (5.52) by

$$d^* = (d-L) + n(\nu)L = d + (n-1)L\,, \qquad (5.57)$$

Assume active medium length *L* with refractive index *n* and intermirror distance *d*

where $n(\nu)$ is the refractive index in the active medium with length L. The refractive index $n(\nu)$ depends on the frequency ν of the oscillating modes, which are within the gain profile of a laser transition where anomalous dispersion is found. Let us at first consider how laser oscillation builds up in an active resonator.

If the pump power is increased continuously, threshold is reached first at those frequencies which have a maximum net gain. According to (5.5) the net gain factor per round trip

$$G(\nu, 2d) = \exp[-2\alpha(\nu)L - \gamma(\nu)]\,, \qquad (5.58)$$

Gain per roundtrip (α(ν) is the gain curve, γ(ν) is the loss rate)

is determined by the amplification factor $\exp[-2\alpha(\nu)L]$ which has the frequency dependence of the gain profile (5.8), and also by the loss factor $\exp(-2\beta d/c) = \exp[-\gamma(\nu)]$ per round trip. While absorption or diffraction losses do not strongly depend on the frequency within the gain profiles of a laser transition, the transmission losses exhibit a strong frequency dependence, which is closely connected to the eigenfrequency spectrum of the resonator. This can be illustrated as follows:

A mathematical description can be derived for the "active resonator"

The "active resonator" is the optical cavity with the active medium enclosed in it

LASER LINEWIDTH AND FINESSE

So, the laser frequency is ultimately defined by the interplay between the gain and the optical cavity

*An important parameter ruling the laser linewidth (**monochromaticity!**) is the **finesse** of the cavity related to the bandwidth of the optical cavity, hence on its Q-factor*

The finesse of an optical resonator (cavity) is defined as its free spectral range divided by the (full width at half-maximum) bandwidth of its resonances. It is fully determined by the resonator losses and is independent of the resonator length. If a fraction ρ of the circulating power is left after one round-trip (i.e., a fraction $1-\alpha$ of the power is lost) when there is no incident field from outside the resonator, the finesse is

$$F = \frac{\pi}{2\arcsin\left(\frac{1-\sqrt{\alpha}}{2\sqrt[4]{\alpha}}\right)} \approx \frac{\pi}{1-\sqrt{\alpha}} \approx \frac{2\pi}{1-\alpha}$$

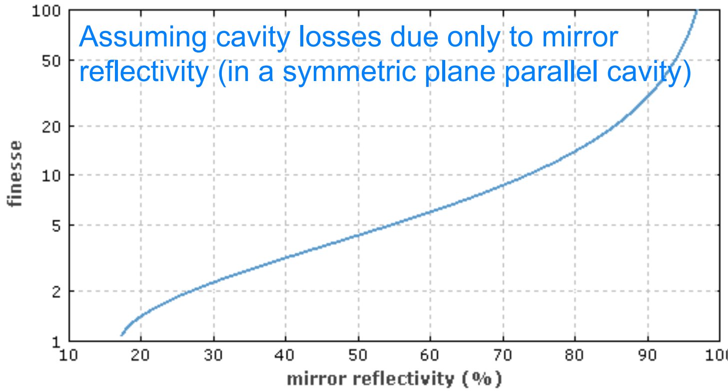
The approximation holds only at low losses, i.e., high finesse

Rather than *Q-factor*, often *Finesse* is considered

Finesse *(F)* =
free spectral range/bandwidth

(*with bandwidth the fwhm of the linewidth of the resonator)*

That is: $Q\text{-factor} = F\,\omega_0/\Delta\omega_{fsr}$



IMPROVING THE *Q*-FACTOR

One possibility is to use an intracavity **etalon** (tilted) active as an additional frequency filter (transmission)

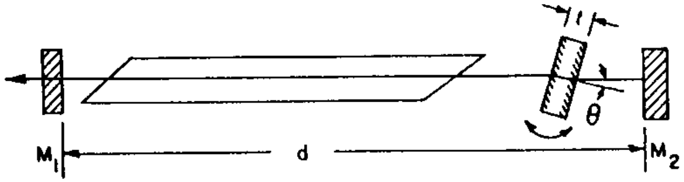
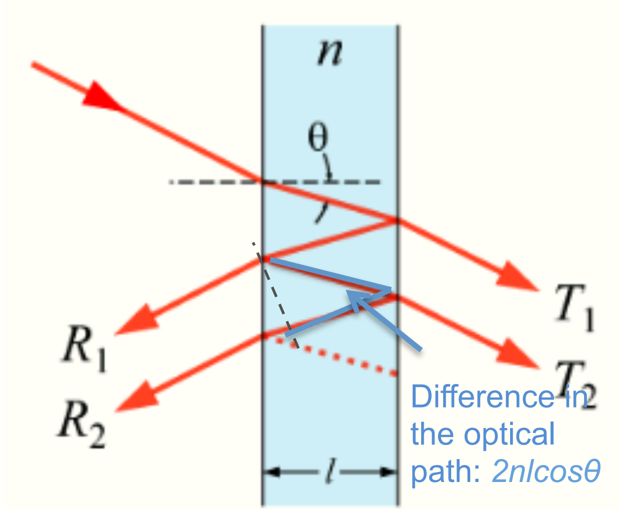


Fig.5.36. Single-mode operation by inserting a tilted etalon inside the laser resonator

In optics, a **Fabry-Pérot interferometer** or **etalon** is typically made of a transparent plate with two reflecting surfaces, or two parallel highly reflecting mirrors. (Technically the former is an etalon and the latter is an interferometer, but the terminology is often used inconsistently.) Its transmission spectrum as a function of wavelength exhibits peaks of large transmission corresponding to resonances of the etalon. It is named after Charles Fabry and Alfred Perot.[1] "Etalon" is from the French *étalon*, meaning "measuring gauge" or "standard".[2]



A Fabry-Pérot etalon. Light enters the etalon and undergoes multiple internal reflections.

The phase difference between each succeeding reflection is given by δ:

$$\delta = \left(\frac{2\pi}{\lambda}\right) 2nl\cos\theta.$$

***Note**:*
this is not that different from "Bragg diffraction"!

If both surfaces have a reflectance R, the transmittance function of the etalon is given by:

$$T_e = \frac{(1-R)^2}{1+R^2-2R\cos(\delta)} = \frac{1}{1+F\sin^2(\frac{\delta}{2})}$$

where $F = \frac{4R}{(1-R)^2}$ is the *coefficient of finesse*.

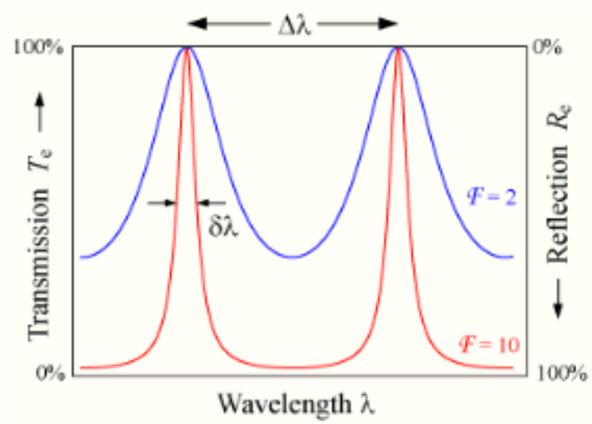
Finesse for a Fabry-Perot etalon

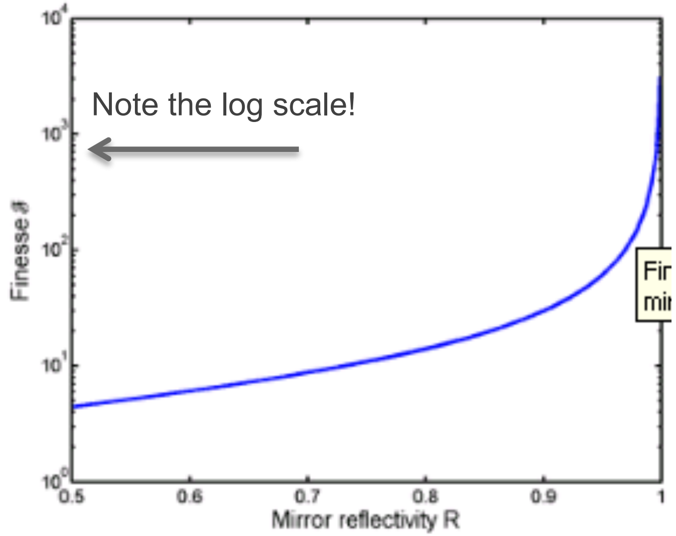
The varying transmission function of an etalon is caused by interference between the multiple reflections of light between the two reflecting surfaces. Constructive interference occurs if the transmitted beams are in phase, and this corresponds to a high-transmission peak of the etalon. If the transmitted beams are out-of-phase, destructive interference occurs and this corresponds to a transmission minimum. Whether the multiply-reflected beams are in-phase or not depends on the wavelength (λ) of the light (in vacuum), the angle the light travels through the etalon (θ), the thickness of the etalon (l) and the refractive index of the material between the reflecting surfaces (n).

HIGH FINESSE FABRY-PEROT ETALONS

$$T_e = \frac{(1-R)^2}{1+R^2-2R\cos(\delta)} = \frac{1}{1+F\sin^2(\frac{\delta}{2})}$$

where $F = \dfrac{4R}{(1-R)^2}$ is the *coefficient of finesse*.





