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INTRODUCTION

HELIUM exists as one of three isotopes, ^{3}He, ^{4}He, and ^{6}He. The last has a half-life of only 0·82 sec, and will not concern us further. Helium 3 and helium 4 combine into liquid and solid phases with quite unusual properties. In this introduction we briefly review the main features of interest.

1.1. The permanent liquids

The helium molecule is monatomic and smaller than that of any other element. The Van der Waals attractive forces which bind together the liquid or solid state are therefore weaker than in all other substances. It follows that the critical and boiling points of helium (Table 1) are the lowest of all. Likewise we expect the freezing point to be very depressed. In fact liquid helium is unique in that it does not solidify at all under its saturated vapour pressure, no matter how low the temperature. Solidification can only be effected by applying a considerable pressure.

TABLE 1

The critical temperature T_C, *and normal boiling point* T_B, *of liquid* ^{3}He *and* ^{4}He

	^{3}He	^{4}He
T_C (°K)	3·32	5·20
T_B (°K)	3·19	4·21$_5$

In nearly all substances, the freezing point is determined by a balance between the Van der Waals forces and the thermal energy. The attractive forces tend to pack the molecules into a regular crystalline lattice, while the thermal energy tends to disrupt an ordered arrangement. However, as already mentioned, the attractive forces between helium atoms are extremely weak. It thus comes about that another contribution to the balance of forces, usually negligible, must be taken into account. This is the effect of the so-called zero point energy.

To a first approximation, we may regard an atom in the solid as an harmonic oscillator, and its ground state energy is given by the Schrödinger equation as $\frac{1}{2}h\nu_0$, where ν_0 is the characteristic frequency and h is Planck's constant. Alternatively, we may picture an atom in the liquid as a free particle contained in a small cage formed by the adjacent atoms; the energy of the lowest state is then $h^2/8mV^{\frac{2}{3}}$, where m is the mass of the atom. and V the volume of the cage. In both cases the energy is lowered if the molar volume increases, hence the effect of this ground state, or zero point, energy is to increase the molar volume. In fact liquid ^{4}He has an equilibrium volume about three times larger than would be expected if the spacing of the atoms were determined principally by the inter-atomic forces, while ^{3}He has a volume over four times larger.

The large molar volumes of both liquid ^{3}He and liquid ^{4}He result in the atoms being so far apart that they do not lock to form a solid phase. The two liquids remain fluid down to absolute zero, and this presents us with a very interesting situation. We commonly associate the spatial disorder of the atoms in a liquid with a considerable contribution to the entropy (this contribution being related to the latent heat of evaporation). Yet condensed helium remains liquid down to absolute zero, where according to the Third Law of Thermodynamics the entropy vanishes. Hence at very low temperatures, we may expect liquid ^{3}He and liquid ^{4}He to exhibit unusual properties.

1.2. Liquid helium 4

As the molar volumes of liquid ^{3}He and liquid ^{4}He are so large, a simple theoretical treatment is to regard them as dense gases of classical particles. (Although, of course, we must augment the kinetic energy of the particles by the zero point energy.) In fact, using the simple kinetic theory of gases we can account, with moderate success, for the thermal and kinetic properties of the liquids near to their boiling points. However, at lower temperatures, both liquids show very marked deviations from this quasi-classical behaviour.

In liquid ^{4}He the change in behaviour is marked by a very characteristic anomaly in the specific heat, the so-called lambda transition (Fig. 1). The specific heat rises to a very high value at the lambda temperature of 2·17°K. The liquid behaves so differently above and below this temperature that we refer to it in the two regions as *helium I* and *helium II* respectively. The difference is apparent visually if we observe a dewar vessel of liquid ^{4}He which is cooled through the lambda point by boiling under reduced pressure. In the helium I region, the liquid is greatly agitated by bubbles of vapour which form throughout the liquid. Yet, immediately the lambda point is passed, the liquid becomes quite calm and still. Further investigation shows that heat transport takes place so readily within the fluid that it is almost impossible to set up appreciable temperature gradients; hence all the evaporation takes place from the free surface, without bubbling.

