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Effective Gauge Symmetry in Supersymmetric Confining Theories.

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Summary. — The role played by composite vector supermultiplets is studied in detail in the context of general supersymmetric confining theories. The structure of the low-energy effective action involving these vector supermultiplets is determined by use of the chiral WT identities. One of the most interesting outcome is the universal appearance of an effective gauge symmetry structure. Another is the emergence of a generationlike structure among the composite « matter » multiplets under this effective gauge symmetry.

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1. – Introduction.

There has been a considerable progress recently in understanding the non-perturbative dynamics in supersymmetric gauge theories, through the study of instanton effects (1-4), anomalies (5) and effective Lagrangians (6-8) as well

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(2) G. C. ROSSI and G. VENEZIANO: *Phys. Lett. B*, **138**, 195 (1984); D. AMATI, G. C. ROSSI and G. VENEZIANO: *Nucl. Phys. B*, **249**, 1 (1985).
(3) I. AFFLECK, M. DINE and N. SEIBERG: *Phys. Rev. Lett.*, **51**, 1026 (1983); IAS preprint (November 1983).
(4) Y. MEURICE and G. VENEZIANO: *Phys. Lett. B*, **141** 69 (1984); I. AFFLECK, M. DINE

as via the index analysis (9). As a result, one seems now in a position to discuss about the possible realizations of chiral symmetries and supersymmetry with some confidence, at least in some classes of models.

Based of this development, we wish to discuss in this paper the role played by certain composite vector supermultiplets in supersymmetric confining theories. (We shall restrict ourselves to gauge theories with $n = 1$ global supersymmetry.)

The vector supermultiplets we shall be interested in are the ones whose D-components are related to the matter kinetic part of the original Lagrangian (see eqs. (2.2) and (2.1)). One of the reasons for introducing these vector supermultiplets is that it allows us to reproduce the chiral WT identities in the effective theory in an exact way. (See subsect. **2˙2** and **2˙3**.) At the same time, the « field-current identity » (10) holds for these vector multiplets, and as a result the structure of the effective action obtained from the consideration of the chiral WT identities can be derived, alternatively, from a generalization of the Lee-Zumino construction (10) (subsect. **2˙1**).

Perhaps the most striking and interesting aspect of the effective action thus obtained is the emergence of an exact gauge symmetry structure (which will be called the *effective gauge symmetry* hereafter): this occurs quite universally in supersymmetric gauge theories with matter. The group of the effective symmetry is the full chiral group G_F, that would be the global symmetry group of the original action if the superpotential and strong U_1 anomalies were both neglected. G_F does not include R-type symmetry groups which do not commute with supersymmetry. For instance, $G_F = U_M \times U_M$ for M-flavoured quantum cromodynamics.

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(6) G. VENEZIANO and S. YANKIELOWICZ: *Phys. Lett. B*, **113**, 321 (1982).

(7) T. R. TAYLOR, G. VENEZIANO and S. YANKIELOWICZ: *Nucl. Phys. B*, **218**, 493 (1983).

(8) M. E. PESKIN: SLAC-PUB 3061 (1983); A. C. DAVIS, M. DINE and N. SEIBERG: *Phys. Lett. B*, **125**, 487 (1983); H. P. NILLES: *Phys. Lett. B*, **129**, 103 (1983); J. M. GERARD and H. P. NILLES: *Phys. Lett. B*, **129**, 243 (1983); see also W. BUCHMULLER, R. PECCEI and H. YANAGIDA: *Nucl. Phys. B*, **227**, 503 (1983).

(9) E. WITTEN: *Nucl. Phys. B*, **202**, 253 (1982); S. CECOTTI and L. GIRARDELLO: *Phys. Lett. B*, **110**, 39 (1982); E. COHEN and G. GOMEZ: *Nucl. Phys. B*, **223**, 183 (1983).

(10) T. D. LEE and B. ZUMINO: *Phys. Rev.*, **163**, 1667 (1967), and references therein; see also, J. WESS and B. ZUMINO: *Phys. Rev.*, **163**, 1727 (1967). Application of the field-current identity in the context of (nonsupersymmetric) composite models of $W^{\pm}$ and Z has been considered recently by N. S. CRAIGIE and J. STERN: ICTP preprint IC/84/36 (1984), and B. SCHREMPP and F. SCHREMPP: DESY preprint, DESY-84-055 (1984).

(R-type symmetries, as global symmetries, will of course be taken into account.)

At first sight, the local G_F symmetry appears to be only approximate, being broken (among others) by vector self-interaction terms of the form

$$\mu^2 \int d^8z \operatorname{Tr} \exp[V], \tag{1.1}$$

where V is a set of composite vector supermultiplets, a matrix of a box-diagonal form in the G_F space, and μ is a parameter with the dimension of mass. Our effective action in fact looks as a straightforward supersymmetric generalization of the one, involving the ρ-mesons, nucleons and pions, given many years ago by LEE and ZUMINO (10).

Our central observation is that in supersymmetric theories, a simple reparametrization of the form

$$\mu^2 \exp[V] = M^\dagger \exp[\hat{V}] M \tag{1.2}$$

(where M is a chiral superfield of a box-diagonal matrix form in G_F space), together with appropriate reparametrization of other composite fields, allows us to bring S_{eff} into an equivalent form which possesses an exact local G_F invariance. Such an equivalence is similar to the one between nongauge and gauge Lagrangians describing the same U_1 Higgs model, first noticed by FAYET (11).

The way the chiral superfield M transforms under this local G_F group is of particular interest. Take, for instance, the case $G_F = U_K$. M behaves under the effective U_K gauge group as K multiplets, $\{\Psi_i\}$,

$$M = ((\Psi_1)(\Psi_2) \dots (\Psi_K)), \tag{1.3}$$

each of which transforms as in the fundamental representation.

For a more general G_F, say $G_F = \prod_i U_{n_i}$, each submatrix M_i of M transforms as n_i fundamental multiplets of the U_{n_i} and singlets of other U_{n_j}'s.

Natural appearance of such a generationlike structure of composite matter multiplets is very interesting in view of an eventual application in a composite model of the presently known « elementary » particles (12).

Another salient feature is that the effective G_F gauge symmetry is exact

(11) P. FAYET: *Nuovo Cimento A*, **34**, 626 (1976); see also G. MAINLAND and K. TANAKA: *Phys. Rev. D*, **12**, 2394 (1975).

(12) For recent reviews on supersymmetric composite models, see, *e.g.*, W. BUCHMULLER: CERN preprint, TH 3873 (1984); R. D. PECCEI: MPI preprint MPI-PAE/PTh 35/84; G. VENEZIANO: *Proceedings of the I Capri Symposium* (1983).

even when the original global G_F symmetry is partially broken by superpotentials and/or strong anomaly. The point is that the composite fields transform differently under the effective gauge group and under the original global group. For instance, if $G_F = U_K$, M transforms as K rows of fundamental multiplets of the global and not like K columns as in eq. (1.3). See sect. **3**.

In this paper, we shall not discuss the question of dynamical supersymmetry breaking (*), although there has been some interesting developments recently ([4]). We believe, however, that the main conclusions of the present paper (sect. **2** and **3**) are independent of (and compatible with) the possible dynamical breaking of supersymmetry.

Another omission is the effects of non-Abelian anomalies ([13]), to whose implications we hope to come back in the near future.

This paper will be organized as follows. In sect. **2**, we study the general structure of the low-energy effective action in a supersymmetric confining theory, by using a generalization of the Lee-Zumino construction of L_{eff} ([10]) (« field-current identity »; see subsect. **2**'1), and by studying the chiral Ward-Takahashi identities associated with unbroken, broken and anomalous global symmetries (subsect. **2**'2 and **2**'3).

The emergence of the effective gauge symmetry and resulting characteristic structure of composite matter multiplets will be discussed in detail in sect. **3**. This section contains the exposition of the main physics ideas of the present paper.

In sect. **4** and **5**, we illustrate the general ideas by constructing the effective Lagrangian explicitly, in two models. In sect. **4**, a chiral SU_6 model will be studied. Minimization of the scalar potential shows that the effective gauge symmetry ($U_2 \times U_1$) gets spontaneously broken, while the original global (SU_2) symmetry remains exact.

The results for supersymmetric QCD with M(flavours) $< N$(colours) will be presented in sect. **5**. We show that the introduction of the composite vector supermultiplets as low-energy degrees of freedom is perfectly compatible with all the known results in SQCD ([2,3,7]). Furthermore our effective Lagrangian leads to new, complementary results involving all possible sets of bilinear condensates.

In particular, the dynamical supersymmetry breaking in the massless limit *à la* PESKIN ([8]) seems to be excluded.

Section **6** contains summary and outlook.

(*) Some of the ideas of the present paper has been first presented in an unpublished note by one of us (KK), which studied a SU_5 model where supersymmetry is believed to be broken dynamically ([4]). Unfortunately, a proper treatment of that model seems to require the introduction of composite superfields which are neither chiral not real, and consequently to require a more involved description than in the effective Lagrangian proposed there. The present paper is a fully revised and extended version of that note.

([13]) J. WESS and B. ZUMINO: *Phys. Lett. B*, **37**, 95 (1971).

2. – Structure of the effective action involving composite vector supermultiplets.

2·1. *Field-current identity.* – In this section we begin to study the structure of the low-energy effective Lagrangian of a general supersymmetric confining theory, involving certain composite vector (real) supermultiplets as low-energy degrees of freedom. We consider a supersymmetric gauge theory (*), with a set of matter chiral (scalar) superfields $\{\Phi\}$ interacting through the action

$$S = \int d^8z\, \Phi^{\dagger i} \exp[V_i]\, \Phi_i + d^6z\, \mathscr{P}(\Phi) + \text{h.c.} + S_{\text{gauge}}, \tag{2.1}$$

where the «flavour» index i labels all the matter fields present (the «colour» indices are, however, kept implicit throughout), $\mathscr{P}$ is the superpotential, and S_{gauge} contains the usual gauge kinetic terms and gauge-fixing and ghost terms. The gauge vector multiplet V enters the first term of S through $V_i = V^a T^a_i$, where T^a_i are the generators of the gauge group (*e.g.*, $a = 1, \ldots, N^2-1$ for the SU_N group) in the basis of the representation appropriate for Φ_i.

The composite vector superfields we shall be interested in are the multiplets (**),

$$R^j_i \equiv \Phi^{\dagger j} \exp[V_i]\, \Phi_i . \tag{2.2}$$

Nondiagonal elements ($i \neq j$) will be considered only between two states belonging to an identical representation of the strong gauge group, for which $V_i = V_j$. Thus R has a box-diagonal matrix form in the flavour space.

Suppose that the Lagrangian possesses a global SU_L symmetry associated with the transformations among Φ_i's, $i = 1, 2, \ldots, L$. The U_1 symmetry related to the common phase rotation of the L Φ fields is broken by the strong anomaly, and it will be taken into account in subsect. 2·3 (***). The properties of the effective action under the full U_L transformations will be important in the discussions of the effective gauge symmetry in sect. 3. For the moment, we study the consequences of an unbroken, nonanomalous SU_L symmetry.

Writing the vector component of R in the Wess-Zumino gauge, one finds that (t^a's are the SU_L generators)

$$(t^a)^i_j (R^j_i)_{\theta\sigma^\mu\bar\theta} \equiv \operatorname{Tr} t^a R|_{\theta\sigma^\mu\bar\theta} = J^a_\mu , \tag{2.3}$$

(*) Throughout, we shall use the notation of ref. (14).

(14) J. WESS and J. BAGGER: *Supersymmetry and Supergravity* (Princeton University Press, Princeton, N. J., 1983).

(**) Such composite vector multiplets have been already considered in the literature: see, *e.g.*, ref. (15).

(15) W. BUCHMULLER, R. D. PECCEI and T. YANAGIDA: *Nucl. Phys. B*, **231**, 53 (1984).

