The g factor of a bound electron in hydrogenlike ions – Status of the theoretical predictions

T. Beier[1,2], H. Persson[1], S. Salomonson[1], P. Sunnergren[1], and I. Lindgren[1]

[1] *Department of Physics, Chalmers University of Technology and Göteborg University, S-412 96 Göteborg, Sweden.*
Tel. +46-31-772-3283, Fax +46-31-772-3496
Email: f3atb@fy.chalmers.se, Website: http://fy.chalmers.se/f3a/theory.html
[2] *GSI, Planckstr. 1, DE-64291 Darmstadt, Germany*

Heavy highly charged ions are a unique tool for the research in Quantum Electrodynamics (QED) of very strong electric and magnetic fields. Up to now, the major fields of experimental investigation have been Lamb shift and Hyperfine structure splitting. In 1998 and 1999, an experiment was carried out to measure the g factor of a bound electron in hydrogen-like carbon. It yielded $g(\mathrm{C}^{5+}) = 2.001041597(1)$ [1] which coincides excellent with the theoretical prediction, $g(\mathrm{C}^{5+}) = 2.001041591(7)$ [2, 3]. Together with the experiment, this prediction at present forms one of the most stringent tests of bound-state QED in a system heavier than hydrogen.

It is planned to continue these experiments with hydrogen-like ions of a higher nuclear charge number [4, 5]. In these experiments, bound-state QED contributions to the electron g factor are likely to become visible up to the order of two internal photons [6]. Measurements of the g factor provide therefore an unique testing ground for QED in the strong fields of hydrogen-like heavy nuclei where a perturbation expansion in the coupling constant to the electric field of the nucleus is no longer applicable. Compared to this, Lamb shift measurements in these systems yield only one-photon QED contributions, since a better experimental precision has not been achieved up to now. Furthermore, the two-photon QED contributions to the Lamb shift are of similar size as those from nuclear corrections to the energy levels [7]. An overview over the sensitivity of the recent experiments to the different theoretical contributions of the electron g factor is provided by Fig. 1.

In our contribution we are going to show the details of the theoretical calculations on the bound-state electron g factor which were carried out so far. The effects of the bound-state QED contributions of order (α/π) are computed by splitting up the vacuum polarization contributions into two parts and the dominant self energy contributions into three parts. By grouping together divergent pieces which cancel each other and by treating the remaining divergences analytically, it is possible to obtain a precision of 10^{-8} for the terms under consideration in high-Z systems.

Our precision has to be compared with the experimental precision which is of the order 10^{-9} at present in low-Z systems. We point out to other contributions to the bound-state electron g factor which come into play at that stage of accuracy, namely the QED diagrams with two internal photon lines, which have not yet been calculated, and those caused by finite nuclear mass (recoil effects) which are available only as an expansion series in α, $(Z\alpha)$, and m_e/M_N up to now [2, 6].

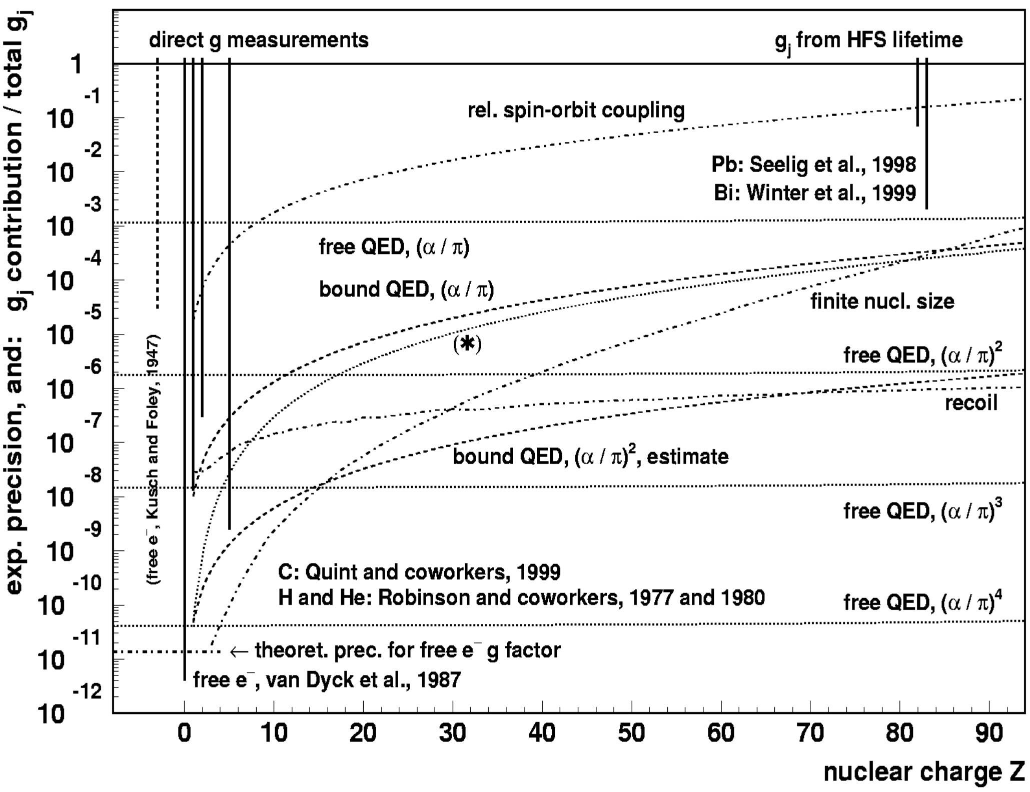

Figure 1: Precision of g-factor experiments compared to the relative value of different theoretical contributions to the bound-electron g factor, denoted by g_J. "Rel. spin-orbit coupling" denotes the bound-state spin-orbit coupling contribution to g, the free-electron QED contributions of orders (α/π) to $(\alpha/\pi)^4$ are indicated as well as the bound-state correction to order (α/π), an estimate of the bound-state correction of order $(\alpha/\pi)^2$, the recoil correction caused by the finite mass of the nucleus, and the correction due to the finite size of the nucleus and (for uranium) its uncertainty. The curve marked $(*)$ indicates the difference between the $(Z\alpha)$ expansion by Grotch [8] and our full bound state QED calculation of order (α/π). The experiment on carbon by Häffner *et al.* is already sensitve to this difference. An overview over the other experiments is given in [3].

[1] H. Häffner and W. Quint, private communication.
[2] H. Persson et al., *Phys. Rev. A* **56** R2499 (1997).
[3] T. Beier et al., *Phys. Rev. A*, submitted.
[4] K. Hermanspahn et al., *Acta Phys. Pol. B* **27** 357 (1996).
[5] M. Diederich et al., in *Trapped charged particles and fundamental physics*, eds. D. Dubin and D. Schneider, AIP conf. proc. 457, p. 43 (AIP, Woodbury, New York).
[6] T. Beier et al., *Hyperfine Int.* **114**, 3 (1998).
[7] P. Mohr et al., *Phys. Rep.* **293**, 227 (1998).
[8] H. Grotch and R. A. Hegstrom, *Phys. Rev. A* **4**, 59 (1971).