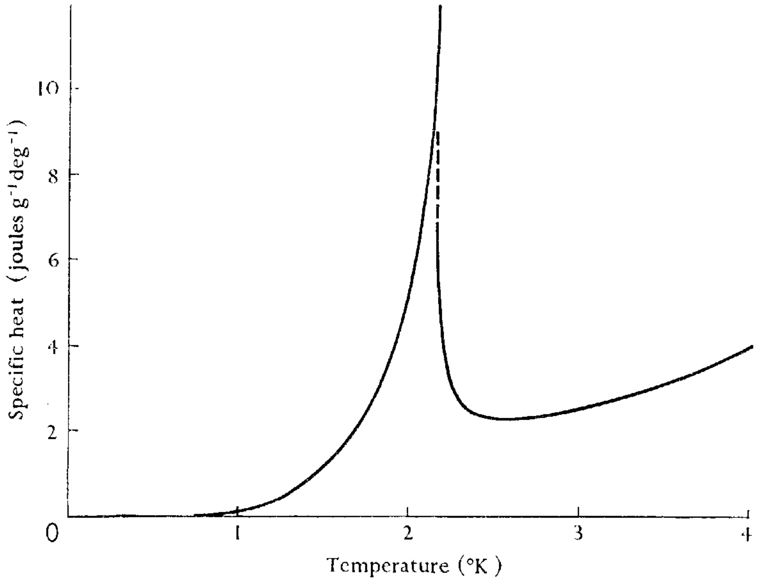

FIG. 1. The specific heat of liquid ^{4}He under the saturated vapour pressure (after Atkins [1]).

The viscosity of helium II, as determined by its rate of flow through narrow slits, is vanishingly small: at least 10^6 times less than the viscosity of helium I. This is the so-called superfluidity of helium II, perhaps its most characteristic feature. Associated with superfluidity is the behaviour exhibited in the remarkable beaker experiment illustrated in Fig. 2b. A small beaker or test tube containing helium II is suspended above a bath of helium II at the same temperature. It is observed that the liquid in the beaker falls to a lower level by flowing up and over the edge of the beaker, then down to the bottom of the tube, where it forms drops which fall off into the bath.

Helium II also exhibits several unusual properties connected with the flow of heat. For example, variations of temperature propagate through the liquid not according to the usual Fourier equation, but as a true wave motion (the so-called second sound). A curious aspect of these thermal properties is illustrated by the simple experiment of Fig. 3. A and B are two vessels joined by a fine capillary. Initially both vessels contain helium II, at the same temperature, and standing at the same level. If an excess pressure ΔP is applied over the helium in vessel A, liquid flows

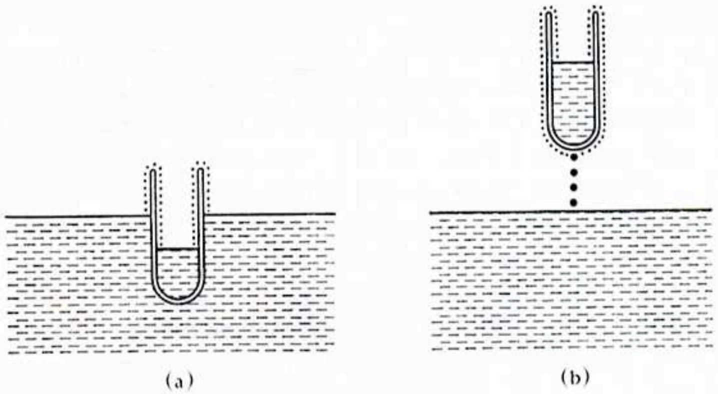

FIG. 2. (a) If an empty beaker is lowered into a bath of helium II, the liquid flows over the surface of the beaker until the levels are equalized. (b) If the beaker is then lifted above the bath, the helium flows out over the rim, drops off the bottom of the beaker, and returns to the bath (after Daunt and Mendelssohn[2].)