(***) This U_1 symmetry may further be broken by the superpotential which (by assymption) respects SU_L. This will be also taken into consideration in subsect. 2·3.

where J^a_μ ($a = 1, \dots, L^2-1$) are the standard conserved Noether currents associated with the global SU_L symmetry. Equation (2.3) is thus the field-current identity ([10]) (and eq. (2.2) its supersymmetric generalization), analogous to that discussed many years ago by LEE and ZUMINO in the context of the hadron physics. There, the assumption that the full isospin currents are proportional to the ρ fields led to an effective Lagrangian involving the ρ-mesons (together with π's and nucleons) which possessed an approximate *local* isospin symmetry, broken by the mass terms of the ρ-mesons only ([10]).

Supersymmetric generalization of the Lee-Zumino construction is best carried out in the superfield formalism. Considering, for the moment, only the composite fields R of eq. (2.2), the effective action can be written in general as

$$S_{\text{eff}} = \int d^6z\,(1/4k)\,\text{Tr}\,W^\alpha W_\alpha + \text{h.c.} + \int d^8z\,F(R)\,, \tag{2.4}$$

where

$$W_\alpha \equiv -(1/4)\bar{D}^2 R^{-1} D_\alpha R\,. \tag{2.5}$$

Notice that we have introduced a matrix notation in the «flavour» $L\times L$ space, which will be used whenever possible hereafter. The first two terms of eq. (2.4) contain the kinetic term for the vector component, and $F(R)$ is a function of R which, by assumption, is invariant under the global SL_L transformations.

The current conservation of the original theory reads (*) (t^a, $a = 1, \dots, L^2$, are the SU_L generators)

$$\bar{D}^2\{\Phi^{\dagger j}\exp[V]\,(t^a)_j^i\Phi_i\} = 0\,, \tag{2.6a}$$

$$D^2\{\Phi^{\dagger j}\exp[V]\,(t^a)_j^i\Phi_i\} = 0\,. \tag{2.6b}$$

The field-current identity, eq. (2.2), then implies the «field conservation» equations

$$\bar{D}^2\,\text{Tr}\,(Rt^a) = 0\,, \tag{2.7a}$$

$$D^2\,\text{Tr}\,(Rt^a) = 0\,. \tag{2.7b}$$

We prove now the following theorem:

Theorem. In order for the «field conservation» equations (2.7) to hold

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(*) The standard current divergence equation is contained in the lowest component of $[D^2\times \text{eq. (2.6}a) - \bar{D}^2\times \text{eq. (2.6}b)]$. In supersymmetric theories, however, eq. (2.6*a*) and eq. (2.6*b*) hold separately; they form natural generalizations of the usual current conservation equations. In what follows we shall refer to eq. (2.6*a*) or eq. (2.6*b*) as « current conservation equation »; no confusion should arise from it.

in the effective theory, $F(R)$ must be of the form (c, ξ are constants)

$$F(R) = c\,\mathrm{Tr}\,(R) + \xi\,\mathrm{Tr}\,(\log R)\,. \tag{2.8}$$

Proof. From the equation of motion of R, one gets

$$-(1/2k)D^{\alpha}(RW_{\alpha}R^{-1})^{k}_{i} + R^{k}_{j}\,\delta F/\delta R^{i}_{j} = 0\,, \tag{2.9}$$

after using the dual identity

$$D^{\alpha}(RW_{\alpha}R^{-1}) = -R\bar{D}_{\dot{\alpha}}(R^{-1}\,\bar{W}^{\dot{\alpha}}R)R^{-1}\,. \tag{2.10}$$

On applying D^2 to eq. (2.9) one finds (for each (k, i)),

$$D^2(R^{k}_{j}\,\delta F/\delta R^{i}_{j}) = 0\,. \tag{2.11}$$

Also, by using the dual identity eqs. (2.10) in (2.9), one gets another relation

$$\bar{D}^2\{(\delta F/\delta R^{j}_{i})R^{j}_{k}\} = 0\,. \tag{2.12}$$

On the other hand, the SU_L invariance of the effective theory implies the associated currents to be conserved. By considering the variation

$$R \to \exp[-i\Lambda^{a\dagger}t^{a}]\,R\,\exp[i\Lambda^{a}t^{a}]\,,$$

and taking the functional derivatives with respect to Λ^a and $\Lambda^{a\dagger}$ independently, we find the current conservation equations

$$D^2\{(t^{a}R)^{i}_{j}\,\delta F/\delta R^{i}_{j}\} = 0\,, \tag{2.13a}$$

$$\bar{D}^2\{(Rt^{a})^{i}_{j}\,\delta F/\delta R^{i}_{j}\} = 0\,. \tag{2.13b}$$

They are indeed satisfied due to eqs. (2.11) and (2.12).

Now, in order for eqs. (2.13) to imply the «field-conservation» equations, eqs. (2.7), F must contain a term whose derivative with respect to R^{i}_{j} gives δ^{j}_{i}. This leads to the first term of eq. (2.8). Furthermore, F may in general contain other terms which, when inserted into eqs. (2.13), give zero identically. It means that the derivative of these extra terms in F with respect to R^{i}_{j} must give either $(R^{-1})^{j}_{i}$ or zero. Such terms can be combined into the form of the second term in eq. (2.8). *Q.e.d.*

Note. Actually, the consideration of SU_L symmetry alone would allow for addition of a general function $f(\det R)$ in eq. (2.8); such a term can be, however, excluded from the consideration of the anomalous U_1 WT identity as discussed in subsect. **2**'3. We have anticipated this result in eq. (2.8).

Let us next consider a more general case in which composite fields other than R are also present. For simplicity we assume all other composite fields are chiral (scalar) superfields, and denote them collectively by $\{T\}$. Let us assume S_{eff} to have the following general form:

$$(2.14) \qquad S_{\text{eff}} = (1/4k)\int d^6z \,\text{Tr}\, W^\alpha W_\alpha + \text{h.c.} + \int d^8z\, F(R) + \int d^8z\, G(T, T^\dagger; R) + \int d^6z\, H(T) + \text{h.c.}\,,$$

where $H(T)$ is a superpotential depending on chiral superfields $\{T\}$ only, $F(R)$ is a function of R only, and G depends on T and $T^\dagger$ (and perhaps also on R). W_α is defined by eq. (2.5) as before, and k is some constant.

Nonrenormalization theorem ([16]), valid to all orders of perturbation, implies that $H(T)$ contains the original superpotential $\mathscr{P}(\Phi)$, formally rewritten as a function of composite superfields; $H(T)$, however, may contain terms which arise due to nonperturbative effects such as instantons ([1,4]). (See also subsect. **2**'**3**. below.)

By assumption, F, G, and H are all invariant under the global SU_L transformations.

From the equation of motion of R (*), it follows that

$$(2.15) \qquad \bar{D}^2\{(\delta F/\delta R^j_i + \delta G/\delta R^j_i)R^j_k\} = 0\,.$$

Moreover, the equation of motion for the l-th T field is

$$(2.16) \qquad -(1/4)\bar{D}^2(\delta G/\delta T_l) + \delta H/\delta T_l = 0\,.$$

By using eqs. (2.15) and (2.16) one sees that the SU_L current conservation equation

$$(2.17) \qquad -(1/4)\bar{D}^2[(\delta F/\delta R^j_i + \delta G/\delta R^j_i)(Rt^a)^j_i + (t^a_T T)_l\,\delta G/\delta T_l] + (t^a_T T)_l\,\delta H/\delta T_l = 0$$

is indeed satisfied. (In eq. (2.17), we denoted the SU_L generators in the basis of the (in general, reducible) representation according to which T transforms, by t^a_T.)

Since F is a function of R only and G depends on T and $T^\dagger$, the only way the field conservation eq. (2.7a) to follow from eq. (2.17) is that G satisfies

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([16]) M. T. GRISARU, W. SIEGEL and M. ROCEK: *Nucl. Phys. B*, **159**, 429 (1979), and references therein.

(*) These equations appear always in a chiral-antichiral pair (as in eqs. (2.6), (2.7), (2.13)). In what follows we shall write only one of them.

(the last term of eq. (2.17) vanishing by the SU_L invariance of H)

$$(2.18)\qquad -(1/4)\bar{D}^2[\delta G/\delta R^j_i(Rt^a)^j_i+(t^a_T T)_l\,\delta G/\delta T_l]=0\,,$$

and that F is given by eq. (2.8) as before. But eq. (2.18) (and its Hermitian conjugate) means that G is a function of T, $T^\dagger$ and R which is invariant under the *local* SU_L transformations.

We are thus led to the following general form of the effective action (*):

$$(2.19)\qquad S_{\text{eff}}=\int d^8z\,\text{Tr}\,R+\xi\int d^8z\,\text{Tr}\,(\log R)+\int d^8z\,G(T,T^\dagger;R)+ \\ +(1/4k)\int d^6z\,\text{Tr}\,W^\alpha W_\alpha+\text{h.c.}+\int d^6z\,H(T)+\text{h.c.}\,,$$

where G and H (hence, all terms of S_{eff} except for the Tr R term) are invariant under the local SU_L transformation

$$(2.20)\qquad \begin{cases} T\ \to \exp[i\Lambda^a t^a_T]\,T\,; \\ T^\dagger \to T^\dagger \exp[-i\Lambda^{a\dagger}t^a_T]\,; \\ R\ \to \exp[-i\Lambda^{a\dagger}t^a]\,R\exp[i\Lambda^a t^a] \end{cases}$$

and ξ (dimensional) and k (dimensionless) are (in principle) calculable constants.

Equation (2.19) is the main result of this subsection. S_{eff} has the structure of an approximate gauge theory; the local SU_L invariance is broken by the Tr R term alone. By writing $R=\mu^2\exp[V_R]$, one sees that eq. (2.19) is quite analogous to the effective Lagrangian containing the ρ and other low-lying hadrons obtained in ref. ([10]).

2·2. *S_{eff} as truncated generating functional Γ*; WT *identities associated with unbroken symmetries.* – In this subsection the structure of S_{eff} in eq. (2.19) will be understood more directly as a consequence of the chiral Ward-Takahashi identities.

Let us again start with the case in which the only composite states considered are the vector supermultiplets R, eq. (2.2). We assume, as before, that the original action S (eq. (2.1)) is invariant under a global SU_L.

Consider the generating functional $Z(J_R)$ of the connected Green's functions,

$$(2.21)\qquad \exp[Z]=\int\mathcal{D}\Phi\,\mathcal{D}\Phi^\dagger\ldots\exp\left[i\left\{S+\int d^8z\,\Phi^{\dagger j}\exp[V]\,\Phi_i(J_R)^i_j\right\}\right],$$

(*) We absorbed the constant c in eq. (2.8) into the R-field; Tr W^2 term is unchanged by this redefinition (recall eq. (2.5) for the definition of W_α).

where S is given by eq. (2.1) and ${J_R}^i{}_j$ are the sources of the composite fields $\Phi^{\dagger j} \exp[V]\, \Phi_i$ (*). Let us define R by

$$\delta Z/\delta {J_R}^j{}_i \equiv R^i_j \tag{2.22}$$

and introduce the following generating functional $\Gamma(R)$ by

$$\Gamma(R) \equiv Z(J_R) - \int d^8z \,\mathrm{Tr}\,(RJ_R)\,. \tag{2.23}$$

It follows from eq. (2.23) that

$$\delta\Gamma/\delta R^j_i = -\,(J_R)^i_j\,. \tag{2.24}$$

We wish to know how $\Gamma(R)$ transforms under the local SU_L transformations

$$R \to \exp[-i\Lambda^{a\dagger}t^a]\, R \exp[i\Lambda^a t^a]\,. \tag{2.25}$$

As usual in supersymmetric theories, we consider the variations with respect to Λ^a and $\Lambda^{a\dagger}$ independently. Under Λ^a transformations, Γ transforms as

$$\delta\Gamma = \int d^8z\, (\delta\Gamma/\delta R^j_i)\cdot(Ri\Lambda^a t^a)^j_i\,, \tag{2.26}$$

hence

$$\delta\Gamma(R\exp[i\Lambda^a t^a])/i\,\delta\Lambda^a|_{\Lambda=0} = (1/4)\bar{D}^2(J_R R t^a)\,, \tag{2.27}$$

where relation (2.24) has been used.

