

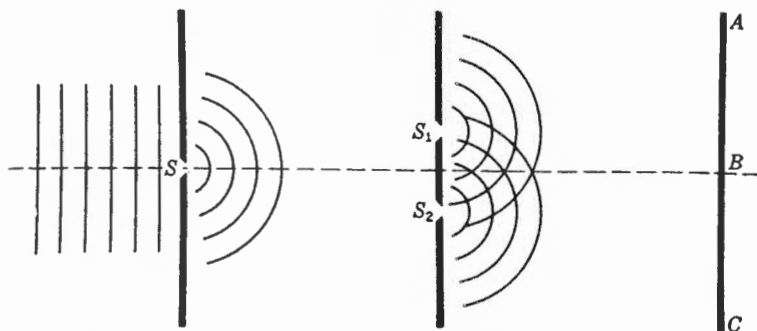


FIGURE 13C
Experimental arrangement for Young's double-slit experiment.

A very simple demonstration of Young's experiment can be accomplished in the laboratory or lecture room by setting up a single-filament lamp L (Fig. 13E) at the front of the room. The straight vertical filament S acts as the source and first slit. Double slits for each observer can be easily made from small photographic plates about 1 to 2 in. square. The slits are made in the photographic emulsion by drawing the point of a penknife across the plate, guided by a straightedge. The plates need not be developed or blackened but can be used as they are. The lamp is now viewed by holding the double slit D close to the eye E and looking at the lamp filament. If the slits are close together, for example, 0.2 mm apart, they give widely spaced fringes, whereas slits farther apart, for example, 1.0 mm, give very narrow fringes. A piece of red glass F placed adjacent to and above another of green glass in front of the lamp will show that the red waves produce wider fringes than the green, which we shall see is due to their greater wavelength.

Frequently one wishes to perform accurate experiments by using more nearly monochromatic light than that obtained by white light and a red or green glass filter. Perhaps the most convenient method is to use the sodium arc now available on the market or a mercury arc plus a filter to isolate the green line, $\lambda 5461$. A suitable filter consists of a combination of didymium glass, to absorb the yellow lines, and a light yellow glass, to absorb the blue and violet lines.

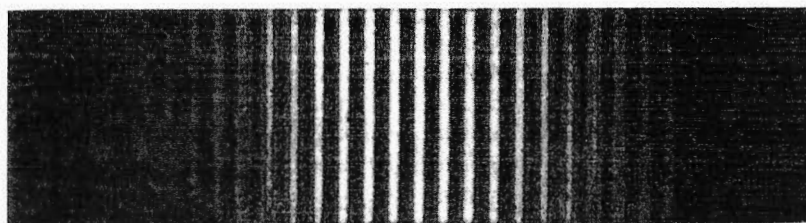


FIGURE 13D
Interference fringes produced by a double slit using the arrangement shown in Fig. 13C.

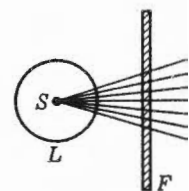


FIGURE 13E
Simple method for Young's experiment.

13.3 INTERFERENCE

We shall now derive an equation for the spacing of the fringes (Fig. 13F) and investigate the spacing of the fringes having traversed different distances. The phase difference given by

It is assumed that the waves from these slits were taken to be equidistant. The amplitudes are practically the same in the two slits and very close together. The problem reduces to that discussed in Section 13.2 for the harmonic motions of the same frequency. The intensity was given by Eq. (13.2),

where a is the amplitude of the waves.

It now remains to evaluate the path difference from the central point D from the slits to the screen. In Fig. 13F, where the distance from P to the screen is D , the path difference between S_1A and S_2A is $d \sin \theta$. As Young's experiment is usually performed with d or λ much larger than d or λ . Hence the angle θ is small. Under these conditions, S_1AS_2 is approximately perpendicular to AS_2 . The path difference becomes $d \sin \theta \approx d \theta$, the sine of the angle equal to the angle itself. We obtain

This is the value of the path difference. The phase difference δ . Now Eq. (13.2),

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Demonstration of single-electron buildup of an interference pattern

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The wave-particle duality of electrons was demonstrated in a kind of two-slit interference experiment using an electron microscope equipped with an electron biprism and a position-sensitive electron-counting system. Such an experiment has been regarded as a pure thought experiment that can never be realized. This article reports an experiment that successfully recorded the actual buildup process of the interference pattern with a series of incoming single electrons in the form of a movie.