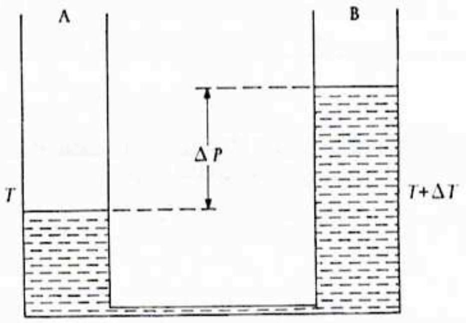

FIG. 3. The vessels A and B are joined by a fine capillary. A displacement of liquid helium II from A to B creates a temperature difference between the two vessels. Alternatively a temperature difference ΔT results in a hydrostatic pressure difference ΔP.

through the capillary into the vessel B, where the level will stand above that in A. In addition, however, the temperature of A rises a little, and that of B falls. Moreover, on releasing the excess pressure, the system returns to its original condition. This result implies, unequivocally, that the liquid which passes through the capillary is different from the bulk liquid in the vessels A and B. In fact the properties of helium II are often described in terms of a 'two fluid model', the liquid being regarded as the sum of two 'fluids', one of which flows through the capillary while the other does not.

Another characteristic feature of helium II is that its behaviour is considerably modified when flowing at a relatively high velocity, or when carrying a relatively high heat current. In the next nine chapters we consider the low velocity region, which is characterized by the absence of any turbulent motion. Then in Chapters 12 and 13 we describe some unusual characteristics of turbulence in helium II.

1.3. Liquid helium 3

The properties of liquid ^{3}He near the boiling point are quite similar to those of liquid ^{4}He near its boiling point. However at lower temperatures all resemblance ceases. Liquid ^{3}He exhibits no lambda anomaly in the specific heat, does not flow through a slit with zero viscosity, and does not support an abnormal transport of heat. Nevertheless its properties indicate a transition from one type of behaviour above 1°K to another quite different type at lower temperatures.

Below about 50 millidegrees K the specific heat varies approximately linearly with the temperature, the nuclear susceptibility no longer follows a Curie law but becomes constant, and the viscosity and thermal conductivity rise rapidly as the temperature falls. These temperature dependences are quite different from those near the boiling point, and resemble those expected for a perfect gas obeying Fermi statistics. In fact the differences between the properties of liquid ^{3}He and ^{4}He arise primarily because they are governed by the Fermi and Bose statistics respectively. The study of liquid helium is therefore an exercise in how simple fluid systems are influenced by the requirements of quantum statistics.

There is some analogy between the behaviour of liquid ^{3}He and ^{4}He, and that of degenerate perfect gases obeying Bose and Fermi statistics. Thus the lambda transition in liquid ^{4}He turns out to be the analogy of the Bose–Einstein condensation in the gas. Yet the parallel is not close. For example, the temperature of the lambda point in liquid ^{4}He is lowered by pressure, whereas the transition temperature in a perfect Bose gas is increased. Similarly the properties of liquid ^{3}He differ considerably from those of a perfect Fermi gas. The main interest of liquid ^{3}He lies in the theoretical interpretation of its behaviour in the degenerate Fermi region.

The ^{3}He nucleus has a spin of $\frac{1}{2}$, which gives rise to a nuclear magnetic susceptibility. As mentioned above, this susceptibility varies according to Curie's law at temperatures near the boiling point, but becomes independent of temperatures at very low temperatures. Measurements of this susceptibility and of the associated relaxation times provide parameters for the study of the liquid, which are not available for liquid ^{4}He. In particular, the susceptibility indicates the influence of exchange forces in a relatively simple system.

Finally, we note that liquid ^{3}He and liquid ^{4}He are miscible over a wide range of concentrations and temperatures. Thus besides making experiments on the pure liquids, we may also study the effects of introducing foreign solute atoms, either ^{3}He in liquid ^{4}He, or ^{4}He in liquid ^{3}He. We shall be concerned chiefly with the properties of pure liquid ^{3}He and pure liquid ^{4}He. However, the behaviour of dilute solutions of ^{3}He in liquid helium II is particularly relevant to the study of pure helium II, and is described in detail in Chapter 9.