By using very high reflectivity mirrors, very high finesse can be achieved

Typical linewidths for continuous wave (CW) lasers:
- Gas lasers (HeNe, Ar^+,...): $\Delta\nu$<1 MHz (down to < 1 kHz for ultrastable cavities)
- Solid state lasers (Nd:YAG, Ti:Sa, ...): $\Delta\nu\approx$1 MHz (down to < 10 kHz if etalon-equipped)
- Dye lasers: $\Delta\nu\approx$100 MHz (down to 100 kHz if etalon equipped)
- Diode lasers: $\Delta\nu\approx$1 MHz (down to < 10 kHz if coupled to an etalon equipped ext cavity)

Warning: thermal and mechanical drifts may produce long-term fluctuations

QUANTUM LIMIT

Schawlow–Townes Linewidth

Definition: linewidth of a single-frequency laser with quantum noise only

Even before the first laser was experimentally demonstrated, Schawlow and Townes calculated the fundamental (quantum) limit for the linewidth of a laser [1]. This lead to the famous *Schawlow–Townes equation*:

$$\Delta\nu_{\text{laser}} = \frac{\pi h\nu\,(\Delta\nu_c)^2}{P_{\text{out}}}$$

Quantum fluctuations (e.g., related to spontaneous emission) pose a **fundamental** lower limit to the laser linewidth

Es.: HeNe, $h\nu$=2eV, $\Delta\nu_C$=1MHz, P_{out}=1mW → $\Delta\nu_{S.T.}$~ **1mHz**!!

with the photon energy $h\nu$, the resonator bandwidth $\Delta\nu_c$ (full width at half-maximum, FWHM), and the output power P_{out}. It has been assumed that there are no parasitic cavity losses. (Compared with the original equation, a factor 4 has been removed because of a different definition of the resonator bandwidth.)

It is often claimed that the phase noise level corresponding to the Schawlow–Townes linewidth is a result of spontaneous emission into the laser mode. Although this picture is intuitive, it is not completely correct. Both the laser gain and the linear losses of the laser resonator contribute equal amounts of quantum noise to the intracavity light field. This means that even when replacing laser gain with some noiseless amplification process, the phase noise would only decrease to half of the Schawlow–Townes value [2].

Carefully constructed solid-state lasers can have very small linewidths in the region of a few kilohertz, which is still significantly above their Schawlow–Townes limit: technical excess noise makes it difficult to reach that limit. The linewidth of semiconductor lasers is also normally much larger than according to the original equation (without the α factor). This, however, is largely caused by amplitude-to-phase coupling effects (which can be quantified with the linewidth enhancement factor), and not by technical excess noise.

Usually, many other "technological" broadening effects occur leading to linewidth >> Schawlow-Townes

AT THE OTHER SIDE: LASER TUNABILITY

In some cases (notably, dye laser, but also Ti:Sa and some diode lasers) the gain curve is broad
→*Tunability over a wavelength range can be achieved*

In these cases, the optical cavity includes a component able to provide with **coarse** frequency filtering, usually joined to a high finesse element (e.g., etalon)

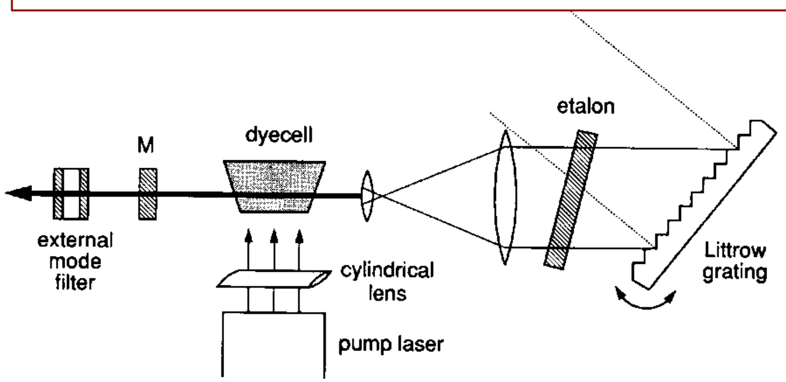
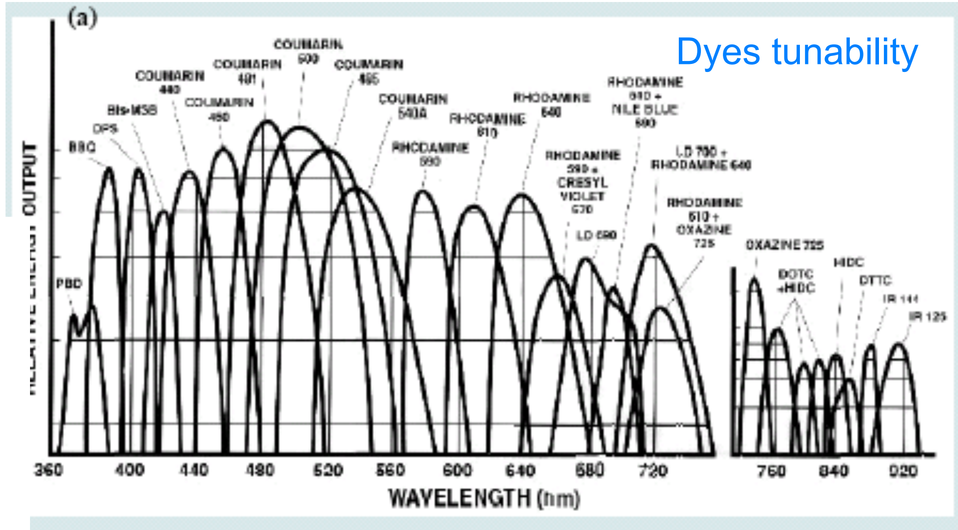


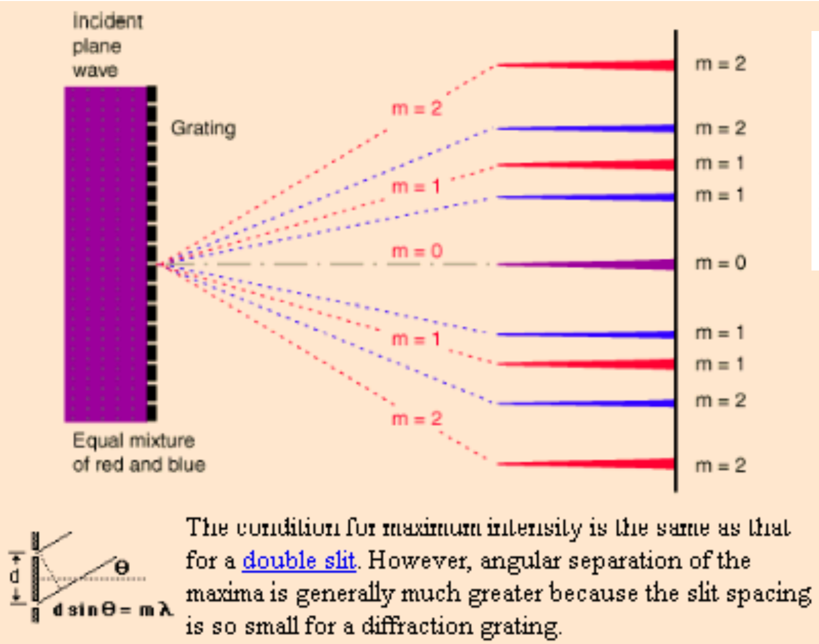
Fig.5.44. Short Hänsch-type dye-laser cavity with Littrow grating and mode selection either with an internal etalon or an external FPI as "mode filter" [5.57]



The "coarse" frequency selection is typically accomplished by using a **diffraction grating**

When there is a need to separate light of different wavelengths with high resolution, then a diffraction grating is most often the tool of choice. This "super prism" aspect of the diffraction grating leads to application for measuring atomic spectra in both laboratory instruments and telescopes. A large number of parallel, closely spaced slits constitutes a diffraction grating. The condition for maximum intensity is the same as that for the double slit or multiple slits, but with a large number of slits the intensity maximum is very sharp and narrow, providing the high resolution for spectroscopic applications. The peak intensities are also much higher for the grating than for the double slit.

DIFFRACTION GRATINGS



$d \ \sin\theta_m = m\lambda.$ Typically, *d = 1/3600-1/600* mm^{-1}

It is straightforward to show that if a plane wave is incident at an angle θ_i, the grating equation becomes

$$\boxed{d\,(\sin\theta_m + \sin\theta_i) = m\lambda.}$$

Rule for the diffraction grating

The light that corresponds to direct transmission (or specular reflection in the case of a reflection grating) is called the zero order, and is denoted $m = 0$. The other maxima occur at angles which are represented by non-zero integers m. Note that m can be positive or negative, resulting in diffracted orders on both sides of the zero order beam.

Reflectivity depends on angle and wavelength!

