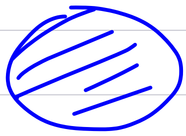
Atomo di idrogeno e orbitali reali

$$\psi_{n,l,m}(r) = R_{n,l}(r)\, Y_{l,m}(\theta,\varphi)$$

(i) orbitali $s \rightarrow l = m = 0$

$$\psi = R_{n,0}(r) \Longrightarrow$$



(ii) orbitali $p \Rightarrow l = 1,\ m = \pm 1,\ 0,$

$$\psi_{n,1,1} = R_{n,1}(r)\cdot\left(-\sqrt{\frac{3}{8\pi}}\right)\sin\theta\, e^{i\varphi} \qquad (m=1)$$

$$= \quad '' \quad \left(\sqrt{\frac{3}{4\pi}}\right)\cos\theta \qquad (m=0)$$

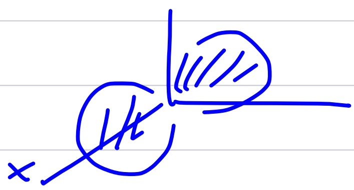
$$= \quad '' \quad \sqrt{\frac{3}{8\pi}}\sin\theta\, e^{-i\varphi} \qquad (m=-1)$$

Osservazioni:

$$\begin{cases} Y_{1,1} \propto -\dfrac{x+iy}{\sqrt{2}\, r}, \\ Y_{1,-1} \propto \dfrac{x+iy}{\sqrt{2}\, r}, \end{cases}$$

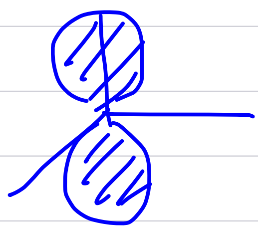
$$Y_{1,0} \propto \frac{z}{r}$$

$$\Rightarrow \quad \psi_{n,1,x} = R_{n,1}\sqrt{\frac{3}{4\pi}}\cdot\frac{x}{r} \rightarrow$$



$$\psi_{n,1,y} = \quad '' \quad '' \quad \frac{y}{r}$$

$$\psi_{n,1,z} = \quad '' \quad '' \quad \frac{z}{r} \rightarrow$$



Una particella che decade in due fermioni identici.

A con $J^P = 1^P$ ← (Parità intrinseca) decade, nel sistema di riposo;
← spin.

$B \longleftarrow A \longrightarrow C$

Parità conservata. $B, C : A = 1/2$.

$S_{tot} = 1, 0$.

$S_{tot} = 1 \Leftrightarrow L = 1, 3, 5, \ldots \quad \psi_L(-r) = -\psi_L(r)$
(S)

$S_{tot} = 0 \Leftrightarrow L = 0, 2, 4, \ldots \quad \psi_L(-r) = \psi_L(r)$.
(A)

Conservazione del mom. ang. ↙ ↙ statistica

$J = 1 \Leftarrow S_{tot} = 1 \Rightarrow L = \cancel{0}, 1, \cancel{2}$

$S_{tot} = 0 \Rightarrow L = \cancel{1}$

$\therefore S_{tot} = 1, \; L = 1$

$\therefore P_A = (-)^L P_B P_C = -1$!

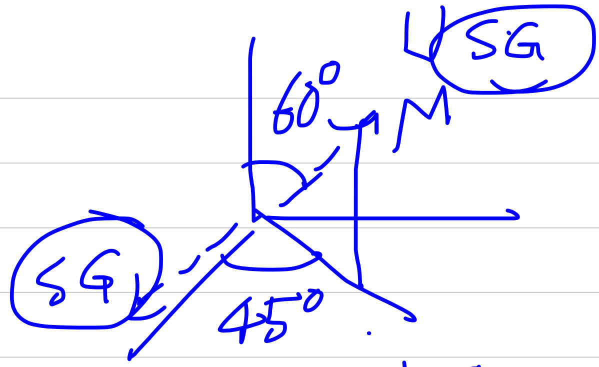
e.g. $\psi^{initial} = |J, J_z\rangle = |1, 1\rangle$

$$\Rightarrow \frac{1}{\sqrt{2}}\left(Y_{1,1} \frac{\uparrow\downarrow + \downarrow\uparrow}{\sqrt{2}} - Y_{1,0} \uparrow\uparrow \right)$$

Supp. che

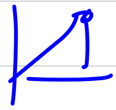
La misura di S_{Bz} abbia

dato $S_{Bz} = 1/2$



$\Rightarrow S_{Az}$?