I. INTRODUCTION

The two-slit interference experiment with electrons is frequently discussed in textbooks on quantum mechanics, and is referred to as "impossible, absolutely impossible to explain in any classical way, and has in it the heart of quantum mechanics."¹ In this experiment (see Fig. 1), electrons incident on a wall with two slits pass through the slits and are detected one by one on a screen behind them. Accumulation of successive single electrons detected at the screen builds up an interference pattern. According to the interpretation in quantum mechanics, a single electron can pass through both of the slits in a wave form called "probability amplitude" when the uncertainty of the electron position in the wall plane covers the two slits, and when no observation is made of the electron at either one of the slits. The electron is then detected as a particle at a point somewhere on the screen according to the probability distribution of the interference pattern. However, if the electron is caught when passing through the slits, it takes place at either one of the two slits, never both, and the probability distribution on the screen will be completely different.

Although in textbooks this experiment is talked about as

a matter of fact, "this experiment has never been done in just this way, since the apparatus would have to be made on an impossibly small scale," as Feynman points out.¹ However, this is not necessarily true. In fact, several attempts have been made up to now; Zeilinger *et al.*² confirmed th

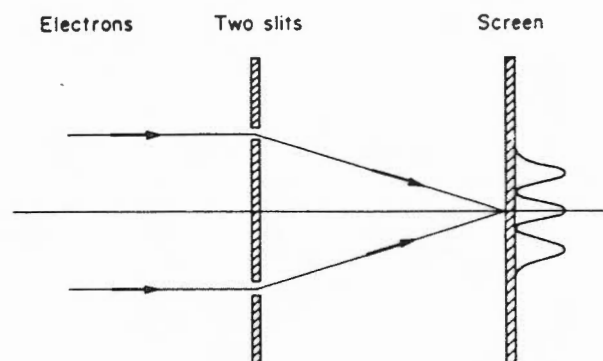


Fig. 1. Two-slit electron interference experiment.

formation of the neutron interference pattern, just as quantum mechanics predicts, by counting arriving neutrons with a scanning counter. In the case of electrons, two groups, one at Tübingen University³ and the other at Bologna University,⁴ demonstrated, in the form of a movie using a highly sensitive TV camera, the observability of the electron interference pattern as it appears when the frequency of incident electrons increases; they showed the electron arrival in each frame without recording the cumulative arrivals. In the case of photons, the buildup process of the interference pattern was recorded on a movie film by Tsuchiya *et al.*⁵ with a position-sensitive counter to accumulate the arrival of single photons on the screen. We note that the typical wavelengths of photons are much larger than those of electrons. Therefore, the difficulty Feynman attributes to the two-slit experiments for electrons does not exist for photons.

The present experiment aims at realizing the two-slit thought experiment for electrons in the form of biprism interference.

II. THEORY OF THE BIPRISM INTERFERENCE PATTERN⁶

The principle of the electron biprism invented by Möllenstedt and Dücker⁷ in 1956 has been investigated from both geometric- and wave-optical aspects.⁷⁻¹⁰ Here, a brief account of the biprism interferometer is given for the reader's convenience.

The biprism consists of two parallel grounded plates with a fine filament between them, the latter having a positive potential relative to the former. If, in the coordinate system shown in Fig. 2, the electrostatic potential is given by $V(x, z)$ and the incoming electron wave by $e^{ik_z z}$, the deflected wave is given by

$$\psi(x, z) = \exp i \left(k_z z - \frac{me}{\hbar^2 k_z} \int_{-\infty}^z V(x, z') dz' \right), \quad (1)$$

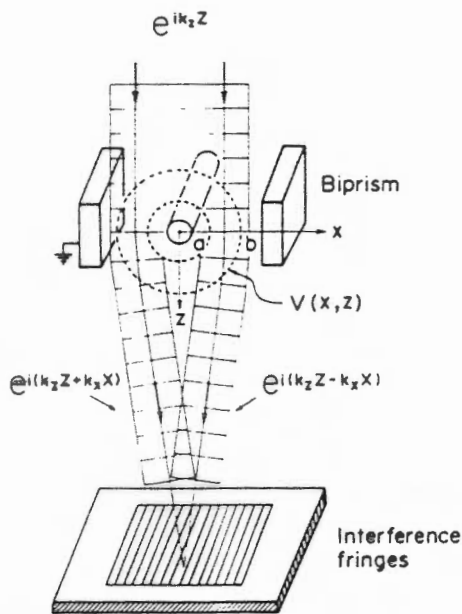


Fig. 2. Deflection of electron waves by biprism—the case of plane-wave incidence.

when $\hbar^2 k_z^2 / 2m \gg e|V(x, z)|$, as is the case in the present experiment.