1.4. Solid helium

We have already noted the reluctance of liquid helium to solidify. However, when the solid is obtained, we find that neither isotope has properties as unexpected as those of liquid helium II. Yet both solids present several unusual features.

As in the liquid state, the zero point energy is responsible for the molar volume being exceptionally large. This effect is most marked in solid ^{3}He, which shows evidence of considerable mobility of the atoms through the lattice by diffusion. As the atoms are widely spaced, both solid ^{3}He and solid ^{4}He are very compressible. Quite moderate pressures produce relatively large changes in the density, and correspondingly large changes in many other properties. We thus have a useful additional parameter, which permits us to make testing comparisons between theory and experiment.

Both solid ^{3}He and ^{4}He exist in three phases with different crystal structures, namely hexagonal close-packed, face-centred cubic, and body-centred cubic. The two former structures have quite similar properties, but the body-centred phase shows rather different behaviour. The explanation of these properties, together with the relative stability of the three phases, presents an elegant exercise in lattice dynamics. Theoretically the problem is made both more interesting and more complex by the extremely large vibrations of the atoms about their equilibrium positions.

The magnetic properties of solid ^{3}He present an additional point of interest. As in liquid ^{3}He, exchange forces will cause the magnetic susceptibility to deviate from Curie's law at sufficiently low temperatures. However these forces are much smaller in the solid state (as the atomic wave functions are more localized), so that Curie's law is obeyed to considerably lower temperatures. There are also possibilities of observing the various spin relaxation times, and the self diffusion of ^{3}He atoms, by magnetic measurements.

1.5. Technical

Our first knowledge of helium concerns helium 4, and dates from 1868 when observations were made of protuberances on the sun during an eclipse. It was seen that the sun's spectrum included lines which had never been observed in the laboratory. These observations were confirmed by further measurements on the sun under non-eclipse conditions. By 1871 the source of the lines was established as a new element named helium, after the Greek word for sun ἥλιος. Finally, in 1895 Ramsay obtained a laboratory sample of gas from a specimen of the mineral cleveite, and identified it as helium from its spectrum. For an interesting account of these historical details see reference [3].

Helium 4 occurs in the atmosphere with a volume concentration of about 1 part in 200 000. It is also found in much greater concentrations in natural gases, particularly in the United States where the concentration may be as high as a few parts per cent. Very large quantities of helium gas are now extracted from this natural gas, for use principally in miscellaneous industrial processes (including welding). Details of the extraction process are given in reference [4].

In 1939 Alvarez and Cornog, using a cyclotron, were able to show that naturally occurring ^{4}He contains a small amount of the light isotope ^{3}He. The concentration of ^{3}He is extremely small: about 1 part in 10^6 for atmospheric helium, and about 1 part in 10^7 for well helium. All the ^{3}He now used in the laboratory is obtained from the radioactive decay of tritium (which is itself manufactured for military purposes). This source is at present sufficient to provide workers in the laboratory with quantities of the order of a litre ^{3}He gas at N.T.P. Although these amounts are quite modest, they are enough to provide one or two cm^3 of liquid which is adequate for most experiments. Details of some of the earliest attempts to isolate ^{3}He are referred to in reference [5].

^{4}He was first liquefied by Kamerlingh Onnes [6] in 1908. For an account of subsequent and present methods of liquefaction, and of the design of cryostats, see for example reference [7]. Being only available in small quantities, ^{3}He is always liquefied by condensation in a vessel cooled by liquid ^{4}He. For details of the design of ^{3}He cryostats see reference [8]. We refer to the design of ^{3}He–^{4}He dilution refrigerators in section 9.4.

Finally we mention that liquid helium is becoming of increasing technological importance. For an account of such applications see references [9] and [10].

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