Information on the right-hand side comes from the definitions, eqs. (2.21) and (2.22). Consider $Z(\exp[-i\Lambda^a t^a]\, J_R)$. On the right-hand side of eq. (2.21), the variation of the source term is compensated by a change of (functional) variables

$$\Phi_i = \{\Phi' \exp[i\Lambda^a t^a]\}_i \qquad (i = 1, \ldots, L)\,, \tag{2.28}$$

which has the form of a local SU_L transformation. All the terms are invariant in the exponent of eq. (2.21) except for the first term of S, which becomes

$$\Phi^{\dagger i} \exp[V]\, \Phi_i = \Phi^\dagger \exp[V]\, \Phi'_k \{\exp[i\Lambda^a t^a]\}^k_i\,. \tag{2.29}$$

Thus

$$\exp\Big[Z(\exp[-i\Lambda^a t^a]\, J_R)\Big] = \Big\{1 + \int d^8z\, i(\Lambda^a t^a)^k_i\, \delta/\delta {J_R}^k{}_i + \ldots\Big\} \exp[Z(J_R)] = \tag{2.30}$$

$$= \exp[Z(J_R)] \Big\{1 + \int d^8z\, i\Lambda^a\, \mathrm{Tr}\,(t^a R) + \ldots\Big\}\,.$$

(*) Notice that $(J_R)^i_j$ are *not* external fields, often introduced to gauge the « flavour » group SU_L. Our J_R are the sources of physical composite particles $\Phi^{\dagger j} \exp[V]\, \Phi_i$.

Comparing the both sides, we get the Ward-Takahashi identity

$$\bar{D}^2\,\mathrm{Tr}\,(t^a J_R R) = -\,\bar{D}^2\,\mathrm{Tr}\,(t^a R)\,. \tag{2.31}$$

Inserting this into eq. (2.27) we finally obtain

$$\delta\Gamma(R\exp[i\Lambda^a t^a])/i\,\delta\Lambda^a|_{\Lambda=0} = -\,(1/4)\bar{D}^2\,\mathrm{Tr}\,(t^a R)\,. \tag{2.32a}$$

An analogous result for the variation with $\Lambda^{a\dagger}$

$$\delta\Gamma(\exp[-\,i\Lambda^{a\dagger}t^a]\,R)/i\,\delta\Lambda^{a\dagger}|_{\Lambda^\dagger=0} = (1/4)D^2\,\mathrm{Tr}\,(t^a R) \tag{2.32b}$$

can be easily found.

Equations (2.32) show how Γ transforms under the local SU_L: it must have the form

$$\Gamma(R) = \int \mathrm{d}^8z\,\mathrm{Tr}\,(R) + \\ + (\text{terms invariant under } R \to \exp[-\,i\Lambda^{a\dagger}t^a]\,R\exp[i\Lambda^a t^a])\,. \tag{2.33}$$

Now, for slowly varying functions (of x) R, $\Gamma(R)$ may be identified as S_{eff} (*). If we keep the lowest-dimensional, ordinary kinetic terms only, we get precisely eqs. (2.4) and (2.8), obtained in subsect. **2·1** (note that the second term of eq. (2.8) is invariant under the local SU_L).

The above discussion can be generalized to the cases in which other composite (chiral) superfields are also present, in a straightforward manner. We introduce the composite chiral fields, $\mathcal{T}_l(\Phi)$, and their chiral sources, J_T^l, and consider

$$\exp[Z(J_R, J_T, J_T^\dagger)] = \\ = \int \mathcal{D}\Phi\,\mathcal{D}\Phi^\dagger \ldots \exp\left[i\left\{S + \int \mathrm{d}^8z\,\Phi^{\dagger i}\exp[V]\,\Phi_i(J_R)^i_j + \mathrm{d}^6z\,\mathcal{T}_l(\Phi)\,J_T^l + \text{h.c.}\right\}\right]. \tag{2.34}$$

Define

$$\delta Z/\delta J_R{}^i{}_j \equiv R^j_i\,, \qquad \delta Z/\delta J_T^l \equiv T_l\,, \qquad \delta Z/\delta J_T^{l\dagger} \equiv T_l^\dagger \tag{2.35}$$

and introduce the new generating functional

$$\Gamma(R;\,T,\,T^\dagger) = Z - \int \mathrm{d}^8z\,\mathrm{Tr}\,(RJ_R) - \int \mathrm{d}^6z\,T_l J_T^l - \text{h.c.} \tag{2.36}$$

(*) Note that for on-shell momenta (of the composition particles), Γ constructed as above gives the correct S-matrix elements. The same comment applies for Γ introduced below, eq. (2.36) and eq. (2.50).

The dual of eqs. (2.35) are:

(2.37) $$\delta\Gamma/\delta R^i_j = -J_R{}^j{}_i\,, \qquad \delta\Gamma/\delta T_l = -J^l_T\,, \text{ etc.}$$

The chiral WT identities of the unbroken SU_L can be obtained, as before, by considering the variation

$$J_R \to \exp[-i\Lambda^a t^a]\, J_R \exp[i\Lambda^{a\dagger} t^a]\,;$$

$$J_T \to \exp[-i\Lambda^a t^a_T]\, J_T\,, \text{ etc.},$$

where t^a_T are the SU_L generators appropriate for $\mathcal{T}(\Phi)$, in eq. (2.34). We find

(2.38) $$-(1/4)\bar{D}^2\,\mathrm{Tr}\,(t^a J_R R) + T_l(t^a_T J_T)^l = (1/4)\bar{D}^2\,\mathrm{Tr}\,(t^a R)\,,$$

which generalizes eq. (2.31).

The way Γ transforms under the local SU_L, as a functional of R, T and $T^\dagger$, is found now by using the WT identities eq. (2.38) and the definition eq. (2.36). We get finally

(2.39) $$\delta\Gamma(R\exp[i\Lambda^a t^a];\, T\exp[i\Lambda^a t^a];\, T^\dagger)/i\,\delta\Lambda^a|_{\Lambda=0} = -(1/4)\bar{D}^2\,\mathrm{Tr}\,(t^a R)$$

and its Hermitian conjugate following from the consideration of variations with $\Lambda^{a\dagger}$.

Equation (2.39) (and its Hermitian conjugate) are the main results of this subsection. If we identify Γ as S_{eff} for slowly varying functions R, T and $T^\dagger$, they provide the constraints how S_{eff} should transform under the local SU_L transformation eq. (2.20). Keeping the simplest kinetic terms for the vector component, we get precisely eq. (2.19), obtained earlier.

2·3. *U_1 anomaly, symmetries broken by superpotentials and* WT *identities.* – In the previous subsections we studied the constraints on S_{eff} following from the presence of an exact global symmetry. Further constraints on S_{eff} come from the U_1 and related anomalies ([5]). In view of the recent developments in this field, in particular due to an improved understanding of nonperturbative effects in supersymmetric gauge theories ([1-4]), it seems to us of crucial importance to take them into account in the construction of effective Lagrangians.

Also, because of the nonrenormalization theorem, the pattern of explicit breaking of symmetries by superpotentials persist to all orders of perturbation. This puts another constraint on the possible form of the low-energy effective Lagrangians.

In order to generalize the discussion of subsect. 2·2 to include these constraints, we need to introduce further sources. First consider the superpotential $\mathcal{P}(\Phi)$. Under the full chiral group G_F of global transformation of *matter* fields,

$$\Phi \to \Phi \exp [i\alpha^a t^a] , \tag{2.40}$$

(t^a = the generators and α^a real constant parameters of G_F), $\mathscr{P}(\Phi)$ may transform in a nontrivial way. However $\mathscr{P}(\Phi)$, being a polynomial of the fundamental fields $\{\Phi\}$, can always be written, by using constant vectors and tensors, as

$$\mathscr{P}(\Phi) = h^i \mathscr{P}_i(\Phi) , \quad h^{ij} \mathscr{P}_{ij}(\Phi) , \quad \text{etc.},$$

or as a sum of such terms. $\mathscr{P}_i$, $\mathscr{P}_{ij}$, etc., transform as irreducible tensors of G_F and h^i, h^{ij}, ... are coupling constants. Let us then write

$$\mathscr{P}(\Phi) = h^A \mathscr{P}_A(\Phi) , \tag{2.41}$$

where $\mathscr{P}_A(\Phi)$ is a (in general, reducible) tensor of G_F. $\mathscr{P}(\Phi)$ then transforms under eq. (2.40) as

$$\mathscr{P} = h^A \mathscr{P}_A \to h^A \mathscr{P}_B (\exp [i\alpha^a t^a_P])^B_A , \tag{2.42}$$

if t^a_P's are the generators of G_F appropriate for $\mathscr{P}_A$.

We shall introduce a (chiral) source J^A_P for each of $\mathscr{P}_A$.

Next consider another composite chiral superfield, $(g_S^2/32\pi^2) W^\alpha_S W_{S,\alpha}$ and introduce its (chiral) source J_S, where W^α_S is the spinor chiral field describing the elementary strong gauge bosons and gauginos (*).

The system may contain of course chiral composite fields other than $\mathscr{P}_A$'s and W^2_S, as low-energy degrees of freedom. Let us denote them by $\mathscr{T}_l(\Phi)$ and their sources by J^l_T as before, and assume that $\mathscr{T}$'s transform under (2.40) as

$$\mathscr{T}_l(\Phi) \to \mathscr{T}_m(\Phi)(\exp [i\alpha^a t^a_T])^m_l , \tag{2.43}$$

where t^a_T are the generators of G_F in the (in general, reducible) representation appropriate for $\mathscr{T}(\Phi)$.

We study then the following generating functional:

$$\begin{aligned} \exp [Z(J_R; J_T; J_P; J_S; J^\dagger_T; J^\dagger_P; J^\dagger_S)] = \\ = \int \mathscr{D}\Phi\, \mathscr{D}\Phi^\dagger \ldots \exp \Big[i \Big\{ S + \int d^8z\, (\Phi^{\dagger j} \exp [V]\, \Phi_i)(J_R)^i_j + \\ + \int d^6z\, \mathscr{T}_l J^l_T + \text{h.c.} + \int d^6z\, \mathscr{P}_A J^A_P + \text{h.c.} + \\ + \int d^6z\, (g_S^2/32\pi^2)\, W_S W_S J_S + \text{h.c.} \Big\} \Big] , \end{aligned} \tag{2.44}$$

where the action S is given in eq. (2.1).

(*) Not to be confused with W^α (eq. (2.5)) which describes the composite vector superfields. g_S is the gauge coupling constant.

In order to derive the WT identities associated with G_F, consider now the local G_F-transformation of the sources (with arbitrary chiral superfields Λ^a's)

$$\begin{cases} J_R{}^i{}_j \to (\exp[-i\Lambda^a t^a])^i_k J_R{}^k{}_j\,, \\ J^l_T \;\to (\exp[-i\Lambda^a t^a_T])^l_m J^m_T\,, \\ J^A_P \;\to (\exp[-i\Lambda^a t^a_P])^A_B J^B_P \end{cases} \tag{2.45}$$

($J^\dagger_T$, $J^\dagger_P$, J_S and $J^\dagger_S$ are kept invariant) in eq. (2.44). On the right-hand side, the variations of the sources terms are compensated if we make a change of functional variables

$$\Phi \to \Phi \exp[i\Lambda^a t^a]\,, \qquad \Phi^\dagger \to \Phi^\dagger\,. \tag{2.46}$$

The total change on the r.h.s. of eq. (2.44) is then given by the variation of the action (which can be obtained from eqs. (2.1) and (2.42)) and by the U_1 anomaly. Thus

$$\begin{aligned}\exp\Big[Z(\exp[-i\Lambda^a t^a]\,J_R;\ \exp[-i\Lambda^a t^a_T]\,J_T;\ \exp[-i\Lambda^a t^a_P]\,J_P;\\ J_S;\ J^\dagger_T;\ J^\dagger_P;\ J^\dagger_S)\Big] = \int \mathscr{D}\Phi\, \mathscr{D}\Phi^\dagger \ldots \exp i\Big[(\text{source terms}) + \\ + \int d^8z\, \Phi^{\dagger i} \exp[V]\, \Phi_j (\exp[i\Lambda^a t^a])^j_i + \int d^6z\, h^A \mathscr{P}_B (\exp[i\Lambda^a t^a_P])^B_A + \\ + \int d^6z\, (g_S^2/32\pi^2)\, W_S W_S\, i\Lambda^a\, \mathrm{Tr}\,(t^a C_\Phi) + O(\Lambda^2)\Big]\,.\end{aligned} \tag{2.47}$$

The last explicit term in the exponent is the U_1 anomaly term (5). C_Φ is the quadratic Casimir operator (*) of the strong gauge group. It is a diagonal matrix in the flavour space G_F with elements $(C_\Phi)_{ii} \equiv \sum_\alpha (T^\alpha_i)^2$. In the functional-integral derivation *à la* FUJIKAWA (17), the anomaly term arises from the Jacobian of the transformation, eq. (2.46) (18).

