EFFECTIVE ACTION FOR DYNAMICAL SUPERSYMMETRY BREAKING

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We propose an effective action describing the low-energy limit of a chiral gauge theory in which instantons are believed to break dynamically supersymmetry. The model is well behaved, provided real superfields, containing the goldstino and giving rise to massive vector exchanges, are coupled to the usual chiral superfields.

Vector-like supersymmetric gauge theories, such as SYM and SQCD, have been analyzed in the last few years by several nonperturbative methods: Witten techniques [1], effective lagrangians [2], Ward–Takahashi (WT) identities [3,4] and, finally, dynamical instanton calculations [5] of correlation functions. A consistent, general picture has emerged for the vacuum structure of these models, confirming that in such theories supersymmetry is unbroken, though, occasionally, the ground state is pathological.

For theories with chiral fermions, the first two techniques mentioned above have so far proved ineffective. However, instanton calculations, coupled to WT identities, have led to the conclusion [6] that in a number of cases supersymmetry should be dynamically broken. (For a somewhat different apparoach, see ref. [7].)

In this note, we shall partially fill the above-mentioned gap by constructing effective actions which explicitly exhibit spontaneous supersymmetry breaking. As for the case of vector-like theories, these effective lagrangians describe the low-energy physics in terms of a set of gauge-invariant composite fields. We find, however, that chiral as well as real superfields are needed in order to recover the absence of flat directions (which is a property of the underlying theory) as well as supersymmetry breaking. We recall that for SQCD chiral superfields were enough.

The effective lagrangian neatly summarizes the various dynamical properties that we believe the theory to possess: spontaneous breaking of supersymmetry with an accompanying goldstino; absence of supersymmetry in the light spectrum; spontaneous breaking of global bosonic symmetries with the accompanying Goldstone bosons; massless fermions needed by 't Hooft consistency arguments because of the conservation of certain chiral symmetries. Our discussion will concentrate on the example of the one-family supersymmetric Georgi–Glashow model. Extension to other cases looks straightforward.

We shall start by recalling the model and the previously given argument for supersymmetry breaking. We shall then construct the effective lagrangian.

The fundamental action of the model is

$$S=\int d^8 z\,(\Phi^\dagger e^V \Phi + X^\dagger e^{\bar{V}} X)$$

$$+\ \text{gauge, gauge-fixing and ghost terms}\,, \qquad (1)$$

where Φ^α and $X_{\alpha\beta}=-X_{\beta\alpha}$ belong to the fundamental (5) and antisymmetric $(\overline{10})$ representations of SU(5). The classical scalar potential has no flat directions. Furthermore, no superpotential can be added to eq. (1).

The global, nonanomalous symmetry of the model consists (apart from supersymmetry itself) of two U(1) symmetries. The first, $U_1(1)$, commutes with supersymmetry and has charges $q_\Phi=3$, $q_X=-1$. The second, $U_2(1)$, is an R symmetry with the transformation law,

$$\Phi \to \exp(-12i\alpha)\,\Phi(x,\theta e^{5i\alpha})\,,$$

$$X \to \exp(14i\alpha)\,X(x,\,\theta e^{5i\alpha})\,,$$

$W \to \exp(-5i\alpha)\, W(x, \theta e^{5i\alpha})$.

The anomalous [U(1)-related] identities read simply [4]

$$\langle \{\bar{Q}_{\dot\alpha}, \bar{\psi}^{\dot\alpha}\phi\}\rangle/2\sqrt{2} = \langle (g^2/16\pi^2)\lambda\lambda\rangle ,$$

$$\langle \{\bar{Q}_{\dot\alpha}, \chi^{\dot\alpha}\eta\}\rangle/2\sqrt{2} = 3\langle (g^2/16\pi^2)\lambda\lambda\rangle , \qquad (2)$$

showing that a nonvanishing gauge-fermion condensate signals spontaneous breaking of supersymmetry.

On the other hand, the nonvanishing instanton result [6]

$$\langle\, |\mathrm{T}\, S(x_1) S(x_2) Y(x_3)\, |0\rangle = \text{const.} \times \Lambda^{13} \neq 0 ,$$

$$S \equiv (g^2/16\pi^2)\lambda\lambda, \qquad Y = W^2 \phi\eta\eta\eta\epsilon , \qquad (3)$$

implies via clustering $\langle S\rangle^2 \cdot \langle Y\rangle = \text{const.} \times \Lambda^{13} \neq 0$. A supersymmetric vacuum would require $\langle Y\rangle = \infty$ [cf. eq. (2)], probably an unacceptable result in view of the absence of flat directions in the classical potential. Thus supersymmetry is dynamically broken.