By ending the optical cavity with a diffraction grating mounted on a rotatable stage, a selection of the frequency supported by the cavity can be done in a generally wide range

When the grating is rotated so that the (unwanted) wavelength is not reflected back

⇩

Losses are artificially (intentionally) introduced at such wavelengths

⇩

The threshold for laser operation is like raised up and viceversa

⇩

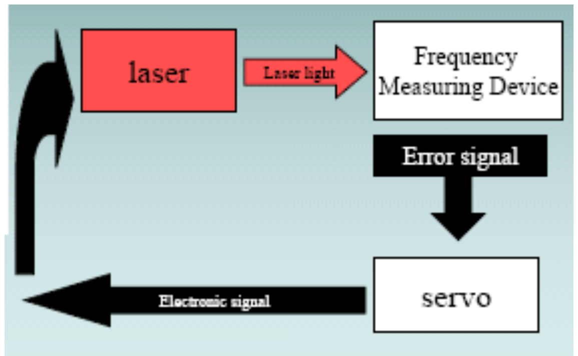
Tunability is achieved

FREQUENCY STABILIZATION

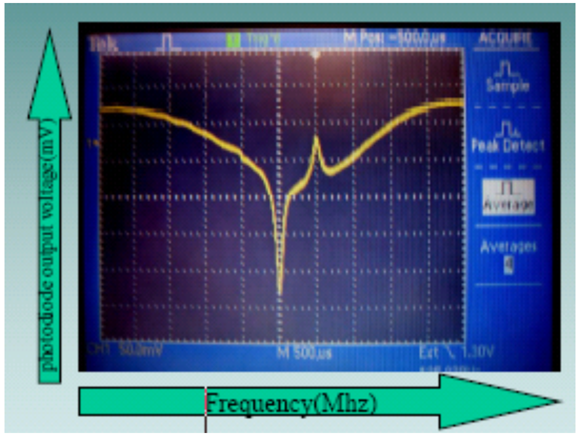
Tunable lasers can then be stabilized, if needed, to emit at a predefined frequency

Stabilization strategy: electronic feedback (PID) on a reference → change of cavity length

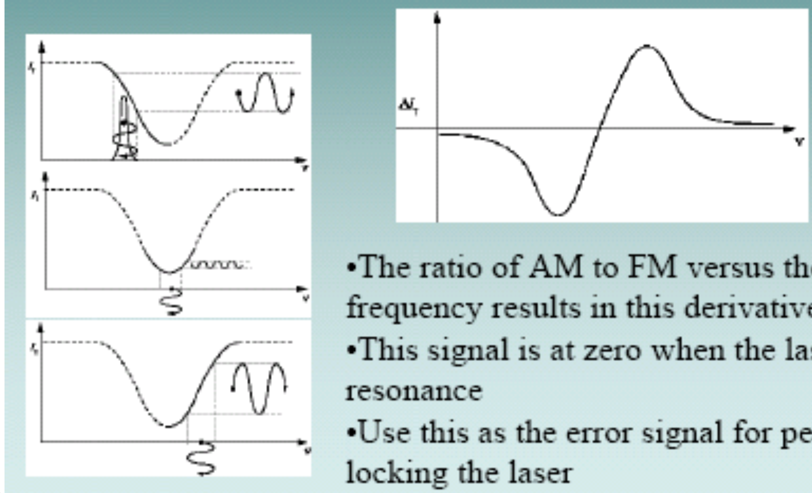
- The optical frequency of a single-frequency laser, or the frequency of one line of the frequency comb from a mode-locked laser, can be stabilized via resonator length control. The feedback signal can be obtained e.g. by recording a beat note with a second laser, by measuring the transmission or reflection of a very stable reference cavity, or by measuring the transmission of some absorption cell (e.g. an iodine cell), possibly using Doppler-free spectroscopy. A frequently used technique for generating an error signal is the Pound–Drever–Hall method [2, 3], using a weak phase modulation of the light which is sent to a reference cavity. A scheme not requiring such modulation is the Hänsch–Couillaud method [1].



Reference cell (absorbs at a predefined frequency, it's a known quantum system!)



Frequency modulation : the wavelength of the laser is scanned across the atomic transition, and the wavelength modulation is seen as varying amplitude modulation.



- The ratio of AM to FM versus the laser frequency results in this derivative signal
- This signal is at zero when the laser is on resonance
- Use this as the error signal for peak locking the laser

Refined stabilization strategies, such as Pound-Drever-Hall, enables stability up to 10^{-10} - 10^{-12} and linewidth < 1Hz (on the short time)

EXAMPLES OF LASER CAVITIES

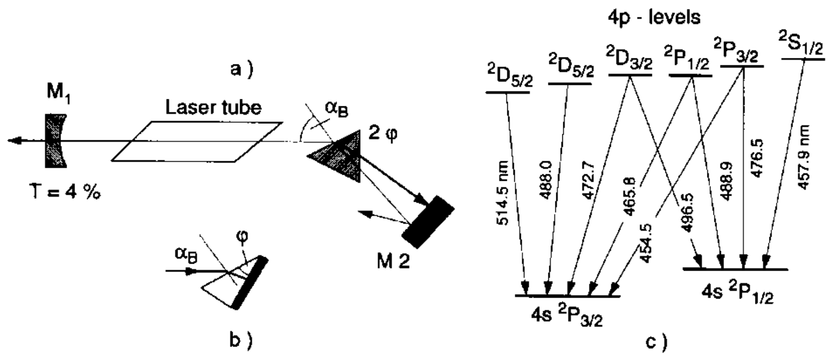
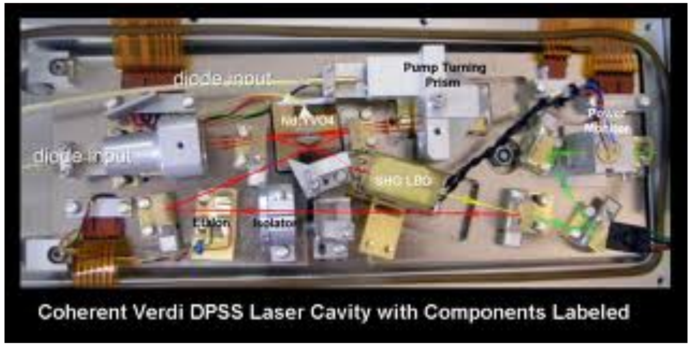
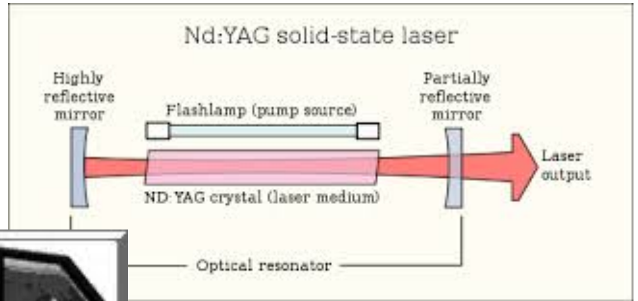
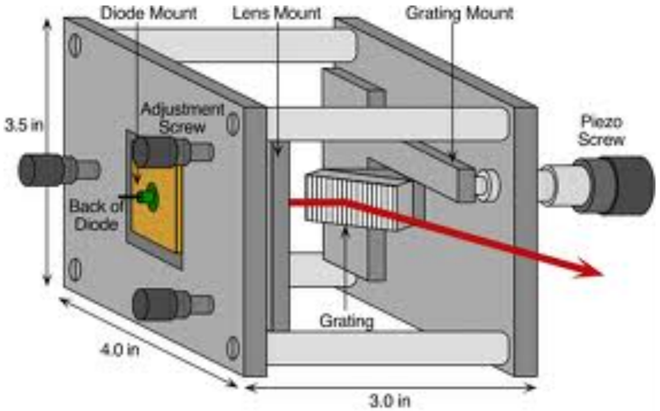
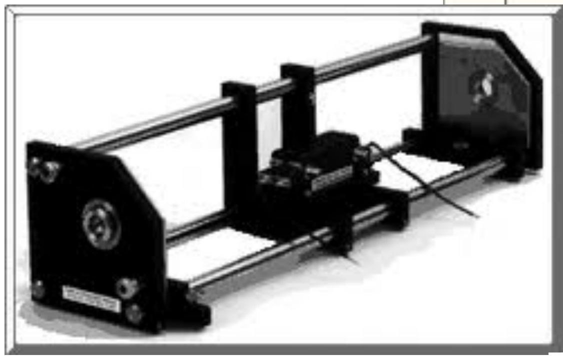


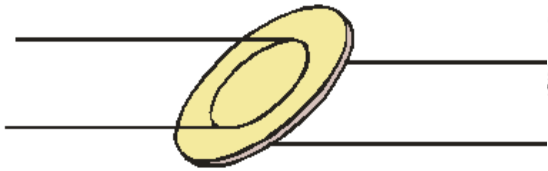
Fig.5.28. Line selection in an argon laser with a Brewster-prism (**a**) or a Littrow-prism reflector (**b**). Term diagram of laser transition in Ar^+ (**c**)

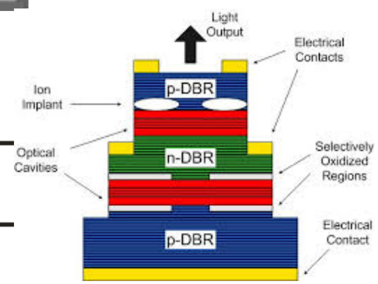












THE ROLE OF TRANSVERSAL MODES

So far, we have only considered the **longitudinal** behavior of the optical cavity, which is responsible for frequency selection

.. but the effects on the transversal direction are relevant as well!