By comparing the terms of first order in Λ^a on both sides of eq. (2.47), we find the chiral WT identities (**)

$$\begin{aligned}(\bar{D}^2/4)\,\mathrm{Tr}\,(t^a J_R R) - T_l (t^a_T J_T)^l - P_A (t^a_P J_P)^A = \\ = -(\bar{D}^2/4)\,\mathrm{Tr}\,(t^a R) + h^A (P t^a_P)_A + \mathrm{Tr}\,(C_\Phi t^a) S\,,\end{aligned} \tag{2.48}$$

(*) Normalized to that in the fundamental representation.

(17) K. FUJIKAWA: *Phys. Rev. Lett.*, **42**, 1195 (1979); *Phys. Rev. D*, **21**, 2848 (1980).

(18) K. KONISHI and K. SHIZUYA: *Nuovo Cimento A*, **90**, 111 (1985); K. KONISHI: *Proceedings of the XIX Rencontre de Moriond* (1984).

(**) This is the most general form of the chiral WT identities containing the effects of explicit symmetry breaking and of the U_1 anomaly. By taking arbitrary number of derivatives with respect to the sources and by setting them to zero, one finds variety where we have introduced the definitions (*),

$$(2.49)\qquad R^i_j \equiv \delta Z/\delta J_{R\,i}^{\;j}\,,\qquad T_l \equiv \delta Z/\delta J^l_T\,,\qquad P_A \equiv \delta Z/\delta J^A_P\,,\qquad S \equiv \delta Z/\delta J_S\,.$$

Another set of WT identities

$$(2.48b)\qquad (D^2/4)\,\mathrm{Tr}\,(RJ_R t^a) - T^\dagger_l (J^\dagger_T t^a_T)^l - P^\dagger_A (J^\dagger_P t^a_P)^A = \\ = -\,(D^2/4)\,\mathrm{Tr}\,(t^a R) + h^{*A}(t^a_P P^\dagger)_A + \mathrm{Tr}\,(t^a C_\Phi)\,S^\dagger\,,$$

which are the Hermitian conjugates of eq. (2.48a), follow from the consideration of variations with respect to $\Lambda^{a\dagger}$'s.

Let us introduce now the generating functional Γ by the Legendre transformation

$$(2.50)\qquad \Gamma(R;\,T;\,P;\,S;\,T^\dagger;\,P^\dagger;\,S^\dagger) = Z(J_R;\,J_T;\,\ldots) - \int \mathrm{d}^8z\,\mathrm{Tr}\,(RJ_R) - \\ -\int \mathrm{d}^6z\,(T_l J^l_T) - \text{h.c.} - \int \mathrm{d}^6z\,P_A J^A_P - \text{h.c.} - \int \mathrm{d}^6z\,SJ_S - \text{h.c.}$$

From this definition it follows that

$$(2.51)\qquad \begin{cases} \delta\Gamma/\delta R^j_i = -\,J_{R\,i}^{\;j}\,, & \delta\Gamma/\delta T_l = -\,J^l_T\,, \\ \delta\Gamma/\delta P_A = -\,J^A_P\,, & \delta\Gamma/\delta S = -\,J_S\,. \end{cases}$$

By using the (by now familiar) procedure, we find the transformation property of Γ from eqs. (2.50), (2.51) and (2.48a):

$$(2.52a)\qquad \delta\Gamma(R\exp[i\Lambda^a t^a];\,T\exp[i\Lambda^a t^a_T];\,P\exp[i\Lambda^a t^a_P];\,S;\,T^\dagger;\,P^\dagger;\,S^\dagger)/i\,\delta\Lambda^a|_{\Lambda=0} = \\ = -\,(\bar{D}^2/4)\,\mathrm{Tr}\,(t^a R) + h^A(Pt^a_P)_A + \mathrm{Tr}\,(C_\Phi t^a)\,S\,.$$

Equation (2.52a) and its Hermitian conjugate,

$$(2.52b)\qquad \delta\Gamma(\exp[-\,i\Lambda^{a\dagger}t^a]\,R;\,T;\,P;\,S;\,\exp[-\,i\Lambda^{A\dagger}t^a_T]\,T^\dagger; \\ \exp[-\,i\Lambda^{a\dagger}t^a_P]\,P^\dagger;\,S^\dagger)/i\,\delta\Lambda^{a\dagger}|_{\Lambda^\dagger=0} = \\ = (D^2/4)\,\mathrm{Tr}\,(t^a R) - h^{*A}(t^a_P P^\dagger)_A - \mathrm{Tr}\,(t^a C_\Phi)\,S^\dagger\,,$$

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of relations involving n-point functions. In simplest cases, they reduce to the ones considered in ref. (4,5,19), etc.

(19) G. VENEZIANO: *Phys. Lett. B*, **128**, 199 (1983); G. SHORE: *Nucl. Phys. B*, **231**, 139 (1984).

(*) There should not be any confusion in using the letter S here, which was used earlier to denote the action.

together with the WT identities eqs. (2.48), are the main formulae of this subsection. If we take R, T, P, S, ... to be slowly varying functions of the space-time and make a re-interpretation, $\Gamma \sim S_{eff}$, eqs. (2.52) specify how S_{eff} must transform under the local G_F transformations.

The first term on the right-hand sides of eqs. (2.52) is the same as that obtained in subsect. **2**'1 and **2**'2. The second term reflects the breaking of G_F by the superpotential; the last the U_1 (and related) anomaly.

It should be stressed that eqs. (2.52) themselves are exact formulae, whether or not the vector supermultiplets $\Phi^{\dagger j} \exp[V] \Phi_i$ are regarded as low-energy effective degrees of freedom. However, it is only when we introduce them as such, that the theory defined by S_{eff} satisfies exactly (formally at least) the required transformation properties, eq. (2.52).

On the contrary, if S_{eff} would be constructed without R fields, R in eqs. (2.52) should be re-expressed as a complicated unknown function of the fields, T, $T^\dagger$, ... Thus in such an effective theory (*) the constraints eqs. (2.52) may or (most likely) may not be satisfied. This is in fact one of the reasons why we think that S_{eff} involving composite vector superfields provides a better approximation to the low-energy dynamics than the simple effective actions containing chiral composite fields only.

Important results follow from eqs. (2.52). First, any subgroup of G_F broken either by the superpotential or by the strong anomaly is broken in the effective action by and only by the so-called F-terms, *i.e.* terms that can be written as

$$\int d^6z\,[...] + \text{h.c.}\,, \tag{2.53}$$

but not as

$$\int d^8z\,[...]\,. \tag{2.54}$$

In other words, only the superpotential terms in S_{eff} reflect the breaking of the global symmetry (**). *Vice versa*, any unbroken, nonanomalous symmetry of the original action is a symmetry of S_{eff} also.

Secondly, no D-terms (*i.e.* terms of the form of eq. (2.54)) other than

$$\int d^8z\, \mathrm{Tr}\,(R) \tag{2.55}$$

(that gives the first term in eq. (2.52)) can break the *local* G_F symmetry. Therefore, any kinetic terms of composite chiral fields T, P and S must be invariant under the local G_F transformations.

(*) Most of effective Lagrangians considered so far [7,8], involving only *chiral* composite fields as low-energy degrees of freedom, are of this type.

(**) The first term of eqs. (2.52) reflects the breaking of *local* G_F symmetry by the kinetic term $\Phi^\dagger \exp[V]\,\Phi|_D$: it does not correspond to the breaking of the global G_F.

These put quite strong constraints on the possible forms of S_{eff}; they are the consequences of WT identities eqs. (2.48) (*).

These results show that the form of S_{eff} found in subsect. **2·1** and **2·2** (eq. (2.19)) is essentially unchanged by the consideration of the full chiral group G_F. Namely, S_{eff} has the form

$$S_{eff}=\int d^8z\,[\mathrm{Tr}\,R+\xi\,\mathrm{Tr}\log R+G(R;\,T,\,T^\dagger;\,P,\,P^\dagger;\,S,\,S^\dagger)]+ +\int d^6z\,H(T;\,P;\,S)+\text{h.c.}+(1/4k)\int d^6z\,\mathrm{Tr}\,(W^\alpha W_\alpha)+\text{h.c.}\,, \tag{2.56}$$

where $W^\alpha=-(1/4)\bar{D}^2R^{-1}D^\alpha R$ and where:

i) $\mathrm{Tr}\log R$, $G(R;\,T,\,T^\dagger;\,...)$, and $\mathrm{Tr}\,(W^\alpha W_\alpha)$ terms are invariant under the local G_F transformations (notice that $\mathrm{Tr}\,WW$ term can be written in the form of eq. (2.52) as well).

ii) $\mathrm{Tr}\,R$ term is invariant under the global G_F transformation; it transforms under the local G_F as in the first terms of eqs. (2.52).

iii) The superpotential $H(T,P,S)$ transforms under the local G_F as in the second and third terms of eqs. (2.52). It transforms also under the global G_F. It is invariant under any global (and local) symmetry group ($\subset G_F$) of the original action.

iv) ξ (dimensional) and k (dimensionless) are (in principle) calculable constants.

We conclude this section with the following remark. The discussion and results of this section neither imply nor require that a local effective Lagrangian always exists, whose space-time integral has the form of eq. (2.56). In particular, the «superpotential» $H(T;\,P;\,S)$ may be a functional, rather than a function, of the fields T, P and S. The possible importance of models of this type will be mentioned later, at the end of sect. **4**.

3. – Effective gauge symmetry.

In this section we shall demonstrate that the effective action eq. (2.56) found in the last section is equivalent to one that possesses an exact local G_F

(*) In fact, we are assuming that the WT identities eqs. (2.48) hold for renormalized composite operators, which are really relevant in S_{eff}. This assumption seems reasonable, since the superpotential is itself invariant under renormalization (this is the content of the nonrenormalization theorem). It is crucial that we deal with supersymmetric theories; in nonsupersymmetric models, WT identities associated with broken symmetries would be of little use, unless the breaking is soft.

invariance. Recall that G_F is the full chiral flavour group of the original action that would be the global symmetry group if the superpotentials and the U_1 anomaly were both neglected.

Let us introduce the definition

$$R \equiv \mu^2 \exp [V_R] \tag{3.1}$$

in eq. (2.54), where V_R is the composite vector supermultiplet of a box-diagonal (form (a $K \times K$ matrix if $G_F = U_K$), and μ is an arbitrary constant with the dimension of mass (*). S_{eff} becomes

$$S_{\text{eff}} = \int d^8z\, [\mu^2 \operatorname{Tr} (\exp [V_R]) + \xi \operatorname{Tr} V_R + G(\mu^2 \exp [V_R];\, T;\, T^\dagger;\, ...)] + \\ + \int d^6z\, H(T;\, P;\, S) + \text{h.c.} + (1/4k)\int d^6z \operatorname{Tr} (W^\alpha W_\alpha) + \text{h.c.}\,, \tag{3.2}$$

where

$$W^\alpha \equiv -(1/4)\bar{D}^2 \exp [-V_R]\, D^\alpha \exp [V_R]\,. \tag{3.3}$$

Although we begin to see a resemblance to a gauge theory, the local G_F invariance is broken in eq. (3.2) by $\operatorname{Tr} \exp [V_R]$ term. (See i)-iii) after eq. (2.56).) (The superpotential H may also break part or whole of the local G_F).