If this is indeed the way this theory is realized, it should be possible, by integrating out the heavy degrees of freedom, to write down a low-energy effective theory exhibiting dynamical sypersymmetry breaking induced by $\lambda\lambda$ condensation. So far this has proved to be a difficult task to accomplish; in particular, the elimination of supersymmetric vacua at infinity has never been successful if only chiral superfields are employed. This leads one to wonder if the absence of flat directions in the underlying theory does indeed imply, as one would guess, the same for the composite fields of the effective theory.

The resolution of this puzzle seems to lie in the correct choice of the low-energy degrees of freedom. Let us start with the goldstino. Eq. (2) leads to

$$f\langle g_{\dot\alpha}\, |\bar{\psi}^{\dot\alpha}\phi|\, 0\rangle = \tfrac{1}{3} f\langle g_{\dot\alpha}\, |\bar{\chi}^{\dot\alpha}\eta|\, 0\rangle$$

$$= 2\sqrt{2}\langle S\rangle \neq 0 , \qquad (4)$$

where $|g\rangle$ is the Goldstone fermion, and f is the strength of supersymmetry breaking, $f\sigma_\mu^{\alpha\dot\alpha} = \langle 0|S_\mu^{\dot\alpha}|g_\alpha\rangle$ (S_μ^α is the supercurrent). It means that the low-energy degrees of freedom must include the real composite supermultiplets

$$R \equiv \Phi^\dagger e^V \Phi , \qquad Q \equiv X^\dagger e^{\tilde V} X , \qquad (5)$$

whose components are associated with the massless Goldstone fermion.

The composite fields R, Q, as well as Y and S of eq. (3) are all singlets of $U_1(1)$. As no condensates breaking this U(1) symmetry are suggested from instanton calculations, one needs a composite superfield associated with the massless 't Hooft-type fermion to saturate the $\{U_1(1)\}^3$ anomaly. A simple candidate is

$$A \equiv (g^2/16\pi^2)(W^2)^\gamma_\alpha \Phi^\alpha \Phi^\beta X_{\beta\gamma} . \qquad (6)$$

The structure of the low-energy effective lagrangian containing R and Q is severely restricted by the fact that these real superfields have in their vector component the original current operators of the (anomalous) $U_\Phi(1)$ and $U_X(1)$ symmetries. The requirement that the anomalous and nonanomalous chiral U(1) identities be correctly reproduced by the low-energy effective theory fixes the form of the effective lagrangian to be almost one of a local gauge theory, as is well known from the earlier studies in conventional (nonsupersymmetric) theories [8]. The supersymmetric generalization of such a construction has been considered in detail in ref. [9], to which we refer for a more complete discussion.

The general structure of S_{eff} is given by

$$S_{\text{eff}} = \int d^8 z\, \{R + Q + \xi_1 \log R + \xi_2 \log Q$$

$$+ f(S^*, S; Y^*, Y; A^*, A; R; Q)\}$$

$$+ \int d^6 z\, \{(1/4k_1)\, W_R^2 + (1/4k_2)\, W_Q^2$$

$$+ S \log S^2 Y\} + \text{h.c.} , \qquad (7)$$

where $(W_R)_\alpha \equiv -\frac{1}{4}\bar{D}^2 R^{-1} D_\alpha R$ and similarly for W_Q; and ξ_1, ξ_2, k_1 and k_2 are (in principle) calculable constants. The dependence of f on Y, Y^*, R and Q is such that f is invariant under the *local* $U_\Phi(1)$ and $U_X(1)$ transformations, namely a function

$$\hat{f}(Y^* R^{-1} Q^{-3} Y) \qquad (8)$$

of the above invariant combination. This follows from the requirement that the $U_\Phi(1)$ and $U_X(1)$ current divergence equations [10,4]

$$\tfrac{1}{4}\bar{D}^2 R + S = 0, \qquad \tfrac{1}{4}\bar{D}^2 Q + 3S = 0 \qquad (9)$$