*First of all, **diffraction** from the end-cavity elements (mirrors) will introduce additional losses --> The problem cannot be considered unidimensional any more!*

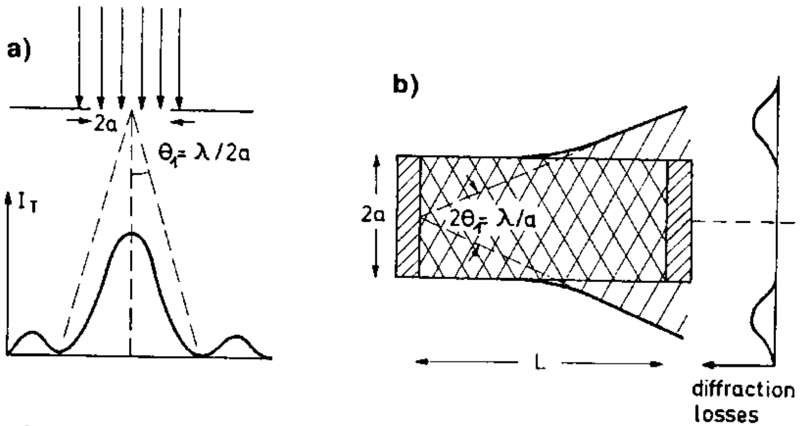


Fig.5.5. Equivalence of diffraction at an aperture (a) and at a mirror of equal size (b). The diffraction pattern of the transmitted light in (a) equals that of the reflected light in (b). The case $\theta_1 d = a \rightarrow N = 0.5$ is shown

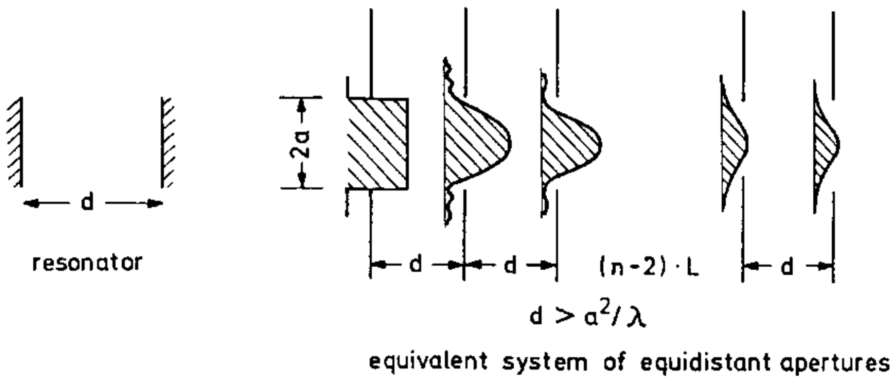


Fig.5.7. The diffraction of an incident plane wave at successive apertures s d is equivalent to the diffraction by successive reflections in a plane-mirr with mirror separation d

Each time a reflection occurs at the end cavity mirrors diffraction takes place

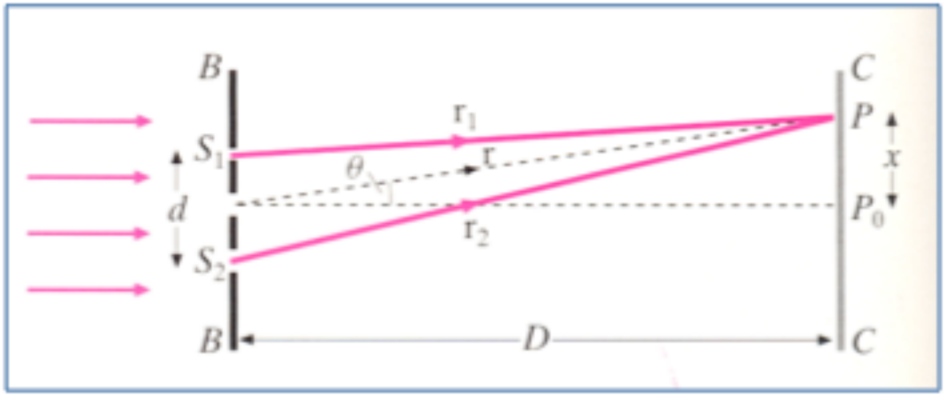
DIFFRACTION I

Let's recall the Young's (double slit) experiment:

Two (small) apertures, i.e., slits, mutually separated by a distance d are placed at distance D from a screen and illuminated by light

Due to "Huygens principle", the slits behave like two distinct and coherent pointlike emitters

Once on the screen, the radiation stemming from the slits will interfere

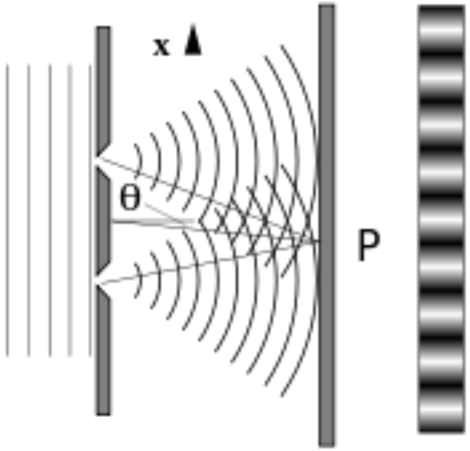


Distance in optical paths: $r_2 - r_1 = d\sin\theta$

Dephasing: $\delta = \dfrac{2\pi}{\lambda} d\sin\theta$

Resulting intensity measured on the screen:

$$I(\theta) = 4I_0 \cos^2\left(\frac{\pi}{\lambda} d\sin\theta\right)$$



Interference fringe spacing:

$$x_{m+1} - x_m = \frac{\lambda D}{d}$$

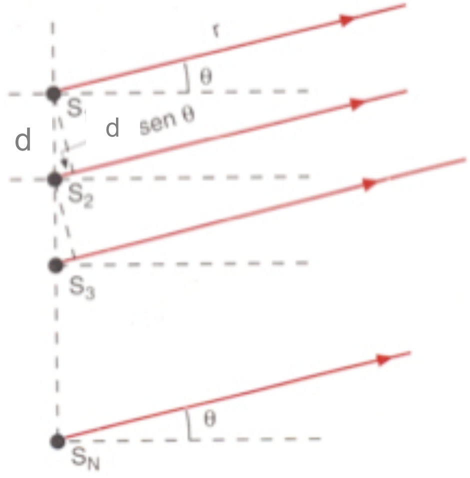
DIFFRACTION II

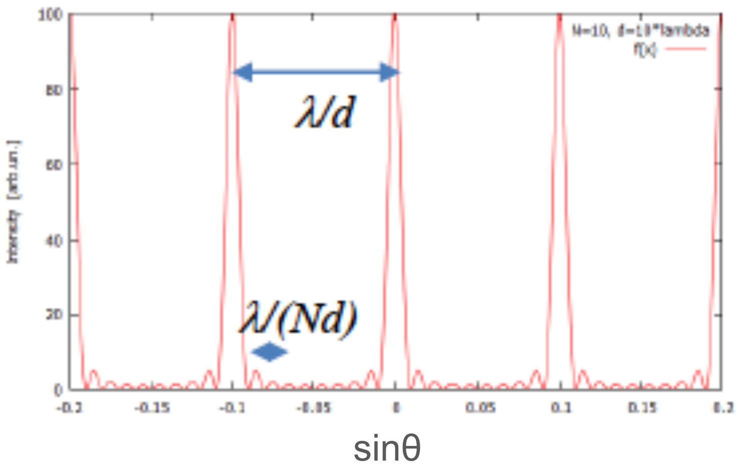
Let's now assume a number *N* of (small) slits:

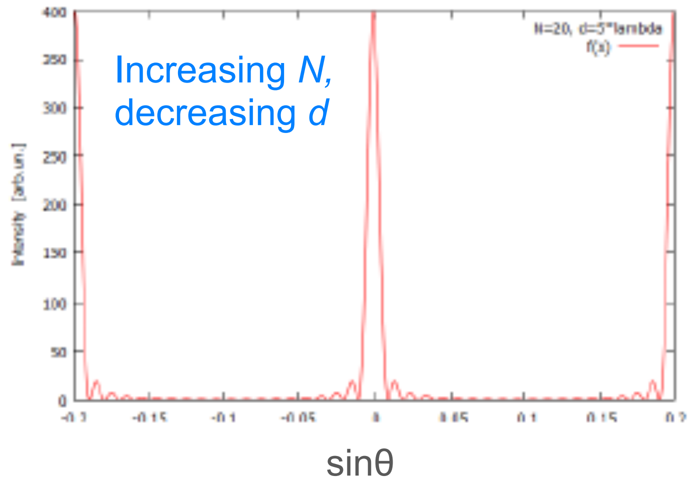
It can be shown (with bloody maths) that the intensity on the screen depends on the angle θ through:

$$I(\theta) \propto \frac{\sin^2(N\gamma)}{\sin^2(\gamma)}$$

$$\gamma = \frac{\pi d}{\lambda}\sin\theta$$







Maxima are separated by *λ/d*

Ripples are separated by *λ/(Nd)* and tend to disappear for large *N*

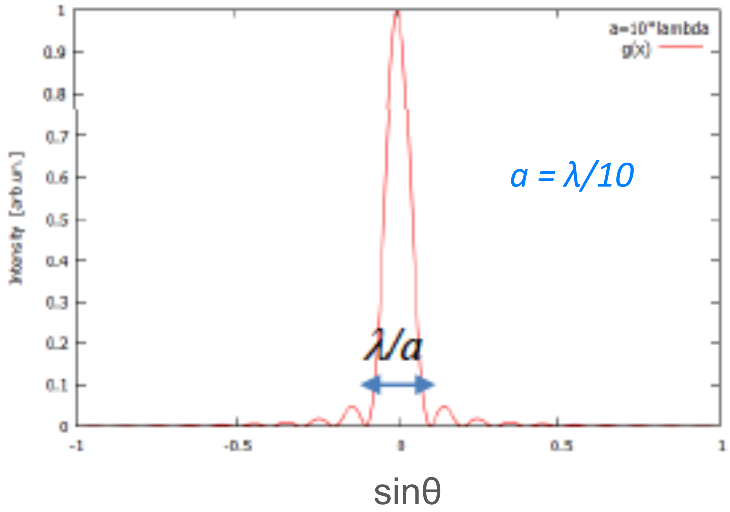
DIFFRACTION III

A single (small) slit can be thought as a very dense (d --> 0, N --> ∞) system of slits