However, let us now make the following reparametrization:

$$R = \mu^2 \exp [V_R] = M^\dagger \exp [\hat{V}_R]\, M\,, \tag{3.4a}$$

$$T = \hat{T} g_T(M)\,, \tag{3.4b}$$

$$P = \hat{P} g_P(M) \tag{3.4c}$$

(S, which is a G_F singlet, is left unchanged) and

$$\hat{V}_R^n = 0 \qquad \text{for } n \geqslant 3. \tag{3.5}$$

(This restriction will be removed in a moment.) Note that this is precisely the transformation needed to diagonalize all the kinetic terms.

In eqs. (3.4), M is a chiral superfield of a box-diagonal matrix form (a $K \times K$ matrix if $G_F = U_K$), and $g_T(M)$, $g_P(M)$ are defined by

$$g_{T,P}(M) \equiv \exp [i\Lambda^a t^a_{T,P}] \tag{3.6}$$

(*) μ is unrelated to the renormalization-group invariant mass scale of the strong gauge theory. In fact, it can always be set to unity by a redefinition of the scalar component of V_R.

if

$$M = \exp [i\Lambda^a t^a] \tag{3.7}$$

for some set of chiral superfields Λ^a, t^a, t^a_T, and t^a_P are appropriate G_F generators.

What we have done in eqs. (3.4) is to separate the « Wess-Zumino gauge » part of V_R as $\hat{V}_R$, and then to call the rest of the degrees of freedom of $\exp [V_R]$ as M. M represents physical degrees of freedom because S_{eff} of eq. (3.2) has no local G_F invariance.

Since eqs. (3.4) have the form of a local G_F transformation and because $\text{Tr}\, V_R$, $G(\mu^2 \exp [V_R], T, ...)$ and $\text{Tr}\, WW$ terms are invariant under local G_F transformations, we get

$$\begin{aligned} S_{\text{eff}} = \int d^8z\, [\text{Tr}\, M^\dagger \exp [\hat{V}_R]\, M + \xi\, \text{Tr}\, \hat{V}_R + \\ + G(\exp [\hat{V}_R]; \hat{T}; \hat{T}^\dagger; \hat{P}; \hat{P}; S; S^\dagger)] + \int d^6z\, H(\hat{T} g_T(M); \hat{P} g_P(M); S) + \text{h.c.} + \\ + (1/4k) \int d^6z\, \text{Tr}\, (\hat{W}^\alpha \hat{W}_\alpha) + \text{h.c.}\,. \end{aligned} \tag{3.8}$$

At this point, we may drop the constraint eq. (3.5), and allow for some arbitrariness in the reparametrization, eqs. (3.4).

But, then, S_{eff} of eq. (3.8) has an *exact local G_F invariance, i.e.* invariance under the local transformations

$$\left\{ \begin{array}{ll} \exp [\hat{V}_R] \to \exp [-\, i\Lambda^{a\dagger} t^a] \exp [\hat{V}_R] \exp [i\Lambda^a t^a]\,, & \\ M \to \exp [-\, i\Lambda^a t^a]\, M\,, & \hat{T} \to \hat{T} \exp [i\Lambda^a t^a_T]\,, \\ \hat{P} \to \hat{P} \exp [i\Lambda^a t^a_P]\,, & S \to S\,, \end{array} \right. \tag{3.9}$$

where Λ^a's are arbitrary chiral fields and t^a, t^a_T, and t^a_P are G_F generators.

Note that, if the simplest possible form is assumed for G, eq. (3.8) has the standard form of a gauge theory with « matter » chiral superfields M, $\hat{T}$, $\hat{P}$ and S; the superpotential H may of course contain nonrenormalizable terms. The constant k plays the role of the (effective) gauge coupling constant squared.

Observe that the Fayet-Iliopoulos–type term (ξ term) is in general expected to be present.

The origin of this effective gauge symmetry lies essentially in the arbitrariness of the reparametrization, eqs. (3.4); however, it is the particular form of the effective action (the Tr (R) term and the local G_F invariance of $G(R; T, ...)$ in eq. (2.56)) that allows S_{eff} to be rewritten in the standard form of a gauge theory. The equivalence between the nongauge model eq. (3.2) and the gauge model eq. (3.8) is a generalization of an analogous one first noticed by FAYET [11] in this study of the U_1 Higgs model.

As in the U_1 Higgs model, this equivalence hinges upon the characteristic structure of supersymmetry multiplets. The original vector V_R contains (for each of the flavour degrees of freedom) one real scalar (1), two two-component spinors (4) and one nongauge vector (3): eight degrees of freedom in all. On the right-hand side of eq. (3.4*a*), with $\hat{V}_R$ constrained by eq. (3.5), the scalar (or chiral) supermultiplet M has one complex scalar (2) and one two-component spinor (2); $\hat{V}_R$ consists of a gauge vector (2) and of its fermion partner (2). Again there are eight physical degrees of freedom.

Of course, one can drop the constraint eq. (3.5); in that case M and $\hat{V}_R$ together will superficially contain twelve degrees of freedom. But just because of the (effective) gauge invariance, the four extra degrees of freedom are gauge freedom; they do not represent physical particles.

In the U_1 Higgs model discussed by FAYET [11],

$$S=\int d^8z\,(\mu^2 \exp[V] + \xi V) + (1/4)\int d^6z\, WW + \text{h.c.}\,, \tag{3.10a}$$

$$S=\int d^8z\,(h^\dagger \exp[\hat{V}]\, h + \xi \hat{V}) + (1/4)\int d^6z\, \hat{W}\hat{W} + \text{h.c.} \tag{3.10b}$$

(an obvious notation has been used; μ and ξ are the parameters of the model), the equivalence of the two versions of the model (*a* and *b*) can be also seen as follows. We choose a particular (« supersymmetric ») gauge in the gauge model of eq. (3.10*b*), in which the matter chiral field h is a constant

$$h \to \exp[i\Lambda]\, h = \text{const} = \mu\,. \tag{3.11}$$

In this gauge, the theory reduces to the nongauge model of eq. (3.10*a*).

Can one understand the equivalence between eq. (3.8) and (3.2) in a similar way? We notice that, since Λ^a are chiral (complex), M as defined in eq. (3.7) is an element of $\prod_i GL(n_i, C)$, not of $\prod_i U(n_i)$, even if t^a's are the generators of the group $G_F = \prod_i U(n_i)$. Hence it is always possible to choose a gauge in which

$$M = \mu \mathbb{1}\,. \tag{3.12}$$

In this gauge, S_{eff} of eq. (3.8) reduces to the original effective action eq. (3.2).

Let us now turn our attention to a few general feature which accompanies the emergence of the effective gauge symmetry. The first is the appearance of a generationlike structure among the « matter » multiplets. Take, for instance, $G_F = U_K$: The chiral superfield M behaves, under the local U_K transformations eq. (3.9), as K multiplets Ψ_i, $i = 1, \dots, K$,

$$M = ((\Psi_1)(\Psi_2) \dots (\Psi_K)) , \tag{3.13}$$

each of which transforms as in the fundamental representation (*).

Tr $(M^\dagger \exp [\hat{V}_R] M)$ can be written as

$$\mathrm{Tr}\,(M^\dagger \exp [\hat{V}_R]\, M) = \sum_i^K \Psi^{\dagger i} \exp [\hat{V}_R]\, \Psi_i . \tag{3.14}$$

For a more general case with $G_\mathrm{F} = \prod_i U(n_i)$, M has the box-diagonal form

$$M = \begin{pmatrix} (M_1) & & 0 & \\ & \ddots & & \\ 0 & & (M_i) & \\ & & & \ddots \end{pmatrix} , \tag{3.15}$$

where M_i transforms as n_i fundamental multiplets of U_{n_i} (and as a singlet of other U_{n_j}'s), like in eq. (3.13).

Natural appearance of such a generationlike structure of (at least part of) the matter fields is very interesting in view of an eventual application of these ideas in the context of a composite model of the quarks, leptons, Higgs scalars and standard gauge bosons.

It should be stressed that the invariance of S_eff under the effective gauge transformation does not require the invariance of the original action under the global G_F. Indeed, the transformation eq. (3.9) acts on the fields differently from the original global group G_F. In the effective action eq. (3.8), the original global G_F transformation can be taken to be

$$M \to M \exp [i\alpha^a t^a] \tag{3.16}$$

(all other fields remaining invariant).

In other words, the group of the effective gauge transformations $(G_\mathrm{F})_\text{effective gauge}$ (eq. (3.9)), and that of the global transformations $(G_\mathrm{F})_\text{global}$ (eq. (3.16)) form a direct product

$$(G_\mathrm{F})_\text{effective gauge} \times (G_\mathrm{F})_\text{global} \tag{3.17}$$

acting on the set of fields M, $\hat{V}_R$, $\hat{T}$, $\hat{P}$ and S. In general, because of the superpotential and strong anomaly, only a subgroup of $(G_\mathrm{F})_\text{global}$, if any, forms a symmetry group of the model.

This means, among others, that in general there is no symmetry under

(*) The fact that M is not a unitary matrix but is an element of $GL_{K,C}$ (recall the comment made before eq. (3.12)) means that $\Psi_1, \Psi_2, \dots, \Psi_K$ are all linearly independent chiral fields.

the exchange of two generations

$$\Psi_i \leftrightarrow \Psi_j .$$

Absence of unnecessary symmetry under the exchange of « generations » is perhaps welcome, if this sort of composite fields should eventually describe the quarks and leptons (*).

The fact that the effective gauge group $(G_F)_{\text{effective gauge}}$ does not constitute any subgroup of $(G_F)_{\text{global}}$, might also be of importance in an eventual realistic model based on the effective gauge symmetry for the following reason. In usual composite models of quarks and leptons, the gauge group $SU_3 \times SU_2 \times U_1$ of the standard strong and electroweak interactions is necessarily a subgroup of the global symmetry group of the original meta-(hyper-, etc.) colour theory. This aspect makes the construction of a realistic model rather difficult, for instance because of the stringent 't Hooft anomaly conditions ([20]).

In contrast, if all or part of the standard $SU_3 \times SU_2 \times U_1$ gauge symmetry is to arise as effective gauge symmetries, then there would be fewer constraints because there are no 't Hooft conditions associated with the effective gauge group. Only the unbroken part of the original global G_F, if any, is relevant for the anomaly matching equations in our case.

A crucial question related to the effective gauge symmetry is whether or not it is spontaneously broken. At first sight it might appear that the first term of eq. (3.2) implies the vector fields to have necessarily a nonzero mass μ. This is, however, false, since $\exp[C_R]$, where C_R is the scalar component of V_R, could get any v.e.v., including zero.

A simple illustration of this point is provided by the U_1 Higgs model, eq. (3.10). In spite of the apparent mass term $\mu^2 V^2/2$ involved in eq. (3.10*a*), the U_1 gauge symmetry is manifest (and the photon massless) for $\xi > 0$. Supersymmetry is broken in this case. The opposite is true (unbroken supersymmetry and broken U_1 gauge symmetry) for $\xi < 0$. This example suggests that, in a general supersymmetric confining model, it is a dynamical question whether or not the effective gauge symmetry is spontaneously broken. The presence of the scalar component in V_R as a physical degree of freedom clearly distinguishes supersymmetric models from nonsupersymmetric ones discussed by LEE and ZUMINO ([10]), in spite of the analogies mentioned in subsect. **2** i).