[eqs. (2) may be regarded as the lowest component of these] be consequences of the equations of motion of R, Q and Y,

$$\tfrac{1}{4}\bar{D}^2\{R+(\partial f/\partial R)R\}=0\,, \quad (10a)$$

$$\tfrac{1}{4}\bar{D}^2\{Q+(\partial f/\partial Q)Q\}=0\,, \quad (10b)$$

$$\tfrac{1}{4}\bar{D}^2(\partial f/\partial Y)Y+S=0\,. \quad (10c)$$

As for the dependence of f on A and A^*, which does not appear in the F term, any single term of the form

$$\hat{f}\sim(A^*R^pQ^qA)^r \quad (11)$$

would do, since its contribution to eq. (10a) [or eq. (10b)] is proportional to the equation of motion of the A field and hence vanishes. (This observation complements the basic results of ref. [9].)

Symmetry considerations alone do not further specify the form of the functions $\hat{f}$ and $\hat{\hat{f}}$, on which the vacuum properties severely depend. Since the potential in the original theory is rising at large ϕ or η in any direction in the field space (true to all orders of perturbation), let us require the same property to hold for the potential following from eq. (7). Also, since there are no constant Green's functions that might suggest nonvanishing condensates for the A field, we require $\langle A\rangle=0$ to follow from eq. (7). These requirements have a simple solution (with possible variants leading to identical physics)

$$S_{\rm eff}=\int d^8z\,\{R+Q+\xi_1\log R+\xi_2\log Q$$
$$+(Y^*R^{-1}Q^{-3}Y)^{-1}+S^*S+A^*RQA\}$$
$$+\int d^6z\,\{(1/4k_1)\,W_R^2+(1/4k_2)\,W_Q^2$$
$$+S\log S^2Y\}+\text{h.c.}\,, \quad (12)$$

with

$$\xi_1\geqslant 0,\qquad \xi_2\geqslant 0\,. \quad (13)$$

Let us now analyze this effective theory.

First we make a simple field reparametrization

$$R=M^\dagger e^{V_R}M\quad (V_R^n=0,\ n\geqslant 3)\,,$$

$$Q=N^\dagger e^{V_Q}N\quad (V_Q^n=0,\ n\geqslant 3)\,, \quad (14a)$$

and introduce the new variables

$$Z=N^3MY^{-1},\qquad B=AMN\,. \quad (14b)$$

This transformation diagonalizes at once all the kinetic terms, and, simultaneously, simplifies the elimination of auxiliary fields. $S_{\rm eff}$ now reads

$$S_{\rm eff}=\int d^8z\,\{M^\dagger e^{V_R}M+N^\dagger e^{V_Q}N+\xi_1V_R+\xi_2V_Q$$
$$+Z^*e^{V_R+3V_Q}Z+B^*e^{V_R+V_Q}B+S^*S\}$$
$$+\int d^6z\,\{(1/4k_1)\,W_R^2+(1/4k_2)\,W_Q^2$$
$$+S\log(S^2MN^3/Z)\}+\text{h.c.} \quad (15)$$

The appearance of the "effective gauge symmetry" structure of $S_{\rm eff}$ is an example of the phenomenon discussed in ref. [9]. Such a structure is crucial for the analysis of $S_{\rm eff}$ as will be seen below. One might object that this gauge symmetry is anomalous due to effective theory loops. Such an anomaly would make parametrization (14a) illegitimate by increasing the number of degrees of freedom in going from $R(Q)$ to M, V_R (N, V_Q). This is, however, not a true anomaly, being equal to the variation of a local (Wess–Zumino-type) counterterm such as $W_R^2\log M$. Addition of such anomaly-cancelling counterterms would not modify our discussion.

The scalar potential which follows from eq. (15) is

$$V_{\rm sc}=|F_M|^2+|F_N|^2+|F_Z|^2+|F_S|^2$$
$$+D_R^2/2k_1+D_Q^2/2k_2, \quad (16)$$

where (denoting the lowest component of the chiral superfields M, N, Z, S, and B by the corresponding small letters),

$$F_M=-s^*/m^*,\qquad F_N=-3s^*/n^*\,,$$

$$F_Z=+s^*/z^*,\qquad F_S=-\ (\log(s^2mn^3/z))^*\,,$$

$$D_R=-\tfrac{1}{2}k_1(\xi_1+|m|^2+|z|^2+|b|^2)\,,$$

$$D_Q=-\tfrac{1}{2}k_1(\xi_1+|n|^2+3|z|^2+|b|^2)\,. \quad (17)$$

It is easy to see that the minimum of $V_{\rm sc}$ occurs at ($\xi_{1,2}>0$)

$$b=0\quad (a=0),$$

$$0<|m|,\ |n|,\ |z|,\ |s|<\infty\,, \quad (18)$$

i.e., no vanishing (except for a) or infinite condensates occur.