The two waves having passed on each side of the filament can be approximated by

$\exp i(k_z z \pm k_x x)$ up to a constant factor, where

$$k_x = -\frac{me}{\hbar^2 k_z} \int_{-\infty}^z \left(\frac{\partial V(x, z')}{\partial x} \right)_{x=0} dz', \quad (2)$$

and the symmetry $V(x, z) = V(-x, z)$ has been taken into account. Therefore, the wave fronts of the two waves are deflected as shown in Fig. 2 and, consequently, the waves propagate toward the center, since $k_x > 0$.

This can be interpreted classically also: $-e[\partial V(x, z')/\partial x]_{x=0}$ is the x component of the force exerted on the electron. Its integral with respect to $dz/v_z = dt$, $v_z = \hbar k_z/m$ gives the impulse imparted to it, which is the same in absolute value but reversed in sign, depending on which side of the filament the electron passes.

If the two waves overlap in the observation plane to give

$$\psi(x, z) = e^{ik_z z} (e^{-ik_x x} + e^{ik_x x}), \quad (3)$$

then this leads to the interference fringes

$$|\psi(x, z)|^2 = 4 \cos^2 k_x x. \quad (4)$$

If the potential in the neighborhood of the filament is approximated by

$$V(x, z) = V_a [\ln(\sqrt{x^2 + z^2}/b) / \ln(a/b)], \quad (5)$$

then

$$k_x = \pi e V_a / \hbar v_z \ln(b/a). \quad (6)$$

For $v_z = c/2 = 1.5 \times 10^8$ m/s, $V_a = 10$ V, $a = 0.5$ μ m, $b = 5$ mm, $k_x = (\pi/900)$ $\text{\AA}^{-1}$, and fringe spacing $d = 900$ $\text{\AA}$. In the actual experiment, a spherical wave instead of a plane wave is incident on the biprism and, consequently, the fringe spacing becomes larger, as described in Sec. III.

III. EXPERIMENTS

Experiments were carried out using an electron microscope equipped with an electron biprism and a position-sensitive electron-counting system.

Coherent electron waves from a sharp field-emission tip were, after collimation, sent to an electron biprism. The biprism interference pattern was enlarged by the electron lenses and the single-electron buildup of the interference pattern was observed in time sequence on the TV monitor of a two-dimensional position-sensitive electron-counting system, which was connected to a storage memory. Electrons could be detected one by one, since the detection efficiency was approximately 100% and the detection error was less than 1%.

The detailed experimental arrangement is shown in Fig. 3. Electrons are emitted from a field-emission tip by an applied electrostatic potential $V_1 = 3 - 5$ kV, and then accelerated to the anode of potential $V_0 = 50$ kV. The electron beam accelerated to V_0 is associated with a wave of wavelength

$$\lambda = h / \sqrt{2meV_0(1 + eV_0/2mc^2)}, \quad (7)$$

which, in the present case, is 0.054 $\text{\AA}$. The total emission current is intentionally limited to ~ 1 μ A, only 10^{-4} of which passes through the anodes. The electrons are focused through the condenser lens into fine probe P_1 , and

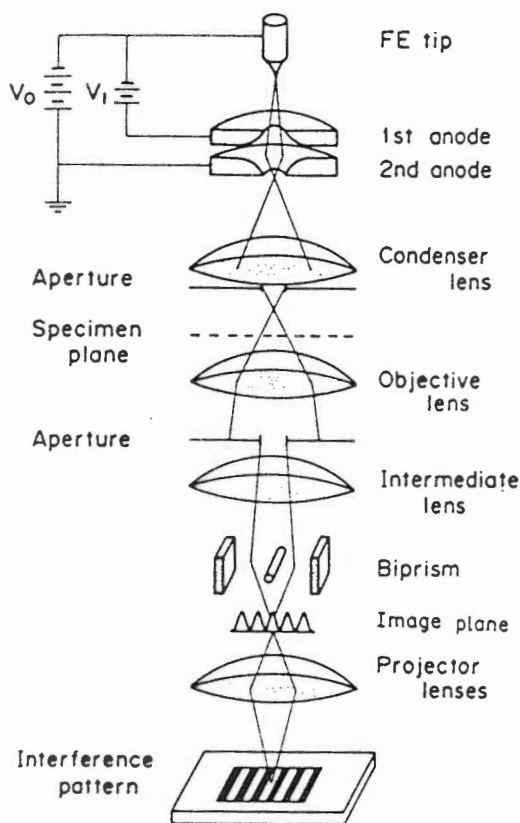


Fig. 3. Electron-optical diagram of the interference experiment.

then illuminate the specimen plane. At this stage, $\frac{1}{10}$ of the electrons pass through the condenser aperture. The electron amplitude in the specimen plane is reproduced through the objective and intermediate lenses onto the image plane of the intermediate lens. Only the central portion of the beam passes through the field-limiting aperture located just above the intermediate lens and is focused by the lens into fine probe P_2 . The total current is controlled by changing the focal length of the intermediate lens and, in a typical case, is 1.6×10^{-16} A or 10^3 electrons/s.