(*) In contrast, in usual approaches in which the generation structure is put in by hand, one tends to have too large symmetries, gauged or not. It is difficult to break these symmetries without encountering some unpleasant features. In conventional GUT schemes, for instance, the intergeneration symmetry is broken by hand in the Yukawa interaction terms. But then the mass relations involving different generations are not computable.

([20]) G. 'T HOOFT: *Cargese Lectures, 1979*, edited by G. 'T HOOFT *et al.* (Plenum Press, New York, N. Y., London, 1969).

In several models including those studied in sect. 4 and 5, we find that the effective gauge symmetry is spontaneously broken. As will be discussed after eq. (4.19), these models are characterized by the existence of a *local* effective Lagrangian with correct anomalous U_1 transformation properties.

On the other hand, there are models (*e.g.*, massless SQCD with M(flavours)$>$ $>N$(colours)) in which no *local* effective Lagrangian can be found that satisfies all the requirements of sect. 2. Whether manifest effective gauge symmetry is a dynamical possibility in such a model is an open question.

A possibly important point in this regard is the fact that $(G)_{\text{effective gauge}}$ and $(G_F)_{\text{global}}$ form a direct product acting on the same set of the fields M, $\hat{V}_R$, $\hat{T}$, $\hat{P}$ and S. This fact is crucial in avoiding the Weinberg-Witten theorem [21] on the existence of massless states of spin 1. Indeed, the effective gauge bosons are neutral (eq. (3.16)) with respect to any unbroken subgroup of $(G_F)_{\text{global}}$, hence the theorem of ref. [21] does not apply here.

Of course, this argument requires that the global chiral transformation be defined as in eq. (3.16), and not as any other combination of $(G_F)_{\text{effective gauge}}$ and $(G_F)_{\text{global}}$. Such different choices are equivalent at tree level of S_{eff}, but are inequivalent in general when loops are taken into account. In fact, the 't Hooft anomaly matching conditions severely restrict the possible definition of $(G_F)_{\text{global}}$ on the transformed fields M, $\hat{V}_R$, $\hat{T}$ and $\hat{P}$.

In fact, let us denote by A_{global} and $A_{\text{effective gauge}}$ the anomaly content of a given field with respect to unbroken subgroup of $(G_F)_{\text{global}}$ and the corresponding subgroup of $(G_F)_{\text{effective gauge}}$. We find from the transformation laws eq. (3.9) and (3.16) the relations

$$\left\{\begin{aligned} A_{\text{effective gauge}}(M) &= -A_{\text{global}}(M)\,,\\ A_{\text{effective gauge}}(\hat{T}) &= A_{\text{global}}(T)\,,\\ A_{\text{effective gauge}}(\hat{P}) &= A_{\text{global}}(P)\,. \end{aligned}\right. \tag{3.18}$$

Furthermore, it is easy to see that the original vector fields V_R never contributes to the anomaly of $(G_F)_{\text{global}}$,

$$A_{\text{global}}(V_R) = 0\,. \tag{3.19}$$

The 't Hooft condition then reads in the original fields

$$\sum_{T,P} A_{\text{global}} = \sum_{\text{fundamental}} A_{\text{global}} \equiv A\,. \tag{3.20}$$

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[21] S. WEINBERG and E. WITTEN: *Phys. Lett. B*, **96**, 59 (1980).

Using eqs. (3.18) and (3.9), this is seen to be equivalent to

$$A = \sum_{\hat{T},\hat{P}} A_{\text{effective gauge}} = \sum_{\hat{T},\hat{P},M,\hat{V}_R} A_{\text{effective gauge}} - \sum_{M} A_{\text{effective gauge}} = \tag{3.21}$$

$$= \sum_{\text{all}} A_{\text{effective gauge}} + \sum_{M} A_{\text{global}} .$$

Now, the possibility of defining the global transformation as in eq. (3.16) means that

$$A = \sum_{M} A_{\text{global}} , \tag{3.22}$$

hence that

$$\sum_{\text{all}} A_{\text{effective gauge}} = 0 . \tag{3.23}$$

Thus the 't Hooft condition and the Weinberg-Witten theorem, taken together, lead to the conclusion that only an anomaly-free subgroup of $(G_F)_{\text{effective gauge}}$, if ever, can be realized manifestly by massless vector bosons.

This conclusion in itself is not surprising since a nonvanishing anomaly, $\sum_{\text{all}} A_{\text{effective gauge}} \neq 0$, would in any case break the exact local symmetry. Nonetheless, the above argument reveals an intriguing interconnection between an unbroken global chiral symmetry (massless fermions) and a possible manifest effective gauge symmetry (massless vector bosons).

An interesting, related question is how the composite fields M, $\hat{V}_R$, $\hat{T}$ and $\hat{P}$ can be expressed in terms of the fundamental fields Φ, $\Phi^\dagger$ and V. One finds that there is no unique way of doing this. It is possible only upon introducing a set of redundant fields: for instance, $M|_{\theta=\bar{\theta}=0}$ can be written as

$$M|_{\theta=\bar{\theta}=0} = (\Phi^\dagger \exp[V]\, \Phi)^{\frac{1}{2}}_{\theta=\bar{\theta}=0} \cdot \exp[i\alpha(x)] , \tag{3.24}$$

where $\alpha(x)$ is an arbitrary complex function on which the physics does not depend. $\alpha(x)$ in fact accounts for two of the four extra degrees of freedom (ignoring the flavour) related to the effective gauge symmetry.

Such a nonuniqueness in the way the field variables are expressed, is precisely what happens in any theory with a local gauge symmetry. What is new here, however, is that this arbitrariness (*i.e.* the effective gauge symmetry), which was not present in the fundamental Lagrangian, appears automatically in the effective Lagrangian describing the composite particles.

This concludes the general discussions on the effective gauge symmetry. In the next two sections the aspects discussed in this section will be illustrated in explicit examples.

4. – A SU_6 model and spontaneous breakdown of the effective gauge symmetry.

As the first explicit example, let us consider a SU_6 gauge theory with the matter chiral superfields

$$\Phi^{\alpha}_{a}\,,\qquad \chi_{\alpha\beta}=-\,\chi_{\beta\alpha}\qquad(\alpha,\beta=1,\dots,6;\ a=1,2)\,, \tag{4.1}$$

namely, two $\underline{6}$'s and one antisymmetric $\underline{15}^*$. The action is given by

$$S=\int d^8z\left(\sum_{a=1}^{2}\Phi^{\dagger a}\exp[V]\,\Phi_a+\chi^{\dagger}\exp[\tilde{V}]\,\chi\right)+ \\ +\,h\int d^6z(\Phi\Phi\chi)+\text{h.c.}+g\int d^6z(\chi\chi\chi)+\text{h.c.}+S_{\text{gauge}}\,, \tag{4.2}$$

where S_{gauge} contains the usual gauge kinetic terms and the gauge-fixing and ghost terms. The most general SU_6-invariant superpotentials

$$h\Phi\Phi\chi\equiv h\varepsilon^{ab}\,\Phi^{\alpha}_{a}\,\Phi^{\beta}_{b}\,\chi_{\alpha\beta}\ \text{ and }\ g\chi\chi\chi\equiv g\varepsilon^{\alpha_1\dots\alpha_6}\,\chi_{\alpha_1\alpha_2}\chi_{\alpha_3\alpha_4}\chi_{\alpha_5\alpha_6} \tag{4.3}$$

are both assumed to be present, with coupling constants h and g, respectively.

The model has a global SU_2 symmetry associated with the Φ fields. Two U_1's of the phase rotations of Φ and χ, and the $U_{R,1}$ are all broken by the superpotentials (for $g\neq0$, $h\neq0$) and by the anomaly.

Before constructing the low-energy effective action, let us make a few general observations on this model, following the method of analysis of ref. [2,4]. The simplest Green's function that can get a nonzero contribution from the one-instanton configuration is of the form (*)

$$G=\langle0|T\prod_{i=1}^{3}\lambda\lambda(x_i)\,\varphi\varphi\eta(x_4)\,\eta\eta\eta(x_5)|0\rangle\,. \tag{4.4}$$

Such a Green's function, involving only the lowest components of chiral superfields, must be a constant (independent of x_i's) in a supersymmetric vacuum [1,2].

We have computed G in the one-instanton approximation, which is presumably reliable at short distances. In fact, perturbative contributions to G vanish to all orders of the strong coupling constant due to the nonzero chirality

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(*) We use the following notation for the component fields:

$$\Phi^{\alpha}_{i}=(\varphi+\sqrt{2}\,\theta\psi+\dots)^{\alpha}_{i}\,,\qquad \chi_{\alpha\beta}=(\eta+\sqrt{2}\,\theta\chi+\dots)_{\alpha\beta}\,,$$

$$W^{\alpha}_{S}=-\,i\lambda^{\alpha}+(i/2)(\sigma^{\mu}\bar{\sigma}^{\nu})^{\alpha\beta}F_{\mu\nu}\theta_{\beta}+\dots$$

Equation (4.4) is a particular case of such Green's functions discussed by MEURICE and VENEZIANO [4].

change required, and furthermore all the integrations over the collective coordinates prove to be finite. The calculation is quite lengthy, but is in principle straightforward, once we use the techniques developed by AMATI, ROSSI and VENEZIANO (2). We have found indeed a nonvanishing and constant result

$$G = \text{const}\, \Lambda_{SU_6}^{15}\,. \tag{4.5}$$

But, then, because of the cluster property of general Green's functions, eq. (4.5) implies the vacuum dominance at large distances,

$$\langle\lambda\lambda\rangle^3 \langle\varphi\varphi\eta\rangle \langle\eta\eta\eta\rangle = \text{const}\, \Lambda_{SU_6}^{15}\,. \tag{4.6}$$

On the other hand, the simplest of the chiral WT identities, eq. (2.48), reads in our case

$$\begin{cases} h\langle\varphi\varphi\eta\rangle - (g_S^2/32\pi^2)\langle\lambda\lambda\rangle = 0\,, \\ h\langle\varphi\varphi\eta\rangle + 3g\langle\eta\eta\eta\rangle - 4(g_S^2/32\pi^2)\langle\lambda\lambda\rangle = 0 \end{cases} \tag{4.7}$$

(where g_S is the strong SU_6 coupling constant). By combining eq. (4.7) with eq. (4.6), we find for the condensates

$$\begin{cases} \langle\lambda\lambda\rangle \sim (gh)^{1/5} \Lambda_{SU_6}^3\,, \\ \langle\varphi\varphi\eta\rangle \sim g^{1/5} h^{-4/5} \Lambda_{SU_6}^3\,, \\ \langle\eta\eta\eta\rangle \sim h^{1/5} g^{-4/5} \Lambda_{SU_6}^3\,, \end{cases} \tag{4.8}$$

if supersymmetry is not dynamically broken. Note that one of the condensates moves to infinity if we let one of g and h to zero. In order to have a well-defined vacuum, we shall keep $g \neq 0$ and $h \neq 0$ in the following.

Let us now write an effective Lagrangian for this model, taking account of all the considerations made in sect. **2** and **3**. The natural choice of the composite chiral superfields are

$$\begin{cases} T \equiv \varepsilon^{ab} \Phi_a^\alpha \Phi_b^\beta \chi_{\alpha\beta}\,, \\ X \equiv \varepsilon^{\alpha_1\ldots\alpha_6} \chi_{\alpha_1\alpha_2}\chi_{\alpha_3\alpha_4}\chi_{\alpha_5\alpha_6}\,, \\ S \equiv (g_S^2/32\pi^2)\, W_S W_S\,. \end{cases} \tag{4.9}$$

As the composite vector superfields we take

$$\begin{cases} R_a^b \equiv \Phi^{\dagger b} \exp[V]\, \Phi_a \equiv \mu^2(\exp[V_R])_a^b & (a, b = 1, 2)\,, \\ Q \equiv \chi^\dagger \exp[\tilde{V}]\, \chi \equiv \mu^2 \exp[V_Q] \end{cases} \tag{4.10}$$

(μ is an arbitrary constant with the dimension of mass).