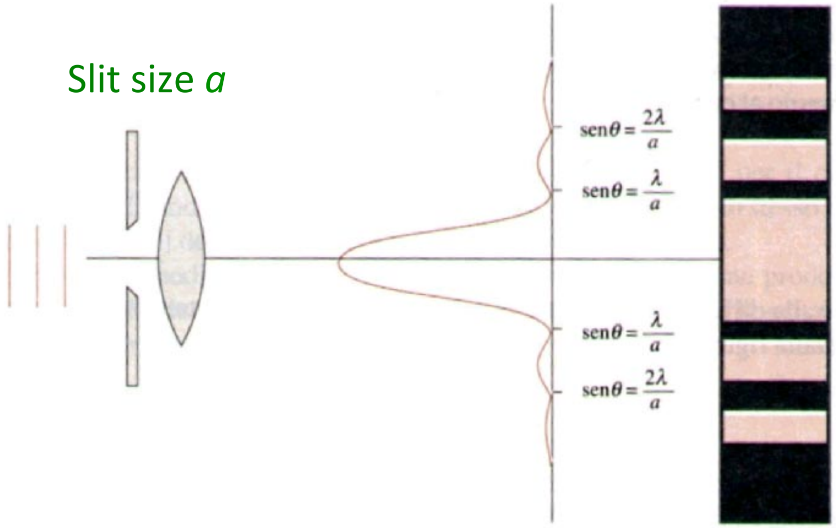
It can be shown that the intensity on the screen depends on the angle θ through:

$$I(\theta) \propto \frac{\sin^2(\beta)}{\beta^2}$$

$$\beta = \frac{\pi a}{\lambda}\sin\theta$$



Slit size a



"***Diffusion cone***":

$$\sin\theta \approx \lambda/a$$

(and ripples, typically not too intense)

“RE-FOCUSING” MIRRORS

5.2.2 Spatial Field Distributions in Open Resonators

In Sect.2.1 we have seen that any stationary field configuration in a closed cavity (called a *mode*) can be composed of plane waves. Because of diffraction, plane waves cannot give stationary fields in open resonators, since the diffraction losses depend on the coordinates (x, y) and increase from the axis of the resonator towards its edges. This implies that the distribution $A(x, y)$, which is independent of x and y for a plane wave, will be altered with each round trip for a wave travelling back and forth between the mirrors of an open resonator until it approaches a stationary distribution. Such a stationary field configuration, called a *mode of the open resonator*, is reached when $A(x, y)$ no longer changes its form, although, of course, the losses result in a decrease of the total amplitude, if they are not compensated by the gain of the active medium.

The mode configurations of open resonators can be obtained by an iterative procedure using the Kirchhoff-Fresnel diffraction theory [5.16]. The resonator with two plane, square mirrors is replaced by the equivalent arrangement of apertures with size $(2a)^2$ and a distance d between successive apertures (Fig.5.7). When an incident plane wave is travelling into the z direction its amplitude distribution is successively altered by diffraction, from a constant amplitude to the final stationary distribution $A_n(x, y)$. The spatial distribution $A_n(x, y)$ in the plane of the n^{th} aperture is determined by the distribution $A_{n-1}(x, y)$ across the previous aperture.

Confocal resonator

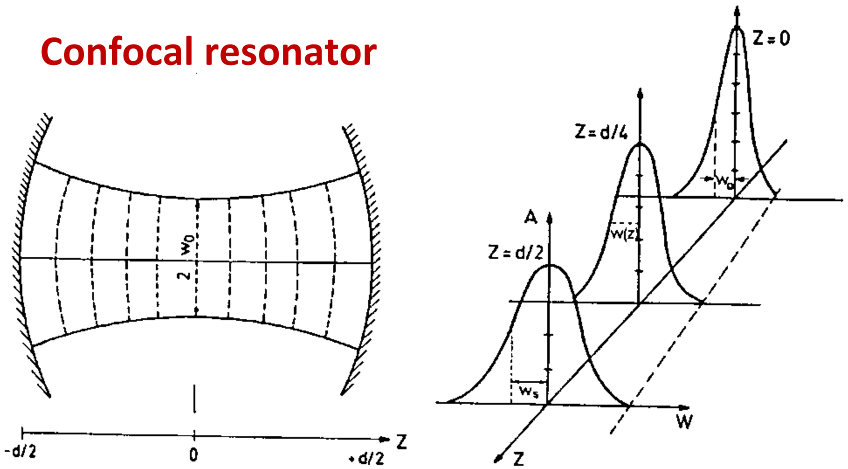


Fig.5.10. Phase fronts and intensity profiles of the fundamental TEM_{00} mode at several locations z in a confocal resonator

From Kirchhoff's diffraction theory we obtain (Fig.5.8)

$$A_n(x, y) = -\frac{i}{2\lambda}\iint A_{n-1}(x', y')\,\frac{1}{\rho}\,e^{-ik\rho}(1 + \cos\theta)\,dx'dy' \,. \tag{5.26}$$

A stationary field distribution is reached if

$$A_n(x, y) = CA_{n-1}(x, y) \quad \text{with} \quad C = \sqrt{1 - \gamma_D}\, e^{i\phi} \,, \tag{5.27}$$

The amplitude attenuation factor C does not depend on x and y. The quantity γ_D represents the diffraction losses and ϕ the corresponding phase shift, caused by diffraction.

Inserting (5.27) into (5.26) gives the following integral equation for the stationary field configuration

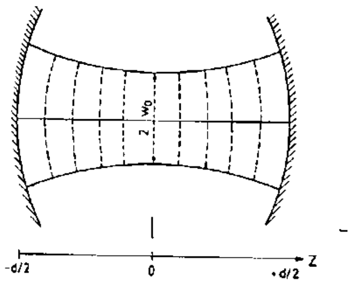
$$A(x, y) = -\frac{i}{2\lambda}(1 - \gamma_D)^{-1/2} e^{-i\phi} \iint A(x', y')\,\frac{1}{\rho}\,e^{-ik\rho}(1 + \cos\theta)\,dx'dy' \,. \tag{5.28}$$

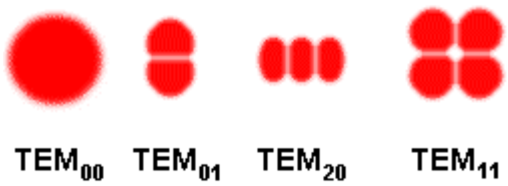
Because the arrangement of successive apertures is equivalent to the plane-mirror resonator, the solutions of this integral equation also represent the stationary modes of the open resonator. The diffraction-dependent phase shifts ϕ for the modes are determined by the condition of resonance, which requires that the mirror separation d equals an integer multiple of $\lambda/2$.

The general integral equation (5.28) cannot be solved analytically and one has to look for approximate methods. For two identical plane mirrors of quadratic shape $(2a)^2$, (5.28) can numerically be solved by splitting it into two one-dimensional equations, one for each coordinate x and y, if the Fresnel number $N = a^2/(d\lambda)$ is small compared with $(d/a)^2$, which means if $a \ll (d^3\lambda)^{1/4}$. Such numerical iterations for the "infinite strip" resonator have been performed by *Fox* and *Li* [5.18]. They showed that stationary field configurations do exist, and computed the field distributions of these modes, their phase shifts and their diffraction losses.

