Generalized Einstein B coefficients for coherently driven three-level systems

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The combination of the Bohr's atomic model with an stochastic conception of the light-matter interaction allowed Einstein (1916-17) to determine the relationship between the rates of absorption and stimulated emission of light by a gas molecule possessing discrete energy levels, in terms of the so-called Einstein B coefficients [1]. In the framework of a perturbation theory, equal B coefficients for absorption and stimulated emission were obtained which, as it is shown in any standard lasers textbook, implies the well known condition of population inversion for light amplification. However, this preliminary picture of the light-matter interaction does not longer hold in the presence of a non-perturbative interaction such as that produced by a laser field. In this case and in the presence of dissipation, the usual procedure to describe the interaction of an intense coherent laser field with an atomic medium consists of using a reduced density-matrix for an averaged ensemble, i.e., the Optical-Bloch equations. Last decade, different stochastic decompositions of the density-matrix based on the interplay of a Schrödinger evolution associated to a coherent interaction, and quantum-jumps associated to dissipation were proposed [2]. In this approach, the time evolution of the wave-function of a single atom, a so-called quantum-trajectory, consists of a series of coherent evolution periods separated by quantum-jumps. Based on this quantum-trajectory formalism, Cohen-Tannoudji *et al.* [3] derived general statistical properties of the coherent evolution periods occurring between two successive quantum-jumps. These statistical tools allow to obtain, without the requisite of explicitly performing a quantum-trajectory or Monte Carlo simulation, (semi)-analytical expressions for the attenuation/amplification of a laser field in interaction with an atomic medium.

In this work, we use the statistical tools developed by Cohen-Tannoudji and coworkers [3] to introduce generalized Einstein B coefficients for one- and two-photon gain and loss processes in coherently driven three-level systems. In order to be more specific, let us consider the cascade scheme shown in Fig. 1(a) where a probe weak laser couples to transition $|a\rangle$-$|b\rangle$ with Rabi frequency α and detuning Δ_α while an intense driving laser couples to the adjacent transition $|b\rangle$-$|c\rangle$ with Rabi frequency β and detuning Δ_β. Dissipative processes are considered by means of the population trasfer rates R_{ij} shown in Fig. 1(b). Let us call B_{ab} (B_{ba}) the generalized Einstein coefficient for one-photon gain (loss) processes and B_{ac} (B_{ca}) that for two-photon gain (loss) processes. In terms of these generalized Einstein B coefficients, the probe attenuation/amplification reads:

$$\left(\frac{d}{dt}n_\alpha\right)^{h,V} = \hbar\omega_\alpha n_\alpha \left(\rho_{aa}B_{ab} + \rho_{aa}B_{ac} - \rho_{bb}B_{ba} - \rho_{cc}B_{ca}\right), \qquad (1)$$

where n_α is the number of photons per unit volume in the probe mode. $n_\alpha \rho_{aa} B_{ab}$ and $n_\alpha \rho_{bb} B_{ba}$ account for the rates of one-photon gain and loss processes, and $n_\alpha \rho_{aa} B_{ac}$ and $n_\alpha \rho_{cc} B_{ca}$ for two-photon gain and loss processes, respectively.

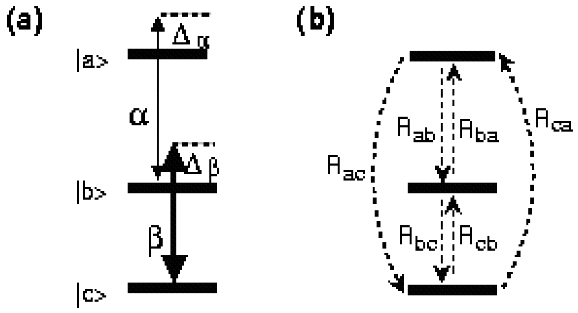

Figure 1: Cascade three-level scheme under consideration.

By using the quantum-jump technique, we have derived some general properties of these generalized Einstein B coefficients. In particular, we have quantified the asymmetry between, on the one-hand, the generalized Einstein coefficients for one-photon processes (i.e., $B_{ab} \neq B_{ba}$), and, on the other hand, for two-photon processes (i.e., $B_{ac} \neq B_{ca}$). For the cascade scheme under consideration, we have obtained:

$$\frac{B_{ab}}{B_{ba}} = 1 + \frac{R_{ac}R_{ba} - R_{ab}R_{ca} + (R_{bc} - R_{cb})(R_{ab} + R_{ac})}{R_{cb}(R_{ab} + R_{ac}) + R_{ab}(R_{ba} + R_{ca})}, \tag{2}$$

$$\frac{B_{ac}}{B_{ca}} = 1 + \frac{(R_{cb} - R_{bc})(R_{ab} + R_{ac}) - R_{ac}R_{ba} + R_{ab}R_{ca}}{R_{bc}(R_{ab} + R_{ac}) + R_{ac}(R_{ba} + R_{ca})}. \tag{3}$$

Clearly, (i) the particular incoherent processes present and their rate value determine the amount and the sign of the asymmetry between the generalized Einstein B coefficients for one- and two-photon processes, and (ii) a "positive" asymmetry between one-photon gain/loss coefficients always comes with a "negative" asymmetry between two-photon gain/loss coefficients and viceversa.

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[2] H. J. Carmichael, *An open systems approach to quantum optics*, (Lectures presented at the Université Libre de Bruxelles, Bruxelles, Belgium, Fall 1991); J. Dalibard, Y. Castin, and K. Mϕlmer, *Phys. Rev. Lett.* **68** 580 (1992); R. Dum, P. Zoller, and H. Ritsch, *Phys. Rev.* A **45** 4879 (1992).

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