By assuming the simplest possible form of the function G of eq. (2.56), and by taking into account the constraints discussed at the end of subsect. **2'3**, we get (*)

$$(4.11)\qquad S_{\text{eff}}=\int \mathrm{d}^8 z\,[\mu^2 \operatorname{Tr}\exp[V_R]+\mu^2\exp[V_Q]+\alpha S^\dagger S+\xi_R \operatorname{Tr} V_R+ + \xi_Q V_Q+\beta\mu^{-6}T^\dagger T\exp[-V_Q-\operatorname{Tr}V_R]+\gamma\mu^{-6}X^\dagger X\exp[-3V_Q]]+ +\int \mathrm{d}^6 z\,[S\log(S^3XT/\Lambda^{15})+hT+gX]+\text{h.c.}+ +\int \mathrm{d}^6 z\,[(1/4k_1)\operatorname{Tr}W_R^2+(1/4k_2)W_Q^2]+\text{h.c.}\,,$$

where $\alpha(\propto\Lambda^{-4})$, $\beta(\propto\Lambda^2)$, $\gamma(\propto\Lambda^2)$, $\xi_{R,Q}(\propto\Lambda^2)$ and $k_i(\propto\Lambda^0)$ are constants, and $W_R^\alpha=-(1/4)\bar{D}^2\exp[-V_R]D^\alpha\exp[V_R]$.

As a result of the simplest possible choice made for G of eq. (2.56), dimensional parameters which break the scale invariance appear in eq. (4.11). Questions related to this point will be discussed in sect. **5**.

Following the discussions of sect. **3**, we introduce the reparametrization

$$(4.12)\qquad \begin{cases} R=\mu^2\exp[V_R]=M^\dagger\exp[\hat{V}_R]\,M\,,\\ Q=\mu^2\exp[V_Q]=N^\dagger\exp[\hat{V}_Q]\,N\,,\\ T=(\det M)\,N\hat{T}\,,\\ X=N^3\hat{X}\,. \end{cases}$$

(M and $\hat{V}_R$ are 2×2 matrices). In terms of these new fields, S_{eff} reads

$$(4.13)\qquad S_{\text{eff}}=\int \mathrm{d}^8 z\,[\operatorname{Tr}(M^\dagger\exp[\hat{V}_R]\,M)+N^\dagger\exp[\hat{V}_Q]\,N+\alpha S^\dagger S+\xi_R\operatorname{Tr}\hat{V}_R+ +\xi_Q\hat{V}_Q+\beta\hat{T}^\dagger\hat{T}\exp[-\operatorname{Tr}\hat{V}_R-\hat{V}_Q]+\gamma\hat{X}^\dagger\hat{X}\exp[-3\hat{V}_Q]]+ +\int \mathrm{d}^6 z\,[S\log\{S^3N^4(\det M)\hat{T}\hat{X}/\Lambda^{15}\}+h(\det M)N\hat{T}+gN^3\hat{X}]+\text{h.c.}+ +\int \mathrm{d}^6 z\,[(1/4k_1)\operatorname{Tr}\hat{W}_R^2+(1/4k_2)\hat{W}_Q^2]+\text{h.c.}$$

As discussed in a general fashion in sect. **3**, this model possesses an exact

(*) In subsect. 2'3, only those symmetries (anomalous or not) which commute with supersymmetry have been considered. In determining the precise form of the anomaly term (the logarithmic term) in eq. (4.11), we have also taken account of the anomalous $U_{R,1}$ (R-symmetry) transformation property of the action as well.

local U_2 symmetry, under which M, $\hat{V}_R$ and $\hat{T}$ transform as

$$(4.14)\qquad \begin{cases} M \to \exp[-i\Lambda^a t^a]\, M\,, \\ M^\dagger \to M^\dagger \exp[i\Lambda^{a\dagger} t^a]\,, \\ \exp[\hat{V}_R] \to \exp[-i\Lambda^{a\dagger} t^a] \exp[\hat{V}_R] \exp[i\Lambda^a t^a]\,, \\ \hat{T} \to \exp[i(\operatorname{Tr}\Lambda^a) t^a]\, \hat{T}\,, \end{cases}$$

and an exact local U_1 invariance under

$$(4.15)\qquad \begin{cases} N \to \exp[-i\Lambda]\, N\,, \\ \exp[\hat{V}_Q] \to \exp[-i\Lambda^\dagger] \exp[\hat{V}_Q] \exp[i\Lambda]\,, \\ \hat{T} \to \exp[i\Lambda]\, \hat{T}\,, \\ \hat{X} \to \exp[3i\Lambda]\, \hat{X}\,. \end{cases}$$

If we write the 2×2 matrix M as

$$(4.16)\qquad M = \big((\Psi_1)(\Psi_2)\big)\,,$$

the chiral superfields Ψ_1 and Ψ_2 both transform like doublets of the local SU_2.

S_{eff} possesses also a global SU_2 symmetry under which M transforms as (with real α^a)

$$(4.17)\qquad M \to M \exp[i\alpha^a t^a]\,, \qquad M^\dagger \to \exp[-i\alpha^a t^a]\, M^\dagger\,,$$

while all other fields remain invariant.

It is straightforward to find the minima of the scalar potential which is obtained by eliminating all the auxiliary fields in eq. (4.13). We find that the ground states are supersymmetric, and that the scalars (denoting the lowest components of $M, N, T = (\det M) N\hat{T}$, $X = N^3\hat{X}$, and S by $A_M, A_N, A_T(=\varphi\varphi\eta)$, $A_X(=\eta\eta\eta)$ and $A_S(=(g_S/32\pi^2)\lambda\lambda)$, acquire the v.e.v.'s

$$(4.18)\qquad \langle A_S\rangle = (gh)^{1/5}\Lambda^3\,, \qquad \langle A_T\rangle = \langle A_S\rangle/h\,, \qquad \langle A_X\rangle = \langle A_S\rangle/g$$

and

$$(4.19)\qquad \langle A_N\rangle = \text{const}\,\Lambda(\neq 0)\,, \qquad \langle A_M\rangle = \text{const}\,\Lambda\cdot\mathbb{1}(\neq 0)\,.$$

The results eq. (4.18) agree with eq. (4.8), obtained from the explicit instanton calculation combined with the chiral WT identities.

Equations (4.19) show that the local $U_2 \times U_1$ is broken spontaneously; on the other hand, a global SU_2 symmetry (the diagonal combination of eq. (4.17) and the global SU_2 included in eq. (4.14)) remains manifest. Indeed, by inserting eq. (4.19) into eq. (4.13), we get for the isotriplet vector masses

$$m_1 = m_2 = m_3 \propto \Lambda \,. \tag{4.20}$$

Natural occurrence of an unbroken global SU_2 symmetry together with a spontaneously broken local SU_2 symmetry in our model, is quite reminiscent of what happens in the standard electroweak model.

We observe that the spontaneous breakdown of the effective local $U_2 \times U_1$ symmetry is inevitable in this model, if the vacuum is to be well defined. This can be seen from the argument of the logarithm (the U_1 anomaly term in eqs. (4.11) and (4.13)), and from the finiteness of the scalar potential. Alternatively, this can be seen as a result of the existence of a nonzero Green's function, eq. (4.4).

A similar argument can be used to show the spontaneous breakdown of the effective gauge symmetry in models in which a nonzero Green's function analogous to eq. (4.4) exists, and in which a local anomaly term can be written.

At the same time, the above observation suggests that the spontaneous breakdown of the effective gauge symmetry is not inevitable in *all* supersymmetric confining models. The effective gauge symmetry might well remain unbroken in models in which no nonzero Green's functions of the type (*) of eq. (4.4) exist. This could open the way to a composite model of the gluons and the photon. Importance of improving our understanding on these questions could hardly be overestimated.

5. – SQCD.

Supersymmetric version of quantum chromodynamics (SQCD) has been studied by many authors [2,3,5,7,8,21]. In particular, recent studies of non-perturbative effects [2,22] strengthened certain conclusions on the low-energy properties of the theory (such as eqs. (5.11) and (5.12) below) obtained earlier in an effective Lagrangian approach [7].

In spite of these existing studies, we feel that the presentation of our results is justified for two reasons at least. First, our result shows that the introduction of composite vector supermultiplets is perfectly compatible with the knowledge

(*) Namely, a n-point function involving only the lowest component of composite chiral superfields that satisfies all the selection rules of one instanton contribution.

(22) M. G. SCHMIDT: *Phys. Lett. B*, **141**, 236 (1984).

we already have: all the most reliable results in SQCD are reproduced by our effective Lagrangian.

Secondly, our S_{eff} gives information on other bilinear condensates (squark-antisquark; quark-antiquark) on which little is known [23].

We shall limit ourselves to the case in which the number of flavours M is less than N, the number of colours.

SQCD is a SU_N gauge theory with two sets of chiral matter fields,

$$\Phi_i^\alpha\,, \qquad \chi_\alpha^j \qquad (\alpha = 1, \ldots, N;\ i, j = 1, \ldots, M;\ M < N)$$

trasforming as N and N^*, respectively. The matter part of the action is given by

$$(5.1)\qquad S_{\text{matter}} = \int d^8z\,(\Phi^{\dagger i} \exp[V]\,\Phi + \chi^i \exp[-V]\,\chi_j^\dagger) + \int d^6z\,(\chi^j m_j^i \Phi_i) + \text{h.c.}\,,$$

where the mass matrix m can be taken to be diagonal

$$(5.2)\qquad m_j^i = \delta_j^i m_i$$

without losing the generality.

As the composite chiral superfields we consider (*)

$$(5.3)\qquad T_i^j \equiv \chi_\alpha^j \Phi_i^\alpha$$

and

$$(5.4)\qquad S \equiv (g_S^2/32\pi^2)\, W_S W_S\,.$$

We also introduce the composite vector superfields

$$(5.5)\qquad \begin{cases} (R_1)_i^j \equiv \Phi^{\dagger j} \exp[V]\,\Phi_i \equiv \mu^2(\exp[V_1])_i^j\,, \\ (R_2)_i^j \equiv \chi^j \exp[-V]\,\chi_i^\dagger \equiv \mu^2(\exp[V_2])_i^j \end{cases}$$

(μ is an arbitrary parameter with dimension of mass).

The effective action involving T, S, R_1, and R_2 can be constructed by following the general discussion of sect. **2**. Assuming the simplest possible form

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[23] See, however, S. Narison: *Phys. Lett. B*, **142**, 168 (1984).

(*) W_S is the usual chiral superfield describing the SU_N gauge bosons and gauginos. For component fields, we shall use the notation

$$\Phi_i^\alpha = (\varphi + \sqrt{2}\,\theta\psi + \ldots)_i^\alpha\,, \qquad \chi_\alpha^j = (\eta + \sqrt{2}\,\theta\chi + \ldots)_\alpha^j\,,$$

$$W_S^\alpha = -i\lambda^\alpha + (i/2)(\sigma^\mu\bar\sigma^\nu)^{\alpha\beta} F_{\mu\nu}\theta_\beta + \ldots\,.$$

for the function G of eq. (2.56), we get

$$(5.6)\qquad S_{\text{eff}} = \int d^8z\,[\operatorname{Tr} R_1 + \operatorname{Tr} R_2 + \xi \operatorname{Tr}\log(R_1R_2/\Lambda^4) + \alpha S^\dagger S + \beta \operatorname{Tr}(R_2^{-1}TR_1^{-1}T^\dagger)] + \int d^6z\,[S\log(S^{N-M}\det T)] + \text{h.c.} + \int d^6z \operatorname{Tr}(mT) + \text{h.c.} + (1/4k)\int d^6z\,[\operatorname{Tr} W_1^2 + \operatorname{Tr} W_2^2] + \text{h.c.}\,,$$

where $W_i^\alpha = -(1/4)\bar{D}^2R_i^{-1}D^\alpha R_i$ $(i = 1, 2)$ and $\alpha(\propto \Lambda^{-4})$, $\beta(\propto \Lambda^2)$ and $k(\propto \Lambda^0)$ are constants.