The effects of diffraction in terms of losses can be minimized by using “re-focusing” mirrors

TRANSVERSE MODES OF THE FIELD





5.2.3 Confocal Resonators

The analysis has been extended by *Boyd*, *Gordon*, and *Kogelnik* to resonators with confocally-spaced, spherical mirrors [5.19, 20] and later by others to general laser resonators [5.21–29]. For the confocal case (mirror separation d is equal to the radius of curvature R), the integral equation (5.28)

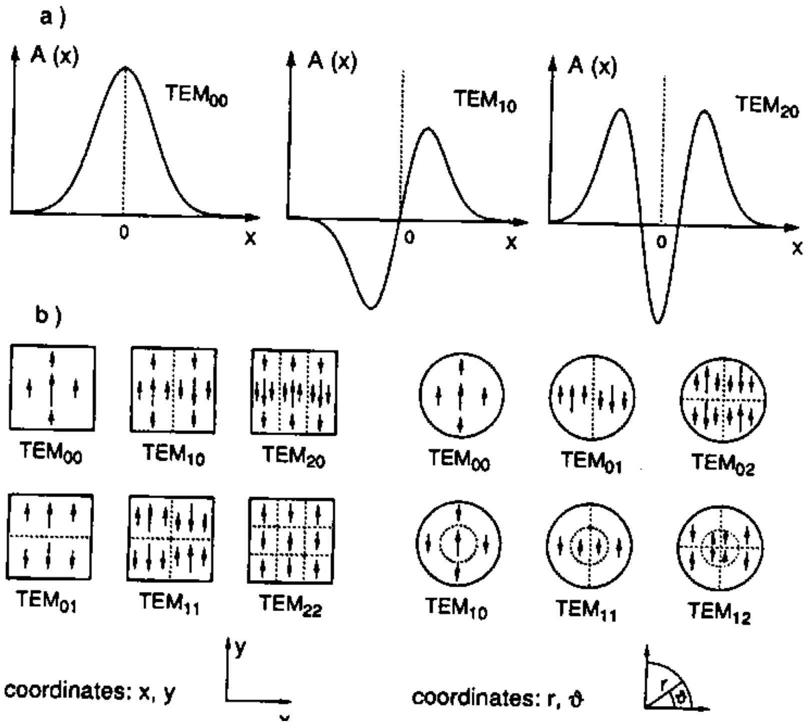


Fig.5.9. (a) Stationary one-dimensional amplitude distributions $A_m(x)$ in a confocal resonator. (b) Two-dimensional presentation of linearly polarized resonator modes for square and circular apertures

can be solved with the acceptable approximation a << d, which implies $\rho \approx$ d in the denominator and $\cos\theta \approx 1$. In the phase term $\exp(-ik\rho)$, the distance ρ cannot be replaced by d, since the phase is sensitive already to small changes in the exponent. One can, however, expand ρ into a power series of xx'/d^2 and yy'/d^2. For the confocal case (d = R) one obtains [5.7, 19]

$$\rho \approx d[1 - (xx'+yy')/R^2]\,. \tag{5.29}$$

Inserting (5.29) into (5.28) allows the two-dimensional equation to be separated into two one-dimensional homogeneous Fredholm equations which can be solved analytically [5.19, 23].

From the solutions, the stationary amplitude distribution in the plane $z = z_0$ vertical to the resonator axis is obtained. For the confocal resonator it can be represented by the product of Hermitian polynomials, a Gaussian function, and a phase factor:

$$A_{mn}(x,y,z) = C^* H_m(x^*) H_n(y^*) \exp(-r^2/w^2) \exp[-i\phi(z,r,R)]\,. \tag{5.30}$$

Here, C^* is a normalization factor. The function H_m is the Hermitian polynomial of m^{th} order. The last factor gives the phase $\phi(z_0,r)$ in the plane $z = z_0$ at a distance $r = (x^2+y^2)^{1/2}$ from the resonator axis. The arguments x^* and y^* depend on the mirror separation d and are related to the coordinates x, y, z by $x^* = \sqrt{2}x/w$ and $y^* = \sqrt{2}y/w$, where

$$w^2(z) = \frac{\lambda d}{2\pi}[1 + (2z/d)^2] \tag{5.31}$$

is a measure for the radial amplitude distribution.

From the definition of the Hermitian polynomials [5.30], one can see that the indices m and n give the number of nodes for the amplitude A(x, y) in the x (or the y) direction. Figures 5.9,10 illustrate some of these "Transverse Electro-Magnetic standing waves" which are called $TEM_{m,n}$ modes. The diffraction effects do not essentially influence the transverse character of the waves. While Fig.5.9a shows the one-dimensional amplitude distribution A(x) for some modes, Fig.5.9b depicts the two-dimensional field amplitude A(x, y) in Cartesian coordinates and $A(r,\theta)$ in polar coordinates. Modes with m = n = 0 are called *fundamental modes* or *axial modes* (often zero-order transverse modes as well), while configurations with m > 0 or n > 0 are transverse modes of higher order. The intensity distribution of the fundamental mode $I_{00} \propto A_{00}A^*_{00}$ can be derived from (5.30). With $H_0(x^*) = H_0(y^*) = 1$ we obtain

$$I_{00}(x,y,z) = I_0 e^{-2r^2/w^2}\,. \tag{5.32}$$

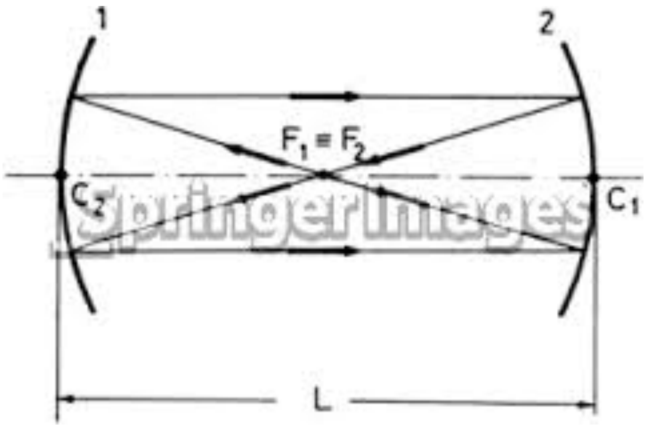
CONFOCAL RESONATORS I

The most convenient configuration is the confocal resonator where the intermirror distance is equal to the radius of curvature

→ each mirror focuses on the position of the other one

→ a Gaussian transverse distribution is supported by the cavity

The TEM_{00} mode (Gaussian distribution of the intensity along the transverse direction) is the one showing less losses, hence is "automatically" selected (*preferred*) by the resonator



Note: it can be demonstrated that the free spectral range of a confocal cavity is one half that of a plane parallel cavity

Equation (5.49) reveals that the frequency spectrum of the confocal resonator is degenerated because the transverse modes with $q = q_1$ and $m+n = 2p$ have the same frequency as the axial mode with $m = n = 0$ and $q = q_1+p$. Between two axial modes there is always another transverse mode with $(m+n+1)$ = odd. The free spectral range of a *confocal resonator* is therefore

$$\delta\nu_{\text{confocal}} = c/4d\ . \qquad (5.51)$$

Free spectral range confocal cavity

CONFOCAL RESONATORS II

The fundamental modes have a Gaussian profile. For $r = w$ the intensity decreases to $1/e^2$ of its maximum value $I_0 = C^{*2}$ on the axis ($r = 0$). The value $r = w$ is called the *beam radius* or *mode radius*. The smallest beam radius w_0 within the confocal resonator is the *beam waist*, which is located at the center $z = 0$. From (5.31) we obtain with $d = R$

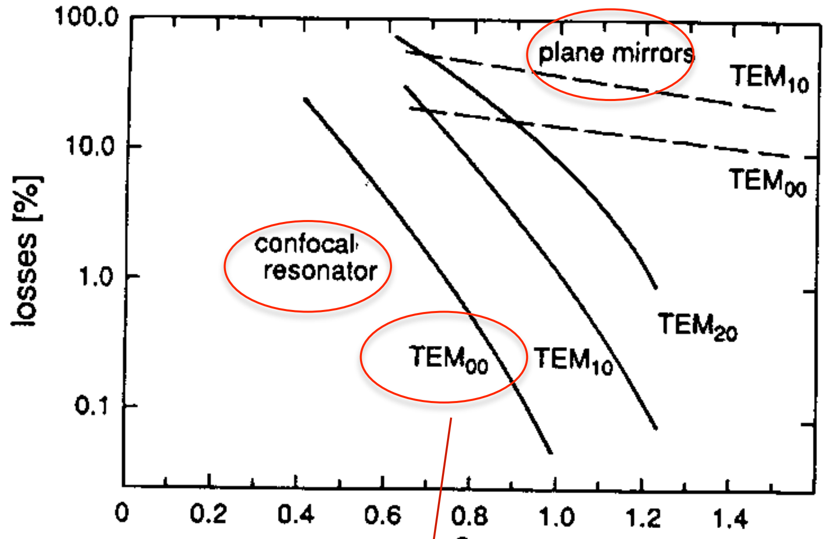
$$w_0 = (\lambda R/2\pi)^{1/2} . \qquad \textbf{Waist} \qquad (5.33)$$

At the mirrors ($z = d/2$) the beam radius $w(d/2) = \sqrt{2}w_0$ is increased by a factor $\sqrt{2}$.

Examples 5.4

a) For a HeNe laser with $\lambda = 633$ nm, $R = d = 30$ cm (5.33) gives $w_0 = 0.17$ mm for the beam waist.

b) For a CO_2 laser with $\lambda = 10\ \mu m$, $R = d = 2$ m is $w_0 = 1.8$ mm.



Fresnel number $N = a^2 / d \cdot \lambda$

Fig.5.11. Diffraction losses of some modes in a confocal and in a plane-mirror resonator, plotted as a function of the Fresnel number N

Typically, the active medium is in the waist of the confocal resonator and little active medium volume is used

A large Fresnel number (well above 1) of a resonator (cavity) means that diffraction losses at the end mirrors are small for typical mode sizes (i.e. not near a stability limit of the resonator, where mode sizes can diverge). This is the usual situation in a stable laser resonator. Conversely, a small Fresnel number means that diffraction losses can be significant – particularly for higher-order modes, so that diffraction-limited operation may be favored.

High Fresnel number→ large diffraction losses

The Gaussian mode, TEM_{00}, enables minimum losses in a confocal resonator

STABILITY OF THE RESONATOR

5.2.6 Stable and Unstable Resonators

In a stable resonator the field amplitude $A(x, y)$ reproduces itself after each round trip apart from a constant factor C which represents the total diffraction losses but does not depend on x or y, see (5.27).

