Quantum Mechanics

A New Introduction

K. Konishi
University of Pisa and INFN, Pisa

G. Paffuti
University of Pisa and INFN, Pisa

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Preface

A student's first encounter with quantum mechanics could be a traumatic one. Instead of the solid differential equation with respect to time (t) which is Newton's classical equation of motion, with its inevitable consequences, she or he learns that the new mechanics predicts as a rule only certain probabilities (!), and that electrons behave like a sort of wave, a bizarre notion—but an empirical fact.[1]

When the student makes some progress in her or his study, however, she (he) will realize that, after all, things are not that bad: the fundamental equation of the new mechanics—the Schrödinger equation—is a well-defined, perfectly respectable linear differential equation in t, and when left alone, the microscopic system evolves in a rigorously deterministic fashion. Not only that, but due to the quantization of finite motions, and intimately related to this, to the existence of a new fundamental constant of Nature, the Planck constant, h, quantum mechanics provides a much sharper (and sometimes, far simpler) explanation of the properties of atoms than does classical mechanics. For instance, all atoms of the same kind, in their normal state, have rigorously identical properties. This fact is fundamental, for instance, to the regular structures, and in the working, of the macroscopic world (solids, crystals, biological phenomena, etc). The advantage of the new mechanics over the classical one is, of course, not limited to atoms. There are many phenomena in our daily life, such as electrical conduction, the laser, electronics, quantum optics, and all other related contemporary technologies, which require quantum mechanics for a proper understanding.

Later on in her or his study, the student might find out that physicists today are debating the validity of the standard model predictions sometimes to the *eleventh* digit, for instance concerning the anomalous magnetic moment of the muon (a kind of heavy electron). Of course, here we are comparing a particular model of Nature with experiments; however, the standard model of fundamental interactions—quantum chromodynamics for strong interactions and the Glashow–Weinberg–Salam theory of electroweak interactions—are all based on *relativistic quantum mechanics.* In atomic physics, the agreement between theory and experiment can be equally good and sometimes even more impressive. All this, finally, will convince her (him) that we are indeed dealing with one of the most precise and perhaps most elegant theories ever known in physics.

One day she or he might become a researcher or a teacher, and may start giving a course on quantum mechanics. Perhaps, after many years,

[1] The background picture on the front-cover page represents electron wave ripples, formed by 50 kV electron beams going through a collodion thin film with tiny holes. The magnification is such that the full page width corresponds to about 0.6 microns. (Courtesy of Dr. Akira Tonomura, Hitachi Advanced Research Laboratory, Saitama, Japan.)

she (he) will continue to marvel at the simplicity and beauty of quantum mechanics, and at the same time its subtle and far-reaching consequences.

One of the main aims of this book is to try to convey this sense of wonder to young students who are starting to appreciate the beauty of physics.

This book is, in fact, meant to be an introductory textbook on quantum mechanics: it should be adequate for those who are learning it for the first time, as well as for slightly more advanced students. Standard courses on classical physics, including classical mechanics, electromagnetism, statistical mechanics and thermodynamics, plus basic mathematics, should provide a sufficient background.

At the same time, however, we hope that this book, with its many examples of solved problems, and the diverse subjects discussed, will be a useful reference tool for more advanced students, active researchers and teachers alike.

Let us illustrate some of the innovative features of this book. We took great pains to try to present quantum mechanics pedagogically, and at the same time with as much logical clarity and organization as possible. Concepts and methods are introduced gradually, and each of them is elaborated better and more precisely as the pages go on. We start, in fact, from the very basic concepts illustrated by elementary applications, and move on to more structural issues such as symmetry, statistics, and formal aspects of quantum mechanics, and then explore several standard approximation methods. Various applications of physical interest are then discussed, taking full advantage of the artillery we have armed ourselves with.

As far as the content goes, for the most part, it is fairly standard, even though some of the discussions in the main text, e.g. in Chapter 12 (systems with general time-dependent Hamiltonians), in Chapter 13 (metastable states), and in Chapter 15 (atoms), and several topics treated in Supplements (Chapter 20), may often not be found in a standard textbook.

At the end of each chapter, there are a number of problems to be solved analytically, as well as some others to be solved by numerical methods. The solutions to both types of problems are provided in an accompanying CD, in the form of PDF files (analytical problems) or in the form of Mathematica notebooks (there are 88 of these). The latter contain self-explanatory expositions of the solutions proposed, as well as an elementary guide to the Mathematica commands used, so that they should be easily usable even by those who are seeing a Mathematica program for the first time. The reader is encouraged to run the program, enjoy observing how the wave functions evolve, for example, modify and extend the problems as she (he) pleases, try to improve the precision of the calculation, etc. (Here are practical tips for the beginner: first, carefully read the ReadmeFirst file before starting; second, make a copy of each nb file before proceeding, and keep the original intact. Use a copy, when actually running the program, and making modifications and extensions.)

In some cases the analysis is pushed a little deeper into the heart of the problem than is ordinarily done in a quantum mechanics textbook (such as the problem of the divergences of perturbation series and resummation; the study of metastable systems; concrete determination of atomic spectra for general elements, etc.), but always in a concrete, physical fashion, never going too much into mathematics.

All in all, this is meant to be a contemporary, but at the same time relatively self-contained and comprehensive, textbook on quantum mechanics.

The book is organized as follows. Part I is an elementary introduction to the basics of quantum mechanics. Together with some initial sections on perturbation theory and variational methods in Part II, Part I could correspond to standard material for an introductory semester course on quantum mechanics in most universities. Part II is dedicated to the three standard methods of approximation, perturbation theory, the variational method, and the semiclassical approximation, through which the concepts in the theory are further developed and the range of applicability vastly increased. In Part III the formalism and methods of analyses developed are applied to various physical situations, from general time-dependent Hamiltonians, general discussions of metastable systems, the motion of electrically charged particles in electromagnetic fields, atoms, the scattering problem, atomic nuclei, and elementary particles.

Part IV is dedicated to two fundamental issues of a conceptual nature: quantum entanglement and the measurement problems.

Part V—the Supplements—is a collection of discussions of various natures, ranging from a review of useful formulas and tables, to some advanced topics, technical issues, and mathematical appendices. They are independent of each other, there is no ordering among them, and many are even independent of the main text, so that each of them can be read at leisure in a convenient moment for each reader.

The accompanying CD, as already anticipated, contains the Mathematica notebooks and PDF files in which the problems proposed at the end of each chapter are solved and discussed. The subfiles for each chapter contain all the notebooks of that chapter, accompanied by a file called Guide-to-NB.nb. In this file a list of all the Mathematica notebooks of that chapter is given, as well as a brief description of each notebook. All analyses have been done by using `Mathematica 6`, Wolfram Research, and tested with `Mathematica 7`, which has just come out.

For updates and corrections, consult our webpages:

`http://www.df.unipi.it/~konishi`
`http://www.df.unipi.it/~paffuti`

We are grateful to Mark Seymour of OUP for his brave attempt at polishing our English and for his invaluable help in improving the look of the whole book. Of course, the responsibility for any errors in the text or formulas, or for any misleading expressions, which may undoubtedly still remain or might have been introduced during the course of corrections, is ours and ours only. Thanks are also due to Charlotte Green of OUP for her crisp approach to editorial help, and to Sonke Adlung, the senior physics editor, for his admirable patience during this book's long period of gestation.

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A final message to all of you (especially to the young):

Read and Enjoy!

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K. Konishi and G. Paffuti

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