Before analysing S_{eff} of eq. (5.6), let us make a digression to discuss possible modifications of S_{eff}. In eq. (5.6) we took the simplest possible form of S_{eff} that is compatible with the constraints eqs. (2.52) following from the chiral WT identities, and with the anomalous global $U_{R,1}$ transformation property. On the other hand, we disregarded the anomalous WT identities involving the $U_{R,1}$, superconformal and dilatation currents [24]. In fact, our S_{eff} contains dimensional parameters α, β and ξ. They are somewhat analogous to F_π appearing in the sigma model.

As in ref. [6,7], one might prefer to make S_{eff} free of Λ_{SQCD}'s (except in the argument of the logarithm): this could be done, *e.g.*, by making the modification

$$(5.7)\qquad \begin{cases} S^\dagger S \to (S^\dagger S)^{1/3}\,, \\ \operatorname{Tr}(R_2^{-1}TR_1^{-1}T^\dagger) \to \operatorname{Tr}(R_2^{-1}TR_1^{-1}T^\dagger)(S^\dagger S)^{1/3}\,, \\ \xi \to 0\,, \end{cases}$$

in eq. (5.6) while all other terms are kept unchanged.

We wish to point out that such a modified S_{eff} (or S_{eff} of ref. [7] for that matter) nonetheless does *not* satisfy correct WT identities associated with the dilation current, because of the anomalous dimensions of the T, S and R fields.

In the pure Yang-Mills model, this problem appears to be avoided [25] by choosing the renormalization-group–invariant composite field, $(\beta(g)/3g)\,W_S W_S$, as the low-energy degree of freedom [6] [*]. Generalization of such a simple recipe to the case of gauge theories with matter is not known at the moment.

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[24] S. FERRARA and B. ZUMINO: *Nucl. Phys. B*, **87**, 207 (1975); W. LANG: *Nucl. Phys. B*, **150**, 201 (1979); O. PIGUET and K. SIBOLD: *Nucl. Phys. B*, **197**, 257, 272 (1982). An explicit calculation of this anomaly in the pure $n = 2$ supersymmetric Yang-Mills theory has been done in P. DI VECCHIA, R. MUSTO, R. NICODEMI and R. PETTORINO: CERN preprint, CERN-TH-3905 (1984).

[25] G. VENEZIANO: private communication.

[*] See, however, ref. [26] for further problems.

[26] V. A. NOVIKOV, M. A. SHIFMAN, A. I. VAINSHTEIN and V. I. ZAKHAROV: ITEP preprint, ITEP-85 (1984).

In any event, we have checked that the modification eq. (5.7) does not change any of the qualitative results obtained from S_{eff} of eq. (5.6) (see after eq. (5.15)).

It may be worthwhile to notice that the modified S_{eff} can also be cast into the standard form (of kinetic terms), by the change of variables

$$S^{1/3} \equiv S' , \qquad TS^{1/3} \equiv T' . \tag{5.8}$$

Coming back to the effective action eq. (5.6), we make the reparametrization

$$\begin{cases} R_1 \equiv \mu^2 \exp [V_1] = M_1^\dagger \exp [\hat{V}_1]\, M_1 , \\ R_2 \equiv \mu^2 \exp [V_2] = M_2 \exp [\hat{V}_2]\, M_2^\dagger , \\ T = M_2 \hat{T} M_1 , \qquad T^\dagger = M_1^\dagger \hat{T}^\dagger M_2^\dagger , \end{cases} \tag{5.9}$$

following the general discussion of sect. **3**. S_{eff} takes now an equivalent form (M_1, M_2, $\hat{V}_1$, $\hat{V}_2$ and $\hat{T}$ are $M \times M$ matrices),

$$\begin{aligned} S_{\text{eff}} = \int \mathrm{d}^8 z \, [&\mathrm{Tr}\,(M_1^\dagger \exp [\hat{V}_1]\, M_1) + \mathrm{Tr}\,(M_2 \exp [\hat{V}_2]\, M_2^\dagger) + \\ &+ \xi \, \mathrm{Tr}\,(\hat{V}_1 + \hat{V}_2) + \alpha S^\dagger S + \beta \, \mathrm{Tr}\,(\exp[-\hat{V}_2]\, \hat{T} \exp[-\hat{V}_1]\, \hat{T}^\dagger)] + \\ &+ \int \mathrm{d}^6 z \, S \log \{S^{N-M} \det (M_2 \hat{T} M_1)\} + \text{h.c.} + \int \mathrm{d}^6 z \, \mathrm{Tr}\,(m M_2 \hat{T} M_1) + \text{h.c.} + \\ &+ (1/4k) \int \mathrm{d}^6 z \, (\mathrm{Tr}\, \hat{W}_1^2 + \mathrm{Tr}\, \hat{W}_2^2) + \text{h.c.} \end{aligned} \tag{5.10}$$

The scalar potential can be readily obtained by eliminating all auxiliary fields. A straightforward minimization of the scalar potential gives N supersymmetric minima, at which (for each i, j)

$$m_i \langle \eta^i \varphi_i \rangle = (g_S^2 / 32\pi^2) \langle \lambda\lambda \rangle = \text{const} \prod_{k=1}^{M} m_k^{1/N} \Lambda_{\text{SQCD}}^{3-M/N} , \tag{5.11}$$

$$\langle \eta^j \varphi_i \rangle = 0 \qquad (i \neq j) \tag{5.12}$$

and

$$\langle \varphi^{\dagger j} \varphi_i \rangle = \langle \eta_j^\dagger \eta^i \rangle = \delta_{ij} f(|\langle \eta^i \varphi_i \rangle|^2) \tag{5.13}$$

where $f(x)$ is the solution of $f^2 - x f^{-1} + \xi f = 0$.

Equations (5.11) and (5.12) agree with the earlier [7], and more recent and model-independent results [2,22].

Equation (5.13) is new, and by combining it with

$$\langle \chi^j \psi_i \rangle = - m_j^* \langle \varphi^{\dagger j} \varphi_i \rangle - m_i^* \langle \eta^j \eta_i^\dagger \rangle \tag{5.14}$$

(which states that the F-component of $\chi\Phi$ has a vanishing v.e.v. in a supersymmetric vacuum), we find that

i) $\langle\eta^j\varphi_i\rangle$, $\langle\varphi^{\dagger j}\varphi_i\rangle$, $\langle\eta_j^\dagger\eta^i\rangle$, $\langle\chi^j\psi_i\rangle$ are all diagonal (in the basis where the mass matrix is chosen diagonal); two matrix elements of each type are equal if the corresponding flavours have equal masses

and

ii) for $m_i \to 0$, $\langle\varphi^{i\dagger}\varphi_i\rangle$ diverges but slowly enough that

$$\langle\chi^i\psi_i\rangle \xrightarrow[m_i\to 0]{} 0\,. \tag{5.15}$$

(When the modified form of S_{eff}, eq. (5.7), is used, eq. (5.13) is changed; but all other results, eqs. (5.11) and (5.12), and i) and ii) above, remain unchanged.)

Equation (5.15) excludes the possible scenario for dynamical supersymmetry breaking discussed by PESKIN and others [8].

Thus our results on SQCD confirm the earlier ones and complement them. The result i) above shows that the vacuum property is dictated by the masses, however small they may be. Such a dramatic difference between the vacuum properties of ordinary and supersymmetric QCD has already been noticed in the earliest study [7].

The effective gauge symmetry group of S_{eff} of eq. (5.10) is $U_M \times U_M$, under which fields transform as

$$\begin{cases} M_1 \to \exp[-i\Lambda_1]\, M_1\,, \\ M_2 \to M_2 \exp[i\Lambda_2]\,, \\ \hat{T} \;\; \to \exp[-i\Lambda_2]\,\hat{T}\exp[i\Lambda_1]\,, \\ \exp[\hat{V}_1] \to \exp[-i\Lambda_1^\dagger]\exp[V_1]\exp[i\Lambda_1]\,, \\ \exp[\hat{V}_2] \to \exp[-i\Lambda_2]\exp[\hat{V}_2]\exp[i\Lambda_2^\dagger]\,, \end{cases} \tag{5.16}$$

where $\Lambda_{1,2}$ are arbitrary chiral superfields of $M\times M$ matrix form. It is completely broken by the condensates eqs. (5.11), (5.13).

6. – Summary and outlook.

We have shown in this paper that in supersymmetric confining theories a gauge symmetry structure appears naturally and quite universally in the low-energy effective action. Such an effective gauge symmetry structure can be regarded as a consequence of the «field-current identity», analogous to that discussed by LEE and ZUMINO [10]. However, in supersymmetric theories, there is a natural reparametrization, similar to the one found by FAYET [11], which brings S_{eff} into the form with an exact local invariance. Existence of such a reparametrization and the possibility that the effective gauge symmetry could be manifest, both arise due to the presence of scalar components as physical degrees of freedom in the vector supermultiplets, a situation new compared to the nonsupersymmetric cases investigated earlier [10].

We have studied a few general features accompanying the emergence of the effective gauge symmetry. Among others, natural appearance of a generationlike structure in part of the composite matter multiplets, seems to be the most interesting one.

The general aspects discussed here suggest an exciting possibility that *all* of the $SU_3 \times SU_2 \times U_1$ gauge symmetries in the standard model appear as effective gauge symmetries, and at the same time *all* of the presently known « elementary » particles (including the gauge bosons) be composite states, bound by the same meta-(hyper-, etc.) colour forces. Before one can decide that such a possibility really exists, however, several important questions must be answered.

The first concerns the question of the possible realization of the effective gauge symmetry. In several models we have studied, the effective gauge symmetry was found to be spontaneously broken. The other possibility, unbroken manifest effective gauge symmetry, has not been explored in this paper, although this is crucial in the application of our ideas to the known exact gauge symmetries, QCD and the electromagnetism.

Secondly, as in any other attempts to construct a realistic model based on supersymmetric theories, a mechanism of supersymmetry breaking must be incorporated. Although we have not discussed here, there has been some progress recently, both concerning dynamical [4] and explicit supersymmetry breaking [27].

Thirdly in a realistic model the pattern of chiral symmetry realization and the ensuing structure of light composite particles, as required by the Goldstone theorem and/or by the anomaly-matching conditions, must fit the observed world of quarks and leptons.

These and other problems, related to the possibility of finding a realistic composite model of the elementary particles within the general scheme discussed above, are presently under study.

* * *

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(27) A. MASIERO and G. VENEZIANO: CERN preprint, TH-3950 (1984).

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Note added in proofs.

The original idea of the field-current identity and the appearance of a gauge-symmetry structure is found in V. I. OGIEVETSKIJ and I. V. POLUBARINOV: *Ann. Phys.* (*N. Y.*), **25**, 358 (1963). Our approach is closer to this work than to those cited in ref. (10). Furthermore, OGIEVETSKIJ and POLUBARINOV discuss explicitly the cases with massless vector bosons as well. We thank L. E. PICASSO for bringing this paper to our attention.

● RIASSUNTO

Si studia in dettaglio il ruolo che i supermultipletti vettoriali composti hanno nell'ambito delle teorie supersimmetriche confinanti. Usando le identità chirali di WT, si determina la struttura della lagrangiana effettiva a bassa energia, che contiene questi supermultipletti vettoriali. Il fatto che generalmente appare una struttura di simmetria di gauge effettiva è uno dei risultati piú interessanti. Un altro è che tra i campi composti di materia, che sono multipletti della simmetria di gauge effettiva, emerge una struttura tipo generazioni.

Эффективная калибровочная симметрия в суперсимметричных удерживащих теориях.

Резюме (*). — В рамках общих суперсимметричных удерживающих теорий подробно исследуется роль составных векторных супермультиплетов. Используя киральные WT-тождества, определяется структура низкоэнергетического действия, которое включает эти векторные супермультиплеты. Один из наиболее интересных результатов — это универсальное появление эффективной калибровочно-симметричной структуры. Другой результат — это возникновение структуры типа размножения между составными мультиплетами « вещества », при условии эффективной калибровочной симметрии.

(*) *Переведено редакцией.*

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