The question is now how the field distribution $A(x, y)$ and the diffraction losses change with varying mirror radii R_1, R_2 and mirror separation d for a general resonator. We will investigate this problem for the fundamental TEM_{00} mode, described by the Gaussian-beam intensity profile. For a stationary field distribution, where the Gaussian beam profile reproduces itself after each round trip one obtains, for a resonator consisting of two spherical mirrors with the radii R_1, R_2, separated by the distance d, the spot sizes πw_1^2 and πw_2^2 on the mirror surfaces [5.1,23]

$$\pi w_1^2 = \lambda d \left(\frac{g_2}{g_1(1 - g_1 g_2)} \right)^{1/2} ; \quad \pi w_2^2 = \lambda d \left(\frac{g_1}{g_2(1 - g_1 g_2)} \right)^{1/2} \qquad (5.42)$$

with the parameters $(i = 1,2)$

$$g_i = 1 - d/R_i \, . \qquad (5.43)$$

This reveals that for $g_1 = 0$ or $g_2 = 0$ and for $g_1 g_2 = 1$ the spot sizes become infinite at one or at both mirror surfaces, which implies that the Gaussian beam diverges: The resonator becomes unstable. An exception is the confocal resonator with $g_1 = g_2 = 0$, which is stable if both parameters g_i are exactly zero. For $g_1 g_2 > 1$ or $g_1 g_2 < 0$ the right-hand sides of (5.42) become imaginary, which means that the resonator is unstable. The condition for a stable resonator is therefore

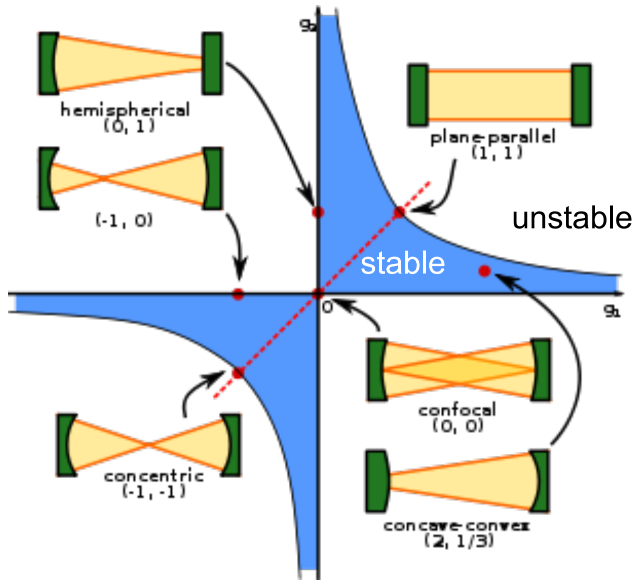
Stable resonators are those limiting the losses for diffraction effects

Unstable resonators are however used when a large volume of active medium must be used
→ High power lasers (and poor optical quality!)

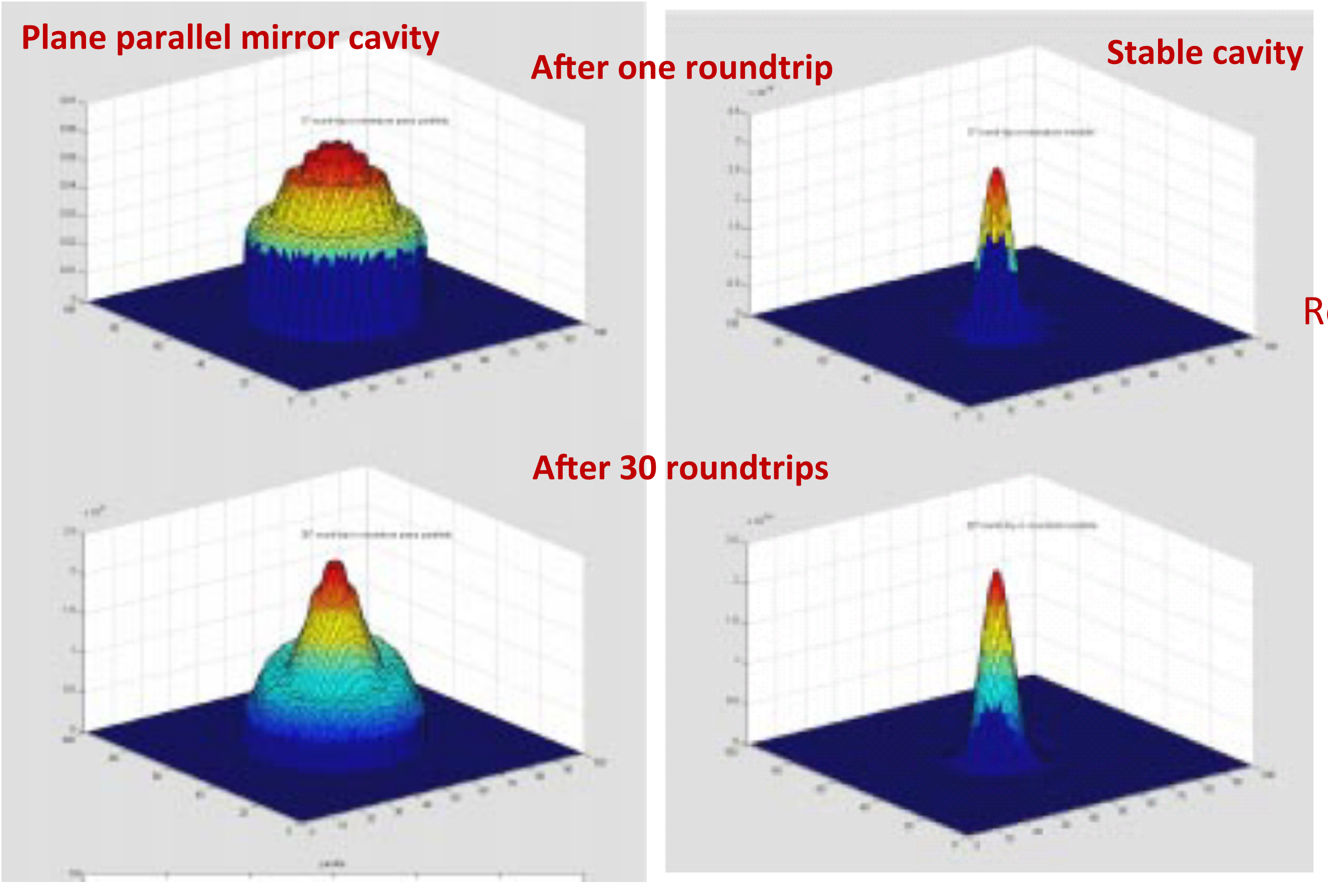
Table 5.1. Some commonly used optical resonators with their stability parameters $g_i = 1 - d/R_i$

Type of resonator	Mirror radii	Stability parameter
Confocal	$R_1 + R_2 = 2d$	$g_1 + g_2 = 2g_1 g_2$
Concentric	$R_1 + R_2 = d$	$g_1 g_2 = 1$
Symmetric	$R_1 = R_2$	$g_1 = g_2 = g$
Symmetric confocal	$R_1 = R_2 = d$	$g_1 = g_2 = 0$
Symmetric concentric	$R_1 = R_2 = \frac{1}{2}d$	$g_1 = g_2 = -1$
Semiconfocal	$R_1 = \infty$	$g_1 = 1, g_2 = \frac{1}{2}$
Plane	$R_1 = R_2 = \infty$	$g_1 = g_2 = +1$

$$\boxed{0 < g_1 g_2 < 1 \quad \text{or} \quad g_1 = g_2 = 0 \, .} \qquad (5.44)$$



QUALITY AND ROUNDTRIPS



Roundtrip time $\tau = L_{tot}/c$
In the vacuum
$\tau\,[\mathrm{ns}] = 3\, L\,[\mathrm{m}]$

Temporal evolution of the e.m. intensity distribution in the cavity

Each roundtrip within the cavity enhances the "optical quality" if diffraction losses are prevented
(*hard to get many roundtrips in pulsed lasers*…)

RESONATOR DESIGN AND LASER BEAM PROPERTIES

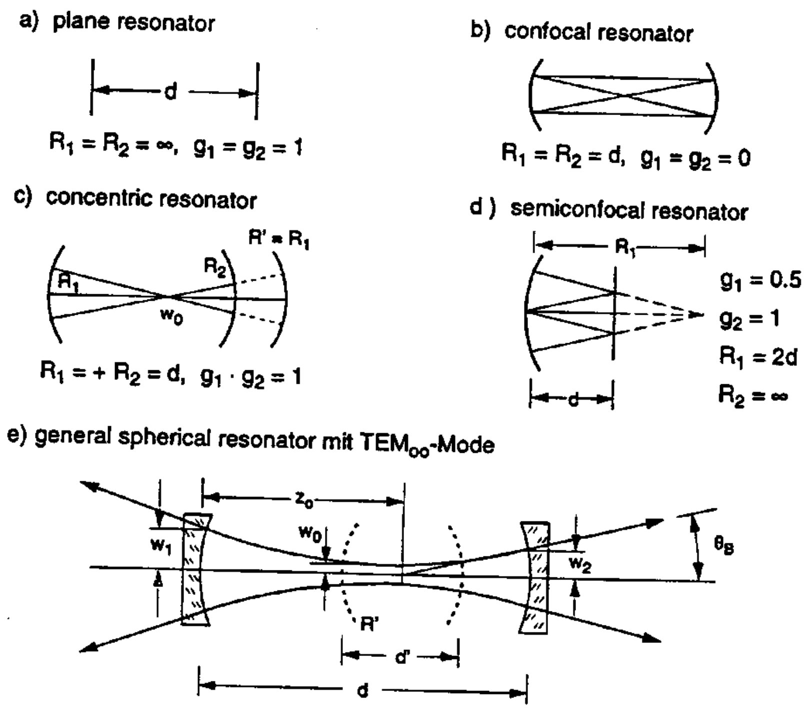


Fig.5.13. Some examples of commonly used ...