At the minimum

$$V_{sc} > 0\,, \qquad (19)$$

and hence supersymmetry is spontaneously broken.

The spectrum can be studied in a straightforward manner. First, the vector boson mass terms are given by

$$M^2_{W_R} = k_1(|m|^2 + |z|^2) > 0,$$

$$M^2_{W_Q} = k_2(|n|^2 + 9|z|^2) > 0\,, \qquad (20)$$

i.e., there are no composite massless vector bosons. The complex scalar boson b (the lowest component of B) gets mass from the D term,

$$m_B^2 = (D_R^2 + D_Q^2) > 0\,. \qquad (21)$$

The moduli of the other scalars m, n, z and s also get masses from the D terms, while their phases describe potentially massless particles. Note that their phases enter V_{sc} only through the $|F_S|^2$ term, thus the three combinations orthogonal to

$$\mathrm{Im}(2s/\langle s\rangle + 3n/\langle n\rangle + m/\langle m\rangle - z/\langle z\rangle)$$

are massless. Not all of them, however, describe true massless particles because the two combinations $\mathrm{Im}(\langle m\rangle m + \langle z\rangle z)$ and $\mathrm{Im}(\langle n\rangle n + 3\langle z\rangle z)$ are absorbed by the vectors V_R and V_Q as in the usual Higgs mechanism. There remains one massless real scalar particle

$$\mathrm{Im}(z/\langle z\rangle - m/\langle m\rangle - 3n/\langle n\rangle + cs/\langle s\rangle) \qquad (22)$$

$[c = \frac{1}{2}\langle s\rangle^2(9/\langle n\rangle^2 + 1/\langle m\rangle^2 + 1/\langle z\rangle^2)]$. It is the Goldstone boson of the spontaneously broken $U_2(1)$ symmetry.

Next consider the fermions. The fermion component of B is massless because $\langle b\rangle = 0$: no mixing with the "gauge" fermions $\lambda_{R,Q}$ (of $V_{R,Q}$) occurs and B has no superpotential. It is the 't Hooft-type fermion and saturates the $\{U_1(1)\}^3$ anomaly of the unbroken $U_1(1)$ symmetry. [Its contribution $5^3 = 125$ matches $3^3 \cdot 5 + (-)^3 \cdot 10$ of the fundamental fermions.]

The massless Goldstone fermion of broken supersymmetry is expected to be

$$\psi \propto \sum_{i=M,N,S,Z} \sqrt{2}\,\langle F_i\rangle \psi_i + \mathrm{i} \sum_{a=R,Q} \langle D_a\rangle \lambda_a\,, \qquad (23)$$

because of the supersymmetry transformation laws

$$\{Q^\alpha, \psi_{i\beta}\} = \sqrt{2}\,F_i \delta^\alpha_\beta \quad (i = M, N, S, Z)\,,$$

$$\{Q^\alpha, \lambda_{a\beta}\} = iD_a \delta^\alpha_\beta\,.$$

ψ of eq. (23) is the combination that transforms inhomogeneously under the supersymmetry. This can be explicitly checked by looking at the 6×6 fermionic mass matrix and using the minimization conditions on the potential as well as the "effective gauge" invariance of the superpotential S log (S^2MN^3/Z).

In conclusion, the model we have presented seems to be well behaved and to exhibit all the features we believe the underlying theory to possess at low energy.

We can ask ourselves why the introduction of real (vector) fields was necessary, since, after all, the vector fields become massive at the end. The fact is that, because of supersymmetry breaking, not all the components of the real supermultiplet can become massive, since the goldstino itself partially lies in R and Q. Thus the full vector supermultiplet cannot be integrated out, while integrating out its massive components would correspond to a nonsupersymmetric approximation to the theory.

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