$e^{i\pi/4} = \cos\pi/4 + i\sin\pi/4$



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(Y_{1,1}(\theta,\varphi)\, \frac{\uparrow\downarrow + \downarrow\uparrow}{\sqrt{2}} - Y_{1,0}(\theta,\varphi)\uparrow\uparrow \right)$$

$$\underset{\substack{\theta=\pi/3\\ \varphi=\pi/4}}{\Longrightarrow} \frac{1}{\sqrt{2}}\left(-\sqrt{\frac{3}{8\pi}}\,\frac{\sqrt{3}}{2}\, e^{i\pi/4} \cdot \frac{\uparrow\downarrow + \downarrow\uparrow}{\sqrt{2}} - \sqrt{\frac{3}{4\pi}}\,\frac{1}{2}\cdot\uparrow\uparrow \right)$$

$$\underset{S_{zB}=1/2}{\Longrightarrow} -\frac{1}{\sqrt{2}}\sqrt{\frac{3}{4\pi}}\left\{ \frac{\sqrt{3}}{2}\, e^{i\pi/4}\,\frac{1}{\sqrt{2}}|\downarrow\rangle_A - \frac{1}{2}|\uparrow\rangle_A \right\}$$

$$= -\frac{1}{2\sqrt{2}}\sqrt{\frac{3}{4\pi}}\left\{ \underbrace{\frac{\sqrt{3}}{2}\, e^{i\pi/4}}_{\hookrightarrow\, 3/4} |\downarrow\rangle_A - \underbrace{|\uparrow\rangle_A}_{\hookrightarrow\, 1} \right\}$$

$$= \sqrt{\frac{4}{7}}\cdot\left\{ \frac{\sqrt{3}}{2}\, e^{i\pi/4}\,|\downarrow\rangle_A - |\uparrow\rangle_A \right\}$$

$$\Rightarrow P_{S_A=1/2} = \frac{4}{7}\,; \quad P_{S_A=-1/2} = \frac{3}{7}.$$

!

atomo di elio in approssimazione di
elettroni indipendenti.

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} - \frac{2e^2}{r_1} - \frac{2e^2}{r_2} \left(+ \frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|} \right)$$

$\Rightarrow$ Lo stato fond.

$$\Psi^0(\mathbf{r}_1, \mathbf{r}_2) = \psi_{100}^{(Z=2)}(\mathbf{r}_1)\, \psi_{100}^{(Z=2)}(\mathbf{r}_2)\, \frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}}$$

1° stato eccitato.

$\Rightarrow 4 \times 4$ volte degenere. ($\# = 4$, $n=2$)

$2lm$

100

$S = S_z = 1 \quad \Psi = A\, \psi_{100}(\mathbf{r}_1)\psi_{2lm}(\mathbf{r}_2) \uparrow\uparrow$

$$= \frac{1}{\sqrt{2}}\left(\psi_{100}(\mathbf{r}_1)\psi_{2lm}(\mathbf{r}_2) - \psi_{2lm}(\mathbf{r}_1)\psi_{100}(\mathbf{r}_2)\right)\uparrow\uparrow \quad 4$$

$S=1, S_z=0,$
$S=1, S_z=-1,$
$S=0, S_z=0 \Rightarrow \psi = A\, \psi_{100}\psi_{2lm} \frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}}$

$$= \frac{1}{\sqrt{2}}\left(\psi_{100}(\mathbf{r}_1)\psi_{2lm}(\mathbf{r}_2) + \psi_{2lm}(\mathbf{r}_1)\psi_{100}(\mathbf{r}_2)\right) \frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}}.$$

In realtà, $S=1$ (orto-elio) ha l'energia più bassa. $\hookrightarrow \psi(\mathbf{r}_1 = \mathbf{r}_2) = 0 \Rightarrow$ meno l'effetto di schermaggio.

Compitino II del 05/02/2015.

Prob 1. Particella A, $J^P = 2^-$, decade, a riposo, in due particelle identiche di spin 1/2.

(i) S_{tot} possibile? $\rightarrow$ $S = 1, 0$

(ii) $S \rightarrow L$? $\rightarrow$ (conserv. mom. angolare) $S = 1$, $L = 3, 2, 1$; $S = 0$, $L = 2$

(iv) Statistica FD $\rightarrow$

Spin che la dinamica del decadimento $\Rightarrow L_{min} = 1$

$|J, J_z\rangle = |2, 1\rangle$ $t=0$.

$S \otimes L = 1 \otimes 1 = \underline{2} \oplus \dots$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(Y_{1,1}\,\chi_{1,0} + Y_{1,0}\,\chi_{1,1}\right)$$

...

Prob. 2 $H = \frac{p^2}{2m} - \frac{e^2}{r} + \frac{f}{r^2} \Rightarrow$

soluzione esatta!

Compitino II del 12/02/16

$\Rightarrow$ facile !