For some laser media, in particular those with large gain, unstable resonators with $g_1 g_2 < 0$ may be more advantageous than stable ones for the following reason: In stable resonators the beam waist $w_0(z)$ of the fundamental mode is given by the mirror radii R_1, R_2 and the mirror separation d, see (5.33), and is generally small (Example 5.4). If the cross section of the active volume is larger than πw^2, only a fraction of all inverted atoms can contribute to the laser emission into the TEM_{00} mode, while in unstable resonators the beam fills the whole active medium. This allows extraction of the maximum output power. One has, however, to pay for this advantage by a large beam divergence.

Let us consider the simple example of a symmetric unstable resonator depicted in Fig.5.14 and formed by two mirrors with radii R_i separated by the distance d. Assume that a spherical wave with its center at F_1 is emerging from mirror M_1. The spherical wave geometrically reflected by M_2 has its center in F_2. If this wave, after ideal reflection at M_1, is again a spherical wave with its center at F_1, the field configuration is stationary and the mirrors image the local point F_1 into F_2, and vice versa.

For the magnification of the beam diameter on the way from mirror M_1 to M_2 we obtain from Fig.5.14 the relation

$$M_{12} = \frac{d + R_1}{R_1}. \qquad (5.45)$$

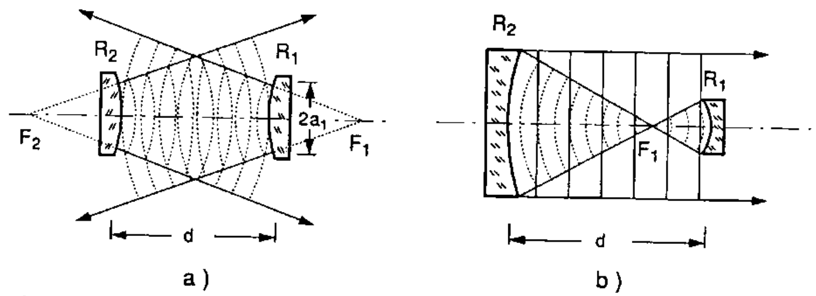


Fig.5.14. (a) Spherical waves in a symmetric unstable resonator, emerging from the virtual focal points F_1 and F_2. (b) Asymmetric unstable resonator with a real focal point between the two mirrors

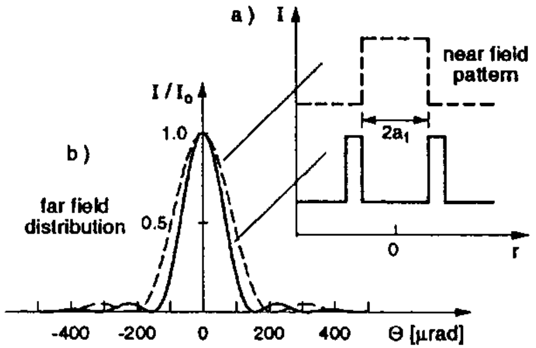


Fig.5.16a,b. Diffraction pattern of the output intensity of a laser with an unstable resonator. (a) Near field just at the output coupler and (b) far-field distribution for a resonator with a = 0.66 cm, g_1 = 1.21, g_2 = 0.85. The pattern obtained with a circular output mirror (*solid curve*) is compared with that of a circular aperture (*dashed curve*)

The optical quality of the produced laser beam depends on the cavity design

CHARACTERIZING THE BEAM PROPERTIES I

Beam Parameter Product (BPP)

Acronym: BPP

Definition: product of the beam radius in a focus and the far-field beam divergence

The beam parameter product (BPP) of a laser beam is defined as the product of beam radius (measured at the beam waist) and the beam divergence half-angle (measured in the far field). The usual units are mm mrad (millimeters times milliradians). The BPP is often used to specify the beam quality of a laser beam: the higher the beam parameter product, the lower is the beam quality.

The BPP can also be defined for non-Gaussian beams. In that case, second moments should be used for the definitions of beam radius and divergence. The smallest possible beam parameter product is then achieved with a diffraction-limited Gaussian beam; it is λ / π. For example, the minimum beam parameter product of a 1064-nm beam is ≈ 0.339 mm mrad.

For non-circular beams, the BPP can be different e.g. in the vertical and horizontal direction.

Note that the BPP remains unchanged when the beam is sent through some non-aberrative optics, such as a thin lens. If that lens generates a focus with smaller beam waist radius, the beam divergence will increase correspondingly. For measuring the BPP, it is thus allowed to form a focus of convenient size, dependent on the equipment used (e.g. a beam profiler) and the available space (which has to extent over several Rayleigh lengths).

Non-ideal optics can "spoil" the beam quality and thus increase the BPP. In some special cases, slight aberrations of an optical element (such as a spherical lens) can somewhat reduce the BPP of a laser beam, if the beam has distortions which can be compensated with that element.

http://www.rp-photonics.com/beam_parameter_product.html

BPP measures the product of beam size and divergence

Typical values:
HeNe 0.5 mm mrad
CO_2 5 mm mrad
Nd:YAG 10 mm mrad
Diode laser 20-100 mm mrad
Excimer laser 500 mm mrad

CHARACTERIZING THE BEAM PROPERTIES II

M² FACTOR

Definition: a parameter for quantifying the beam quality of laser beams

The M^2 factor, also called *beam quality factor* or *beam propagation factor*, is a common measure of the beam quality of a laser beam. According to ISO Standard 11146 [4], it is defined as the beam parameter product divided by λ/π, the latter being the beam parameter product for a diffraction-limited Gaussian beam with the same wavelength. In other words, the half-angle beam divergence is

$$\theta = M^2 \frac{\lambda}{\pi w_0}$$

where w_0 is the beam radius at the beam waist and λ the wavelength. A laser beam is often said to be "M^2 times diffraction-limited". A diffraction-limited beam has an M^2 factor of 1, and is a Gaussian beam. Smaller values of M^2 are physically not possible. A Hermite–Gaussian beam, related to a TEM_{nm} resonator mode, has an M^2 factor of $(2n+1)$ in the x direction, and $(2m+1)$ in the y direction.

The M^2 factor of a laser beam limits the degree to which the beam can be focused for a given beam divergence angle, which is often limited by the numerical aperture of the focusing lens. Together with the optical power, the beam quality factor determines the brightness (more precisely, the radiance) of a laser beam.

For not circularly symmetric beams, the M^2 factor can be different for two directions orthogonal to the beam axis and to each other. This is particularly the case for the output of diode bars, where the M^2 factor is fairly low for the *fast axis* and much higher for the *slow axis*.

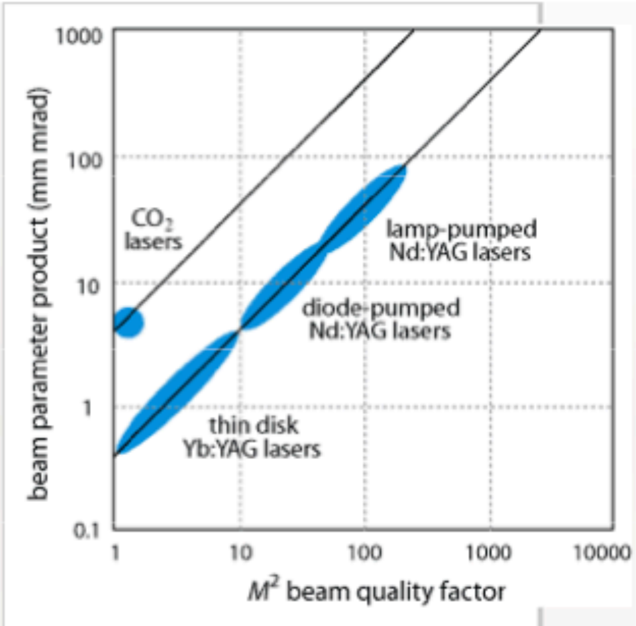


Figure 1: Beam parameter product and M^2 values of various laser types. Due to the longer wavelength, CO_2 lasers have a larger beam parameter product than diffraction-limited 1-μm solid-state lasers, but still compare favorably with lamp-pumped systems.

CONCLUSIONS

- ✓ The optical cavity, or resonator, is a key component of any laser just as the active medium is
- ✓ The boundary conditions set the possible (sustained) longitudinal modes
- ✓ The interplay between the frequency of the allowed modes and the gain curve ultimately determines the laser frequency and linewidth
- ✓ The transverse behavior is relevant as well to:
  - control the diffraction losses
  - rule the laser beam optical properties (divergence)
- ✓ Many schemes have been realized in practical laser systems in order to either improve the linewidth, the tunability, or the beam optical properties

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