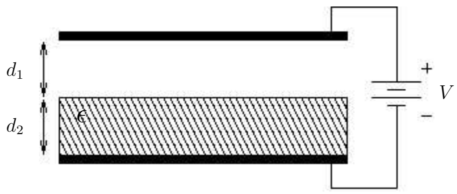
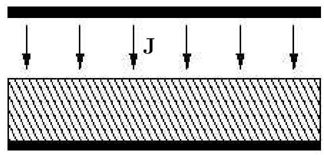
Dielectric barrier discharge



The plates of a plane capacitor are kept at a constant voltage difference V. Inside the capacitor there are two layers, a first layer of thickness d_1 filled with a gas of negligible dielectrical susceptivity ($\chi = 0$, $\epsilon = \epsilon_0$), and a second layer of thickness d_2 filled with a dielectric material having dielectric permittivity $\epsilon = \epsilon_r \epsilon_0$ with $\epsilon_r > 1$. Boundary effects can be neglected.

a) Find the electric field inside the capacitor.



At $t = 0$, inside the first layer an ionization discharge occurs, and the gas instantaneously becomes conducting. We assume the ionized gas to be modeled for $t > 0$ as an Ohmic conductor of constant and uniform resistivity ρ.

b) After a sufficiently long time we observe that the current does not flow anymore in the gas, and the system is in a steady state (i.e. all physical quantities are constant). At this stage, find the electric field in the capacitor and the surface density of free charge density between the two layers.

c) Find the temporal behavior of the field in the transient stage ($t > 0$) and the typical time needed for the system to reach a steady state condition.

Solution

a) Let E_1 and E_2 be the fields respectively in the gas (1) and dielectric (2) layers. The voltage drop between the plates is V, thus

$$E_1 d_1 + E_2 d_2 = V \,,$$

while the continuity of $D = \epsilon E$ gives

$$\epsilon_0 E_1 = \epsilon E_2 \,.$$

From these relations we obtain

$$E_1 = \frac{\epsilon V}{\epsilon d_1 + \epsilon_0 d_2} \,, \qquad E_2 = \frac{\epsilon_0 V}{\epsilon d_1 + \epsilon_0 d_2} \,.$$

b) In steady conditions the current must be zero, otherwise the charge would keep piling up on the surface of the dielectric. Since $J = E_1/\rho$, we must have $E_1 = 0$. Since $E_1 d_1 + E_2 d_2 = V$ still holds, we obtain $E_2 = V/d_2$.

The free charge density is equal to the jump of D at the surface:

$$\sigma = \epsilon E_2 - \epsilon_0 E_1 = \epsilon E_2 = \epsilon V/d_2 \,.$$

c) From the continuity equation we have

$$\partial_t \sigma = J = \frac{E_1}{\rho} \,. \tag{1}$$

Using the already introduced equations

$$\sigma = \epsilon E_2 - \epsilon_0 E_1 \,, \qquad E_1 d_1 + E_2 d_2 = V \,,$$

we may eliminate E_1 and obtain

$$\partial_t \sigma = -\frac{d_2}{\rho(\epsilon_0 d_2 + \epsilon d_1)} (\sigma - \epsilon V/d_2) \,.$$

Defining $\tau = \rho(\epsilon_0 d_2 + \epsilon d_1)/d_2$, the solution satisfying the initial condition $\sigma = 0$ is

$$\sigma = (\epsilon V/d_2)(1 - e^{-t/\tau}) \,.$$

This problem shows the concept of the "*dielectric barrier discharge*". This scheme, where the dielectric layer acts as a current limiter, is used in various electrical discharge devices, for example in plasma TV screens where the discharge acts as an ultraviolet micro-source to activate the phospors on the screen.