Compitino I del 18/12/09. H.O. con $\mathbf{B}=(0\,0\,B)$

$$H = \frac{(\mathbf{p}-\frac{q}{c}\mathbf{A})^2}{2m} + \frac{m\omega^2}{2}r^2 .$$

con $\mathbf{A} = \left(-\frac{By}{2}, \frac{Bx}{2}, 0\right)$

(i) $\mathbf{B}=0 \Rightarrow$ I primi tre livelli ? Degenerazione ? Quali L ?

(ii) B piccolo $O(B^2)$

$$H = \frac{1}{2m}\left[\left(p_x + \frac{qB}{2c}y\right)^2 + \left(p_y - \frac{qB}{2c}x\right)^2 + p_z^2\right]$$

$$+ \frac{m\omega^2}{2}(x^2+y^2+z^2)$$

$$= \underbrace{\frac{1}{2m}p_x^2 + \frac{m\omega^2}{2}x^2 + \frac{1}{2m}\left(p_y^2 + \frac{m\omega^2}{2}y^2\right) + \frac{1}{2m}\left(p_z + \frac{m\omega^2}{2}z^2\right)} + \frac{qB}{2mc}(y p_x - x p_y)$$

$$= H^{O.H.} - \frac{qB}{2mc}L_z$$

$$\Rightarrow E_0 = \frac{3}{2}\omega\hbar \quad (L=0 \Rightarrow L_z = 0)$$

$$E_1^{L_z} = \frac{5}{2}\omega\hbar - \frac{qB\hbar}{2mc} L_z \quad (L_z = 1, 0, -1)$$

$$E_2^{L_z} = \frac{7}{5}\omega\hbar - \frac{qB\hbar}{2mc} L_z \quad (L_z = 2, 1, 0, -1, -2)$$

$\rightsquigarrow L_z = 0$ doppio $\begin{pmatrix} L=2 \\ L=0 \end{pmatrix}$.

(iii) Tenendo conto anche di B^2. $\Rightarrow$.

$$H = \frac{p^2}{2m} + \frac{m\omega^2 r^2}{2} + \frac{q^2B^2}{8mc^2}(x^2+y^2) - \frac{qB\hbar}{2mc} L_z$$

$[L_z, H] = 0$ ancora !

Sol. dei primi tre termini. $\Rightarrow$

$$E_{\tilde n_1 \tilde n_2, n_3} = \tilde\omega\hbar(\tilde n_1 + \tilde n_2 + 1) + \omega\hbar\left(n_3 + \frac{1}{2}\right)$$

$$\tilde\omega \equiv \sqrt{\omega^2 + \frac{q^2B^2}{4m^2c^2}}$$

(i) $\tilde n_1 = \tilde n_2 = n_3 = 0 \rightarrow L_z = 0$

$$E_0 = \tilde\omega\hbar + \frac{1}{2}\omega\hbar$$

(ii) $N = 1$ si divide in

$\underbrace{(1\,0\,0), (0,1,0)}$ e $(0,0,1)$.

$$E^{0,0,1} = \tilde{\omega}\hbar + \frac{5}{2}\omega\hbar \quad (L_z = 0)$$

$$\psi_{100} \pm \psi_{010} \propto Y_{1,\pm1} \quad (L_z = \pm 1)$$

$$\Rightarrow E_1^{L_z=1} = 2\tilde{\omega}\hbar + \frac{\omega\hbar}{2} - \frac{qB\hbar}{2mc} \quad (L_z = +1)$$

$$E_1^{L_z=-1} = \text{''} \quad \text{''} \quad + \quad \text{''} \quad (L_z = -1)$$

iii) $\underline{N=2}$ si divide in

$$\psi_{002} \quad (L_z = 0),$$

$$\psi_{101} \pm \psi_{011} \quad (L_z = \pm 1)$$

$$\psi_{200} + \psi_{020} \quad (L_z = 0),$$

$$\psi_{200} - \psi_{020} \pm \sqrt{2}\,\psi_{1,1,0} \quad (L_z = \pm 2)$$

con relativa energia,

$$\tilde{\omega}\hbar + \frac{5}{2}\omega\hbar, \quad (L_z = 0)^*$$

$$2\tilde{\omega}\hbar + \frac{3}{2}\omega\hbar \mp \frac{qB\hbar}{2mc} \quad L_z = \pm 1$$

$$3\tilde{\omega}\hbar + \frac{\omega\hbar}{2} \quad (L_z = 0)^*$$

$$3\tilde{\omega}\hbar + \frac{\omega\hbar}{2} \mp \frac{qB\hbar}{mc} \quad (L_z = \pm 2)$$

La degen nel caso $O(B^2)$ è eliminata.