Diffraction of Gaussian atomic wave-packets by a standing wave

A. M. Ishkhanyan

Engineering Center of Armenian National Academy of Sciences
Ashtarak-2, 378410 Armenia
Tel +374-43-36373, Fax +374-2-151251; E-mail: artur@ec.sci.am

Recently, a number of unusual effects taking place at coherent scattering of atoms in the field of a standing wave has been discovered that do not meet the established representations on the diffraction of atoms by standing waves [1]. For instance, it was shown that the diffraction pattern can be strongly asymmetric and oscillatory (depending on the detuning of the field frequency) [2]-[3]. An other non-trivial example is the strong narrowing of the well-known interference fringes of the diffraction pattern [3].

These anomalies take their origin from the preparation of atoms in specific states prior to the standing wave scattering, namely, the initial splitting of the atomic wave packet in the momentum space. This stresses the necessity to study systematically the different types of preliminary splitting of the atomic wave packet to clear up, as a minimum, the spectrum of all the possible initial state preparation effects. In the present paper, we study the role of the initial momentum distribution of amplitudes in the standing wave scattering in the case of Gaussian distribution.

We consider a coherent interaction of a two-state quantum system with a standing wave at initial conditions of the form

$$a_1(0) = \sum_{m=-\infty}^{+\infty} \alpha_{2m} e^{i2mkz} \cdot \varphi(z), \qquad a_2(0) = \sum_{m=-\infty}^{+\infty} \beta_{2m+1} e^{i(2m+1)kz} \cdot \varphi(z), \tag{1}$$

and show that in the case of exact resonance the solution of the diffraction problem, in the Raman-Nath approximation, is written as ($s_m = a_m + \beta_m$)

$$W_n(t) = \left| \sum_m i^m s_m J_{n-m}(2Ut) \right|^2 . \tag{2}$$

Then we consider a Gaussian momentum distribution:

$$s_m = e^{i(\alpha - \frac{\pi}{2})m} e^{-m^2/(2M)} / (\pi M)^{1/4}, \tag{3}$$

and show that the n-th order diffraction probability (2) in the first approximation is given as

$$W_n \approx \frac{1}{\sqrt{\pi M(1 + \sin^2\alpha \cdot (2Ut/M)^2)}} \cdot e^{-\frac{(n - \cos\alpha \cdot 2Ut)^2}{M(1+\sin^2\alpha \cdot (2Ut/M)^2)}} . \tag{4}$$

As is seen from this expression, the distribution shape during the time evolution always remains Gaussian, broadening if $\sin a \neq 0$ and moving in the momentum space if $\cos a \neq 0$.

The solution in the next approximation is expressed by the Airy function of the first kind:

$$W_n \approx W_{00} \left| e^{hN} Ai(N + h^2) \right|^2, \tag{5}$$

where the constant W_{00} is defined from the normalization condition and

$$N = \left(\frac{2}{\cos\alpha \cdot u} \right)^{1/3} \cdot (n - \cos\alpha \cdot u), \qquad \mathrm{h} = \frac{\mathrm{M} - \mathrm{i}\sin\alpha \cdot \mathrm{u}}{(2\cos\alpha \cdot \mathrm{u})^{2/3}}\,. \tag{6}$$

Since the behavior of the Airy function is oscillatory at $N + h^2 < 0$ and exponentially decreasing at $N + h^2 > 0$, it is seen that when the distribution peak displaces, some oscillations arise at the distribution wing that is close to the coordinate origin (see Fig.1).

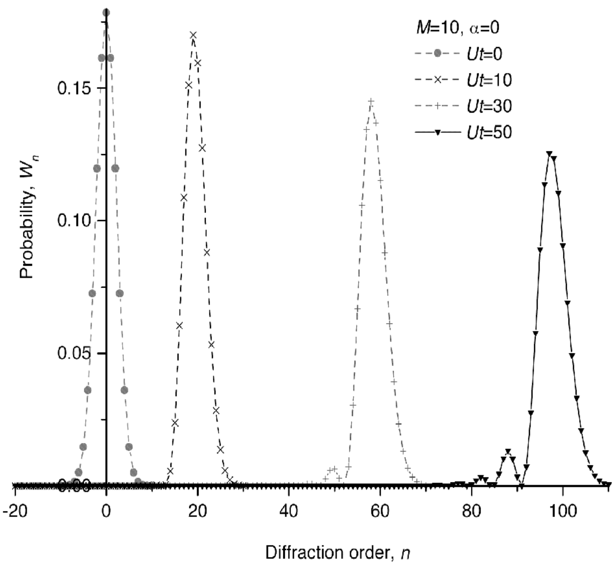

Figure 1: Time evolution of an atomic wave packet with Gaussian initial momentum distribution of amplitudes ($M = 10$) in the field of a standing wave at $\alpha = 0$ (minimal broadening of the wave packet).

Thus, the Gaussian atomic wave packet in the field of a standing wave behaves approximately as a non-decaying quasi-particle. Depending on the phases of the initial momentum distribution components, the number of absorbed photons may vary from a maximal possible value (determined by the limiting speed of the stimulated photon re-emission acts) to zero. Consequently, the coherent diffraction of Gaussian wave packets presents refraction to a controllable refraction angle.

[1] V. G. Minogin and V. S. Letokhov, *Pressure of Laser Radiation on Atoms* (Nauka, Moscow, 1986); A. P. Kazantsev, G. I. Surdutovich, and V. P. Yakovlev, *Mechanical Action of Light on Atoms* (World Scientific, Singapore, 1990).

[2] V. A. Grinchuk et al., *Laser Physics* **6**, 150 (1996); I. V. Romanenko and L. P. Yatsenko, *Pis'ma Zh. Eksp. Teor. Fiz.* **63**, 920 (1996); A. M. Ishkhanyan, *Laser Physics* **7**, 1225 (1997).

[3] A. M. Ishkhanyan, *Phys. Rev A* **61** (2000) (in press).