The beam issuing from the probe is then incident on the biprism, and the two partial beams are deflected by the angles $\pm k_x/k_z$ [see Eq. (2)] on each side of the biprism. The divergence angle 2α of the incident beam is approximately 4×10^{-8} rad. Consequently, the transverse coherence length given by $\lambda/2\alpha$ is $140 \mu\text{m}$, which is larger than the diameter of the biprism filament ($< 1 \mu\text{m}$) but smaller than the distance between the two grounded electrodes ($\sim 10 \text{ mm}$). The two beams interfere on the image plane to form interference fringes, one-half of the angle β between the interfering beams being given by

$$\beta = 1/(l+l')(k_x/k_z), \quad (8)$$

and the fringe spacing by

$$d = \lambda/2\beta, \quad (9)$$

which is larger by a factor $(l+l')/l$ than the value given at the end of Sec. II.

In this experiment, $\lambda = 0.054 \text{ \AA}$, $V_0 = 10 \text{ V}$, $l'/l = 6$, and $2\beta = 8 \times 10^{-6} \text{ rad}$, so that $d = 7000 \text{ \AA}$. The interference pattern is finally magnified 2000 times through two projector lenses onto the detector plane. The detector is

approximately 12 mm in diameter and the enlarged fringe spacing is 1.4 mm.

Electrons are detected by a two-dimensional position-sensitive electron-counting system, which is schematically illustrated in Fig. 4. This system is a combination of a fluorescent film and the photon-counting image acquisition system (PIAS) produced by Hamamatsu Photonics K.K. We paid special attention to suppressing both counting loss and detection noise to less than 1%. When a 50-kV electron hits the fluorescent film, approximately 500 photons are produced from the spot. The photons excite the photo cathode through the fiber plate and photo electrons are produced. They are accelerated to 3 kV through the electrostatic lens and the point image of electrons is formed at the upper surface of the multichannel plate (MCP). The number of electrons is multiplied there and the position is then measured by the position sensor. The signal of the electron arrival at each channel is transferred to the storage memory and the accumulated electron image is displayed on the TV monitor.

The experiment was performed at the electron arrival rate of approximately 10^3 electrons/s in the whole field of view so that the interference fringes could be formed in a reasonable time, say, 20 min. The distance from the source to the screen is 1.5 m, while the average interval of successive electrons is 150 km. In addition, the length of the electron wave packet is as short as $\sim 1 \mu\text{m}$. Therefore, there is very little chance for two electrons to be present simultaneously between the source and the detector, and much less chance for two wave packets to overlap.

An example of the buildup process of the interference pattern is shown in Fig. 5 in the form of a time series of photographs. The photographs were taken from single frames in a TV display. Electrons were detected one by one, and the total number of accumulated electrons increases with time. At first, electrons appear to be distributed quite at random. A dim figure of the biprism fringes begins to emerge in Fig. 5(c). The fringes can finally be clearly observed in Fig. 5(e), where the total number of electrons is approximately 70 000, i.e., 14 000 electrons per fringe.

These results unambiguously demonstrate the wave-particle duality of electrons. On the one hand, a single electron passes through the two slits as a wave and forms a probability interference pattern; electron-electron interaction plays no role in this process since the subsequent elec-

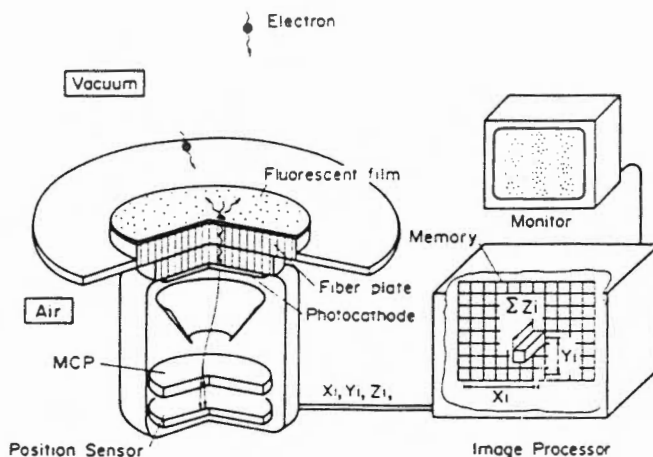


Fig. 4. Schematic diagram of position-sensitive electron-counting system.